Asynchronous Class for Algebra I Preparation

Hunter Garretson
heg31@uakron.edu

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Bridging the Gap: Designing an Asynchronous Algebra I Preparation Course to Address Summer Learning Loss

Hunter Garretson

The Lebron James Family Foundation School of Education

Buchtel College of Arts and Sciences, The University of Akron
Abstract

Algebra I is the first high school mathematics class students take making it an important building block in the student’s growing mathematical foundation. Mitigating any losses in knowledge over the summer by using educational review tools has been proven helpful in previous studies. Every student comes from a different background, socioeconomic class, ethnicity, culture, etc., allowing each student to have a different summer experience. A meta-analysis looking into the concern of Summer Learning Loss (SLL) found that while some of its sample size would make summer knowledge gains, more than half of the sample size was projected to suffer from SLL (Atteberry & McEachin, 2020). This emphasizes the need to close the gap between these two groups. One study focusing on younger students, found that after implementing a math app regime for twelve weeks, the children demonstrated greater math learning improvements compared to the control of no app usage (Outhwaite et al., 2019). This information can be applied to Algebra I. Utilizing a summer mathematical course can be hypothesized to mitigate SLL in pre-algebraic content and leave students better prepared for Algebra I. Based on the Common Core State Standards and the Ohio Learning Standards for Mathematics, a basic outline of an asynchronous Algebra I preparation summer course was Developed. The course uses Desmos Classroom to deliver 6 lessons. These lessons are created to review knowledge learned throughout middle school mathematics to prepare students for Algebra I and to give teachers insight into any gaps in knowledge the students may have.

$\textit{Keywords:}$ Common Core, Mathematics, Algebra I, Summer Learning, Online Course, Desmos
Bridging the Gap: Designing an Asynchronous Algebra I Preparation Course to Address Summer Learning Loss

Algebra is often referred to as the “gateway” into the higher levels of mathematics, specifically, Algebra I, as it lays the basis of thinking abstractly and relating abstract thinking to mathematics. Without a solid understanding of the basics of mathematics, it is difficult to comprehend abstract thinking and abstract variables. Mathematics is a constant building of knowledge and when gaps in student knowledge occur, they need to be addressed so the building and deeper understanding of mathematics can continue to grow.

These gaps occur in many ways. Every student comes from a unique culture, demographic, and socioeconomic background which all include different variables that can affect each student. For example, students can miss a week of school for family reasons or vacations. The most recent widespread factor affecting every student’s learning curve was the COVID-19 pandemic that swept across the United States and the world. This pandemic affected learning across all ages of students when the pandemic struck between 2020-2022 shutting down schools and forcing remote education. According to the National Science Board, “The main NAEP math assessment, which started in 1990, shows a 5-point drop among 4th graders and an 8-point drop among 8th graders from 2019 to 2022.” (Jeffers, 2023). This drop in scores shows how the pandemic had a widespread effect on student learning. To counteract this drop in scores and gaps of knowledge, the time during the summer when students are not in school could be used to review and solidify mathematical concepts as well as give teachers incite into what area students could use even more review.

According to the meta-analysis performed by (Atteberry & McEachin, 2020), there are students who both gain mathematical and other academic knowledge over summer vacation
while other students lose some of the progress they made the previous school year. This difference in summer learning is due to each student’s different background and individual learning needs. Summer programs can limit the loss of knowledge or can even help students gain knowledge. In turn, this would put students ahead of where they were academically when they left school in the spring (Munro, 2022). Additionally, summer programs or courses keep students' brains engaged in academics. As with all skills, if a person does not practice or execute a certain task often, then their skill level will decrease. While most think about this concept with physical skills, it also pertains to mental skills and knowledge.

Creating an interactive online course can act as a factor in “bridging the gap” for these students. With asynchronous learning, feedback for students is important. To give students immediate feedback, the course lessons were created on the online learning platform called Desmos Classroom. Desmos Classroom allows students to investigate and complete lessons at their own pace while giving teachers an organized platform to review students' work and explanatory answers. The most outstanding feature of Desmos Classroom that makes it the platform of choice is what they call the Computational Layer. The Computational Layer allows teachers to code and create specific features within the lesson, like feedback. This makes it possible for students to get specific feedback based on their answers or get additional help or tips if they are struggling with a particular question. With students being able to complete these lessons anywhere on the internet, Desmos Classroom was the platform chosen for this course.

The purpose of the creation of the Desmos summer course is to properly bridge the gap between the students who are at risk of suffering SLL and who currently suffer from SLL and those who are on track. This design, based on the Common Core State Standards and Ohio Learning Standards for Mathematics, intends to mitigate the SLL of mathematical understanding
of students who are preparing to begin their math journey of Algebra I that fall. The course is
meant to be inclusive to all groups as well as easily accessible to ensure that all students,
especially the ones who need these materials, can access them during the summer when school is
not in session.
Methods

To limit the degradation of student knowledge in mathematics and to prepare students for an Algebra I class, this section describes the process used to create an asynchronous Algebra I preparation course. To make an Algebra I preparation course, the Common Core State Standards were obtained from https://www.thecorestandards.org/Math. For high school-aged students, these standards are broken down into 6 areas of concentration: Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability. The Common Core State Standards are a set of standards and concepts implemented by the US government as a way to unify education across America and a majority of states do follow these standards. Since Ohio adopted the Common Core State Standards in 2010 (Education: All-in-one, 2023), the Ohio Learning Standards for Mathematics were also used in the creation of the course and can be found at education.ohio.gov. Coincidently, the Ohio Learning Standards for Mathematics coincide with the Common Core State Standards other than a few different conventions and naming within the standards organization.

Unlike the Common Core State Standards, the Ohio Learning Standards for Mathematics do have, however, a guide for which standards should be covered in an Algebra I course over the school year. Thus, as seen in APPENDIX 2, a list of the standards covered in an Algebra I course was made. These include standards from three of the categories, Number and Quantity, Algebra, and Functions. With the help of the Coherence Map from Achievethecore.org, standards previously learned in 6th-grade, 7th-grade, and 8th-grade mathematics were aligned to the Algebra I standards, and this alignment can be seen in APPENDIX 1. Additionally, a description of each of the 6th-grade, 7th-grade, and 8th-grade mathematics standards that were aligned to an Algebra I standard can be found in APPENDIX 3. As the course is designed to be
taken the summer before the students enter Algebra I, these 6th-grade, 7th-grade, and 8th-grade mathematics standards are the focus of the lessons.

Each lesson in the course highlights a particular area of concentration of Algebra I and is based on the 6th-grade, 7th-grade, and/or 8th-grade mathematics standards that align with each area of concentration. In addition to creating the interconnected web of standards of their Coherence Map, Achievethecore.org also gave example tasks for some of the mathematics standards used in this course. These example tasks come from Illustrative Mathematics, a non-profit mathematics curriculum creator for kindergarten to 12th grade (Illustrative Mathematics K–12 Math, 2024). A conglomeration of the Ohio Learning Standards of Mathematics and these example tasks, along with self-made problems, were used to create each lesson of the course.

Lesson one focuses on the Numbers and Quantities concentration, specifically the sub-group of “Quantities” and covers the following standards: 6.RP.A.3, 7.RP.A.1, and 8.EE.A.4. The standards learned in Algebra I from this concentration consist of being able to use units and measurements within real-world problems and situations. Being proficient in topics such as units, ratios, rates, fractions, and scientific notation, is necessary to be successful in Algebra I and thus these topics are reviewed in this lesson. The lesson starts with a Ticket Booth problem in which the students have to find unit rates to determine what is the best deal. Next, the lesson continues to compare different rates and ratios as different ways to represent how fast an elevator is moving. Then, scientific notation is explicitly reviewed with an example and then students do a card sort to further practice scientific notation. Finally, the lesson ends with the students solving a complex real-world problem in which they use scientific notation to calculate the
storage capacity of a phone. This lesson can be found and explored at the following URL:


Lesson two focuses on the Algebra concentration, specifically the sub-group of “Seeing Structure in Mathematics” and covers the following standards: 6.EE.A.2, 7.EE.A.1, 7.EE.A.2, and 7.RP.A.3. To give students a warm up into the lesson, the lesson starts with having the students evaluate expressions without any variables to practice order of operations. After the warmup, the students are then asked to simplify expressions with variables. The students then use these simplification skills to find different expressions that are equivalent in value. Next, the lesson has the students apply expression manipulation to create expressions to determine the distance students commute with their bicycles over a four-week period. To round out the end of this lesson, the students have to explain how two different given expressions represent the area of the picture. This will give the teacher insight into how the students think about and understand expressions. This lesson can be found and explored at the following URL:

https://teacher.desmos.com/activitybuilder/custom/6602fed1ae298e035b243955.

Lesson three also focuses on the Algebra concentration, but pertains to the sub-group of “Arithmetic with Polynomials and Rational Expressions” and covers the 8th-grade standard, 8.EE.A.1. Polynomial expressions and equations are not taught in the grades leading up to Algebra I, thus this section is all about exponents and properties of integer exponents. Knowledge of all exponential properties gives students a conceptual understanding of exponents which leads to learning polynomials including quadratic equations in Algebra I. To start the lesson, students do a card sort activity in which they match the different exponential problems with expressions and equations that use that property. They then practice simplifying different expressions that include exponents and negative exponents. These concepts are then used by the
students to complete an activity in which they investigate the exponential growth of bacteria. They investigate the growth by filling out a table and explaining their reasoning. The lesson finishes with them using an equation to find the population of the bacteria prior to when the population size was first measured. This lesson can be found and explored at the following URL: https://teacher.desmos.com/activitybuilder/custom/661343c51280350c439d126c.

Lesson four is the last lesson that focuses on the Algebra concentration, and it focuses on the two sub-group of “Creating Equations” and “Reasoning with Equations and Inequalities” and covers the following standards: 6.EE.B.5, 7.EE.A.1, 7.EE.A.2, 7.EE.B.4, 8.EE.A.2, 8.EE.B.5, 8.EE.B.6, 8.EE.C.7, 8.EE.C.8 and 8.F.B.4. This lesson consists of mostly word problems so students can practice using mathematics in real-world scenarios. It starts with a police station question in which the students have to choose the correct representation of how many police officers the station can afford. This is done using multiple choice answers so that students can see and remember how using variables within expressions are formatted. The lesson then pivots to solving simple, one-variable, linear equations. To round out the lesson, students are presented with three different tasks: Chocolate Bars, Furnace Troubles, and Wordy Fruit Salad.

The Chocolate Bars task has students use a table that they complete to help them create a linear equation that represents the money made from selling chocolate bars based on how many sold. This equation is then used to find how much money would be made from 100 boxes of chocolate bars and how many boxes of chocolate bars need to be sold to fundraise a certain amount of money. The Furnace Troubles asks the question, which of the three companies costs the least amount of money to fix a furnace? The students answer this question by first creating three equations with each equation representing the cost of each company for a given amount of time. The students then compare these equations for different amounts of time and choose the
company that is the cheapest. The final task, Wordy Fruit Salad, has students calculate the pieces of fruit within a fruit salad. They are given the ratios between the different types of fruit and the number of one specific fruit from which they can calculate the number of all other fruits. This lesson can be found and explored at the following URL:  

Lesson five focuses on the Functions concentration, and covers the following standards: 7.EE.A.1, 7.RP.A.3, 8.F.A.1, 8.F.A.2, 8.F.A.3, 8.F.B.4, and 8.F.B.5. Similar to lesson 4, this lesson consists of only word problems. The lesson starts with a similar task in which students fill in tables relating the amount of time allowed to park at a meter with how many dimes they put in the meter. They must calculate the time allowed to park given a certain amount of dimes and also the amount of dimes needed to park a certain amount of time. This leads into the second activity in which students compare and interpret two line graphs to find the winner of a bike race. The next activity relates to battery charging. Students use a table showing the battery level over time as a device is being charged to determine how much charge the device will have in an hour. Then, students complete a task in which they have to create a function to model the money made from stuffing envelopes, and then they use this equation to calculate how many envelopes they need to stuff to earn $42. Finally, the lesson ends with students interpreting a line graph to determine the amount of low and high tides during a 24 hour period. This lesson can be found and explored at the following URL:  

Lesson six is the last lesson and focuses on the Functions concentration, specifically the sub-group of “Interpreting Categorical and Quantitative Data” and covers the following standards: 6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 7.SP.B.3, 7.SP.B.4, 8.SP.A.2, 8.SP.A.3,
8.SP.A.4, and 8.F.B.4. This lesson reviews the concepts, mean, median, mode, and range. The lesson starts with explanations and examples of each of these concepts independently, starting with mean and ending with range. To practice these concepts, the students then calculate the mean, median, mode, and range of quiz scores, something they can apply to their current career in school. To finish off the lesson, the students will again calculate the mean, median, mode, and range of the number of goals scored at a game. This lesson can be found and explored at the following URL:

https://teacher.desmos.com/activitybuilder/custom/6628640bd63b33631147c0e1.
Discussion

The purpose of creating these lessons is to keep students engaged in mathematics during the summer to limit the “summer slide” and prevent students from suffering from Summer Learning Loss (SLL). While the content of these lessons should not be new to the students, reviewing concepts previously learned throughout their middle school mathematical career can give them an advantage as they progress into Algebra I. If students are able to enter the beginning of Algebra I with all the knowledge they learned throughout middle school, both the students and the teachers will be able to spend more time on Algebra I concepts to create a deeper understanding of the topic and not have to spend valuable time reviewing previous concepts.

The future applications of this course could be implemented in any Ohio high school as it is based on the Ohio Learning Standards for Mathematics. Since these learning standards align with the Common Core State Standards, these lessons would also have relevance in any state that has adopted the Common Core State Standards. The first trial of this course will occur in the summer of 2024 with Dr. Jodi Burgess at Springfield High School and Junior High. Dr. Burgess will assign these summer lessons to her students who will have just completed Pre-Algebra with her in the spring of 2024 as 7th graders. These students will then begin to take Algebra I with Dr. Burgess in the fall of 2024. The trial run of the course will allow feedback to be received from both Dr. Burgess and the students taking the course to improve its form and functionality. These changes could include, for example, more help on certain problems which were more difficult for the students than expected.

In the future, if I am in the position to teach Algebra I, not only will I use this course to try to limit students’ loss of knowledge over the summer, but this course can also be used to
assess the student's previous knowledge before entering my class. Thus, this knowledge will allow me to better suit the needs of my students once the school year starts.
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https://doi.org/10.1037/edu0000286
## Appendix A

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<td>N.Q.1</td>
<td>6.RP.A.3 7.RP.A.1 8.EE.A.4</td>
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<td>N.Q.3</td>
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<tr>
<td>Algebra Standards</td>
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<td>A.SSE.2</td>
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<td>A.APR.1</td>
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<td>CREATING EQUATIONS Create equations that describe numbers or relationships.</td>
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<td></td>
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<td>A.CED.2</td>
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<td>A.CED.3</td>
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<td>A.CED.4</td>
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<td>REASONING WITH EQUATIONS AND INEQUALITIES Understand solving equations as a process of reasoning and explain the reasoning.</td>
<td>A.REI.1</td>
<td>6.EE.B.5 8.EE.C.7</td>
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<tr>
<td>Functions Standards</td>
<td>INTERPRETING FUNCTIONS</td>
<td>BUILDING FUNCTIONS</td>
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<tr>
<td>Understand the concept of a function and use function notation.</td>
<td>F.IF.1 8.F.A.1</td>
<td>F.BF.1 8.F.B.4</td>
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<td>Interpret functions that arise in applications in terms of the context.</td>
<td>F.IF.2</td>
<td>F.BF.2 8.F.B.4</td>
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<td>Analyze functions using different representations.</td>
<td>F.IF.3</td>
<td>F.BF.3</td>
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<td>Represent and solve equations and inequalities graphically.</td>
<td>F.IF.4 8.F.B.5</td>
<td>F.BF.4</td>
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<td>Solve systems of equations.</td>
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<td>Solve equations and inequalities in one variable.</td>
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<td>F.BF.1 8.F.B.4</td>
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<td>A.REI.11</td>
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<td>A.REI.12 7.EE.B.4 8.EE.C.8</td>
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</table>
| LINEAR, QUADRATIC, AND EXPONENTIAL MODELS | F.LE.1 | 8.F.A.3  
| Construct and compare linear, quadratic, and exponential models, and solve problems. | 8.F.B.4 | 8.F.B.5 |
| F.LE.2 | 8.F.B.4 |
| F.LE.3 | 8.F.B.5 |
| F.LE.5 |

| INTERPRETING CATEGORICAL AND QUANTITATIVE DATA | S.ID.1 | 6.SP.B.4 |
| Summarize, represent, and interpret data on a single count or measurement variable. | S.ID.2 | 6.SP.A.2  
| | 6.SP.A.3 |
| | 6.SP.B.5 |
| | 7.SP.B.3 |
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| S.ID.8 | 8.SP.A.2 |
Appendix B

Number and Quantity Standards

Reason quantitatively and use units to solve problems.
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★
N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. ★
N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★

Algebra Standards

Seeing structure in expressions

Interpret the structure of expressions.
A.SSE.1. Interpret expressions that represent a quantity in terms of its context. ★
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, to factor 3x(x - 5) + 2(x - 5), students should recognize that the "x - 5" is common to both expressions being added, so it simplifies to (3x + 2)(x - 5); or see x4 - y4 as (x2)2 - (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 - y2)(x2 + y2). Write expressions in equivalent forms to solve problems.
A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   c. Use the properties of exponents to transform expressions for exponential functions. For example, 8t can be written as 23t.

Arithmetic with polynomials and rational expressions

Perform arithmetic operations on polynomials.
A.APR.1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
   a. Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2)

Creating equations

Create equations that describe numbers or relationships.
A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. ★
  a. Focus on applying linear and simple exponential expressions. (A1, M1)
  b. Focus on applying simple quadratic expressions. (A1, M2)
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★
  a. Focus on applying linear and simple exponential expressions. (A1, M1)
  b. Focus on applying simple quadratic expressions. (A1, M2)
A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, it represents inequalities describing nutritional and cost constraints on combinations of different foods.★ (A1, M1)
A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★
  a. Focus on formulas in which the variable of interest is linear or square. For example, rearrange Ohm's law V = IR to highlight resistance R, or rearrange the formula for the area of a circle, \( A = (\pi)r^2 \) to highlight radius \( r \). (A1)

**REASONING WITH EQUATIONS AND INEQUALITIES**

*Understand solving equations as a process of reasoning and explain the reasoning.*
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted in the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.  

*Solve equations and inequalities in one variable.*
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A.REI.4 Solve quadratic equations in one variable.
  a. Use the method of completing the square to transform any quadratic equation in \( x \) into an equation of the form \( (x - p)^2 = q \) that has the same solutions.
  b. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for \( x^2 = 49 \); taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring. (+)
  c. Derive the quadratic formula using the method of completing the square.

*Solve systems of equations.*
A.REI.5 Verify that, given a system of two equations in two variables, replacing one equation with the sum of that equation and a multiple of the other produces a system with the same solutions.
A.REI.6 Solve systems of linear equations algebraically and graphically. a. Limit to pairs of linear equations in two variables. (A1, M1)
A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Represent and solve equations and inequalities graphically.

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A.REI.11 Explain why the $x$-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.

A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions Standards

INTERPRETING FUNCTIONS

Understand the concept of a function, and use function notation.

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

F.IF.2 Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of context.

F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.

Interpret functions that arise in applications in terms of the context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★(A2, M3) b. Focus on linear, quadratic, and exponential functions. (A1, M2)

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.★

b. Focus on linear, quadratic, and exponential functions. (A1, M2)

Analyze functions using different representations.

F.IF.7 Graph functions are expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include
applications and how key features relate to characteristics of a situation, making the selection of a particular type of function model appropriate.★

a. Graph linear functions and indicate intercepts. (A1, M1)
b. Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2)
e. Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1)

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of context. (A2, M3)

i. Focus on completing the square to quadratic functions with the leading coefficient of 1. (A1)
b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify the percent rate of change $G$ in functions such as $y = (1.02)^t$, and $y = (0.97)^t$ and classify them as representing exponential growth or decay. (A2, M3)

i. Focus on exponential functions evaluated at integer inputs. (A1, M2)

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (A2, M3)

b. Focus on linear, quadratic, and exponential functions. (A1, M2)

BUILDING FUNCTIONS

*Build a function that models a relationship between two quantities.*

F.BF.1 Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from context.

i. Focus on linear and exponential functions. (A1, M1)

ii. Focus on situations that exhibit quadratic or exponential relationships. (A1, M2)

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★ Build new functions from existing functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3)

a. Focus on transformations of graphs of quadratic functions, except for $f(kx)$; (A1, M2)

F.BF.4 Find inverse functions. a. Informally determine the input of a function when the output is known. (A1, M1)

LINEAR, QUADRATIC, AND EXPONENTIAL MODELS
Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★
   a. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).★

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ★ (A1, M2) Interpret expressions for functions in terms of the situation they model.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of context.★

INTERPRETING CATEGORICAL AND QUANTITATIVE DATA

Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots) in the context of real-world applications using the GAISE model.★

S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare the center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets. ★

S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★ Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★

S.ID.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related.★
   c. Fit a linear function for a scatterplot that suggests a linear association. (A1, M1) Interpret linear models.

S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★
<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.RP.A.3</td>
<td>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</td>
</tr>
<tr>
<td>6.EE.A.2</td>
<td>Write, read, and evaluate expressions in which letters stand for numbers.</td>
</tr>
<tr>
<td>6.EE.B.5</td>
<td>Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.</td>
</tr>
<tr>
<td>6.SP.A.2</td>
<td>Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape.</td>
</tr>
<tr>
<td>6.SP.A.3</td>
<td>Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</td>
</tr>
<tr>
<td>6.SP.B.4</td>
<td>Display numerical data in plots on a number line, including dot plots, histograms, and box plots.</td>
</tr>
<tr>
<td>6.SP.B.5</td>
<td>Summarize numerical data sets in relation to their context, such as by: their center, spread, and overall shape.</td>
</tr>
<tr>
<td>7.RP.A.1</td>
<td>Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. For example, if a person walks $1/2$ mile in each $1/4$ hour, compute the unit rate as the complex fraction $1/2/1/4$ miles per hour, equivalently $2$ miles per hour.</td>
</tr>
<tr>
<td>7.EE.A.1</td>
<td>Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.</td>
</tr>
<tr>
<td>7.EE.A.2</td>
<td>Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”</td>
</tr>
<tr>
<td>7.EE.B.4</td>
<td>Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities.</td>
</tr>
<tr>
<td>7.RP.A.3</td>
<td>Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.</td>
</tr>
<tr>
<td>7.SP.B.3</td>
<td>Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.</td>
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<tr>
<td>7.SP.B.4</td>
<td>Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.</td>
</tr>
<tr>
<td>8.EE.A.1</td>
<td>Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, (3^2 \times 3^{-5} = 3^{-3} = 1/33 = 1/27).</td>
</tr>
<tr>
<td>8.EE.A.2</td>
<td>Use square root and cube root symbols to represent solutions to equations of the form (x^2 = p) and (x^3 = p), where (p) is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that (\sqrt{2}) is irrational.</td>
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<tr>
<td>8.EE.A.4</td>
<td>Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.</td>
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<tr>
<td>8.EE.B.5</td>
<td>Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</td>
</tr>
<tr>
<td>8.EE.B.6</td>
<td>Use similar triangles to explain why the slope (m) is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation (y = mx) for a line through the origin and the equation (y = mx + b) for a line intercepting the vertical axis at (b).</td>
</tr>
<tr>
<td>8.EE.C.7</td>
<td>Solve linear equations in one variable.</td>
</tr>
<tr>
<td>8.EE.C.8</td>
<td>Analyze and solve pairs of simultaneous linear equations.</td>
</tr>
<tr>
<td>8.F.A.1</td>
<td>Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. *Function notation is not required in Grade 8.</td>
</tr>
<tr>
<td>8.F.A.2</td>
<td>Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For</td>
</tr>
</tbody>
</table>
example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

| 8.F.A.3 | Interpret the equation \( y = mx + b \) as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function \( A = s^2 \) giving the area of a square as a function of its side length is not linear because its graph contains the points \((1,1), (2,4)\) and \((3,9)\), which are not on a straight line. |
| 8.F.B.4 | Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two \((x, y)\) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values. |
| 8.F.B.5 | Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. |
| 8.SP.A.3 | Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height. |
| 8.SP.A.4 | Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible associations between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores? |