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Obstructive Wiring Patterns to Circular Planarity in Electrical Networks

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OBSTRUCTIVE WIRING PATTERNS TO CIRCULAR PLANARITY IN ELECTRICAL NETWORKS

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OBSTRUCTIVE WIRING PATTERNS TO CIRCULAR PLANARITY IN ELECTRICAL NETWORKS

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ABSTRACT

In order for an electrical network to be printed on a flat surface without changing the network's input or output, it is important to consider if any wires will cross and if this problem can be avoided. If a circular network can be printed so that no wires cross, the network is said to be circular planar. In this paper, we identify a number of wiring patterns that make circular planarity impossible. We find exactly 3 wiring patterns using circular pairs with sets of two nodes, and we find exactly 78 wiring patterns using circular pairs with sets of three nodes.

ACKNOWLEDGEMENTS

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CHAPTER I

INTRODUCTION

Electrical networks with inputs and outputs seem to be everywhere in the modern world. From tiny integrated circuits in smartphones to large regional power grids, they vary greatly in size and application.

In some cases, an electrical network can be represented mathematically as a circular graph, or circular network, where wires are represented by edges and various components are represented by nodes. Edge weights represent the conductance of the wire, and if the wire has no conductance, the edge is not drawn. Figure 1.1 illustrates an example of a circular network.

In some contexts, one may want to know if such an electrical network can be printed on a flat surface without changing the input or output of the network. In order for a network to be printed in this way, it is important to consider if any wires will cross and if this problem can be avoided. If the circular network can be printed so that no wires cross, the network is said to be circular planar. In this paper, we identify a number of wiring patterns that make this task impossible.

We reference the work of Curtis and Morrow in their book, *Inverse Problems for Electrical Networks*. In it, they discuss solutions to the problem of reconstructing an electrical network given its response matrix, M, and its boundary nodes. We

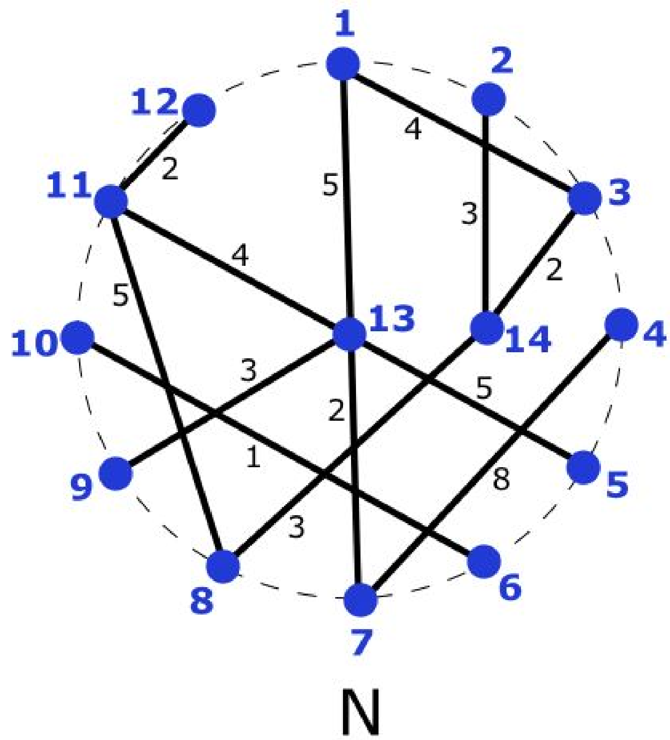

Figure 1.1: Circular network N, an electrical network, contains twelve boundary nodes, labeled 1 through 12, and two interior nodes, labeled 13 and 14. Wires are labeled with their conductances.

also reference the work of Dörfler and Bullo in "Kron Reduction of Graphs with Applications to Electrical Networks", in which they discuss the usefulness of applying the Kron reduction method to this area.

CHAPTER II

PRELIMINARIES

The following terms are defined in the book *Inverse Problems For Electrical Networks* by Curtis and Morrow [1].

2.1 Circular Network

A circular network consists of a collection of ordered nodes with connecting edges. Some nodes are arranged on a circle defined as a boundary, and are called boundary nodes; the rest are called interior nodes. Edges have values associated with them. In an electrical network, an edge value represents a wire's conductance and will be positive (or zero, if there is no conductance on the wire). In the event that an edge value is zero, we will not draw that edge. Figure 1.1 shows an example.

2.2 Laplacian Matrix

Let N be a circular network with m nodes. The Laplacian matrix, $L(N)$, is an $m \times m$ symmetric matrix as follows:

- Off-diagonal entry L_{ij} is the conductance of the edge $[i, j]$ or 0 if there is no edge between nodes i and j.

- Diagonal entry L_{ii} is the negative value which makes the row and column both sum to zero, that is, L_{ii} is the negative of the sum of conductances of the edges adjacent to node i.

2.3 Response Matrix

Let N be a circular network with m nodes and n boundary nodes. The Response matrix, $M(N)$, is an $n \times n$ matrix defined as

$$M = A - BC^{-1}B^T$$

where:

- A is the $n \times n$ submatrix corresponding to the boundary nodes,
- B is the $n \times (m-n)$ submatrix corresponding to edges between the boundary nodes and the interior nodes, and
- C is the $(m-n) \times (m-n)$ submatrix corresponding to the interior nodes.

Note that if N has no interior nodes, then $L(N) = M(N)$.

2.4 Network Equivalence

Let N and N' be circular networks. If $M(N) = M(N')$, then the two networks are said to be equivalent. This is denoted as $N \cong N'$ and is easily seen to be an equivalence relation.

2.4.1 Example

Let N be a circular network with four boundary nodes and no interior nodes, and let N' be a circular network with four boundary nodes and one interior node, as shown in Figure 2.1.

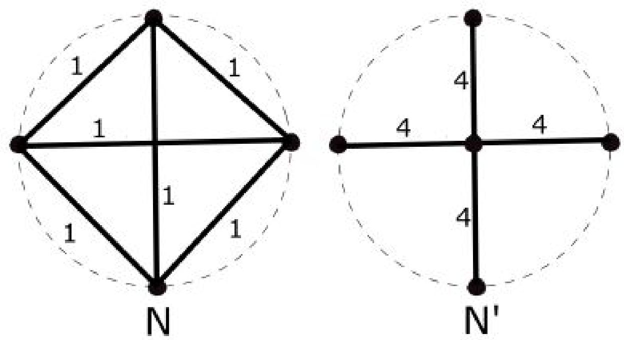

Figure 2.1: Network N with four nodes and conductances of 1, and Network N' with fives nodes and conductances of 4.

Then we have:

$$L(N) = M(N) = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 1 & -3 & 1 & 1 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & 1 & -3 \end{bmatrix}$$

Thus, the response matrix is easily obtained for N. Turning to N', we first find the Laplacian matrix:

$$L(N') = \begin{bmatrix} -4 & 0 & 0 & 0 & 4 \\ 0 & -4 & 0 & 0 & 4 \\ 0 & 0 & -4 & 0 & 4 \\ 0 & 0 & 0 & -4 & 4 \\ 4 & 4 & 4 & 4 & -16 \end{bmatrix}$$

From this, we calculate the response matrix:

$$M(N') = \begin{bmatrix} -4 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix} - \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \end{bmatrix} \left(\frac{-1}{16}\right) \begin{bmatrix} 4 & 4 & 4 & 4 \end{bmatrix}$$

Therefore,

$$M(N') = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 1 & -3 & 1 & 1 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & 1 & -3 \end{bmatrix}$$

Thus, we can observe that $M(N) = M(N')$ and the two networks are equivalent. It is interesting to note that N' does not have any edges crossing and visually appears to be planar. However, N does have edges that cross and visually appears to be nonplanar. Without finding the response matrices, one could not predict that these two networks are equivalent.

2.5 Circular Planar

For an equivalence class of circular networks, if any representative circular network can be drawn inside the boundary circle with no edges crossing, the equivalence class of circular networks is said to be circular planar.

2.6 Circular Pair

A circular pair is a pair of ordered lists of boundary nodes of network N

$$(p_1, p_2, ..., p_k; q_1, q_2, ..., q_k)$$

such that combining the two lists as

$$(p_1, p_2, ..., p_k, q_k, ..., q_2, q_1)$$

produces a list that respects the circular order, i.e., goes around the circle in a clockwise direction only once. We denote $[p_1, p_2, ..., p_k]$ as P and $[q_1, q_2, ..., q_k]$ as Q.

2.6.1 Example

In Figure 2.2, N is a circular network with twelve boundary nodes. At left is the original network, and at right, we have selected the circular pair $(2, 3, 6, 7; 1, 12, 11, 10)$.

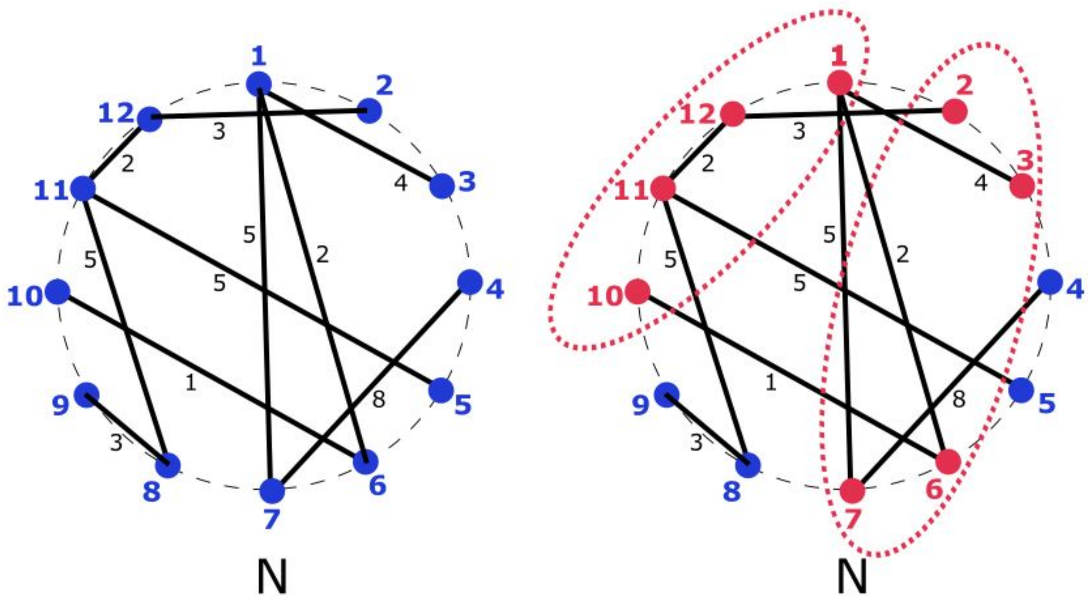

Figure 2.2: Circular Network N with a circular pair of size $k = 4$ selected. For clarity, each set of nodes in the pair is circled with a dashed line.

2.7 Circular Minor

The circular submatrix for pair $(P; Q)$ is the submatrix of $M(N)$ using rows $p_1, ..., p_k$ and columns $q_1, ..., q_k$, in that order. The circular minor for $(P; Q)$ is the determinant of that submatrix.

2.7.1 Example

In Figure 2.2, with circular pair $(2, 3, 6, 7; 1, 12, 11, 10)$, the corresponding circular submatrix is:

$$\begin{bmatrix} 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 \end{bmatrix}$$

Then the circular minor for this circular pair is the determinant of this matrix, which is zero.

Theorem 2.7.1 *A response matrix $M(N)$ for an electrical network N has all non-negative circular minors if and only if N is circular planar [2].*

2.8 Bipartite Graph

A graph N is said to be bipartite if the set of nodes V may be partitioned into two subsets X and Y such that each edge of N contains one node in X and one node in Y. Figure 2.3 shows an example of a bipartite network.

2.9 Complete Bipartite Graph

A bipartite graph in which every node of one subset shares an edge with every node of the other subset is said to be a complete bipartite graph. A complete bipartite graph is notated as $K_{r,s}$, where one subset contains r nodes and the other

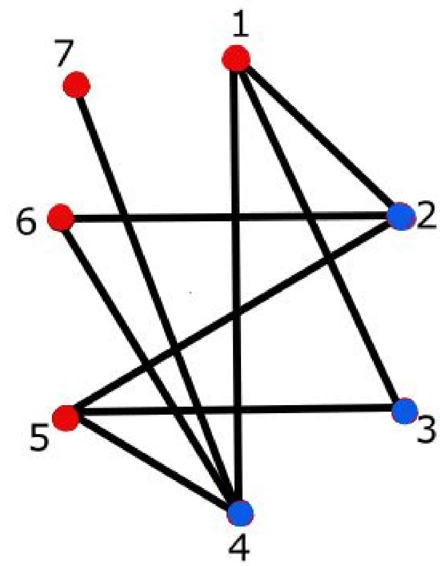

Figure 2.3: In this bipartite network, nodes 2, 3, and 4 share edges only with nodes 1, 5, 6, and 7.

subset contains s nodes. Figure 2.4 shows an example of a complete bipartite network.

2.10 Kron Reduction of a Network

The Kron reduction of a circular network N is a network $K(N)$ whose Laplacian matrix is $M(N)$. Therefore, $K(N) \cong N$ and in fact, $K(N)$ is the only member of the equivalence class of N that has no interior nodes. To construct $K(N)$, use the boundary nodes from N, but remove the interior nodes. Place an edge between two boundary nodes if there is a path between them that does not include any other

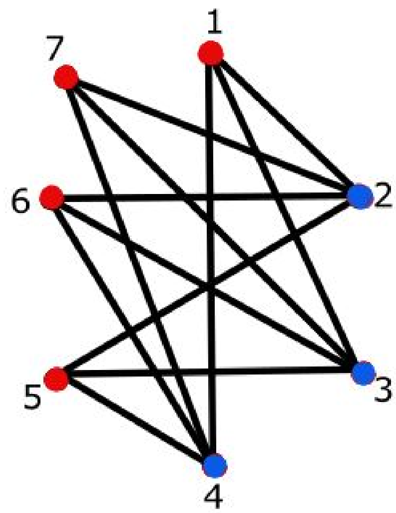

Figure 2.4: $K_{3,4}$

boundary nodes. Edge conductances can be obtained from $M(N)$ [3]. In Figure 2.1, $N = K(N')$.

CHAPTER III

OBSTRUCTIONS TO PLANARITY

We now consider circular networks where equivalence to circular planarity is impossible. Recall that a network is circular planar if and only if all circular minors are nonnegative. Thus, a network is circular non-planar if there exists a negative circular minor associated with that network. If applied to electrical networks, a network calculated to be circular non-planar would be impossible to construct in two dimensions without wires in the network crossing. We will identify some of these obstructive wiring patterns.

Recall that the Kron reduction of any circular network can be found, and the Kron reduction of a network is equivalent to the network (that is, their response matrices are equal). Because of this, when looking for obstructions to planarity, we will consider only the Kron reductions of networks. We can consider networks with no interior nodes and be assured that our results would apply to their equivalent networks containing interior nodes. When considering a circular network N with no interior nodes in the calculations that follow, the process is made easier since $L(N) = M(N)$.

3.1 Circular Pairs of Size k=1

Trivially, we can consider the case where only one node is chosen for P and one node is chosen for Q from a circular network N. Then, there is only one element in the submatrix of $M(N)$, representing the conductance of the edge between the two nodes (or 0 if there is no edge there). Since we have a matrix containing only one element and the element is greater than or equal to 0, the determinant is nonnegative. Thus, the circular minor could never be negative, and we cannot find an obstructive wiring configuration.

3.2 Circular Pairs of Size k=2

Next, let us consider the case where the circular pair contains sets of two nodes. Suppose we have a circular network N containing four or more boundary nodes. Let us consider a complete graph with four nodes as shown in Figure 3.1. Let us select a circular pair $(P; Q)$ of size $k = 2$. Let us arbitrarily select the circular pair to be $(2, 3; 1, 4)$. Then the resulting circular submatrix is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Calculating the determinant of this matrix yields the expression $ad - bc$, and we seek to make this expression negative for all positive edge values. There are three ways to guarantee that this expression is always negative: $a = 0$, $d = 0$, or $a = d = 0$. Thus, there are three configurations that give a negative circular minor.

Note that the edge between node 2 and node 3 and the edge between node 1 and node 4, with conductances of f and e, respectively, have no effect on the circular

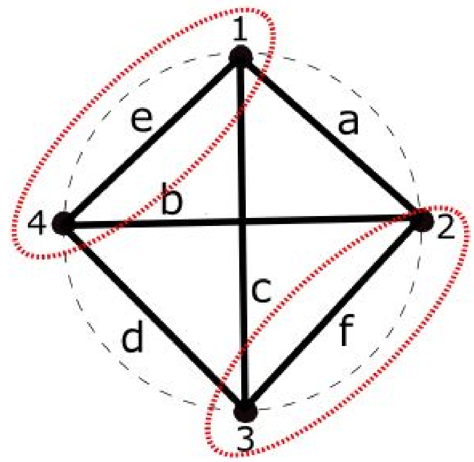

Figure 3.1: A complete circular network with four nodes and circular pair selected. Edge conductance values are indicated by variables a through f.

minor calculation, since only edge values for edges from a node in one set to a node in the other set are used to form the circular submatrix. Therefore, we can limit our focus to the complete bipartite network such that the circular pair gives the partition of the nodes, which we will call $N_{2,2}$. This is network shown in Figure 3.2.

With the unnecessary edges omitted, our three configurations are subnetworks of $N_{2,2}$. We will refer to these as "obstructive networks" and display them in Figure 3.3. Notice that the first and third pictures are reflections of one another, so there are two unique obstructive networks up to reflection.

Definition 3.2.1 *A pair-induced subnetwork of a network N for a circular pair $(P;Q)$ of nodes is the subnetwork induced by $(P;Q)$, minus edges that have both endpoints in P or both in Q.*

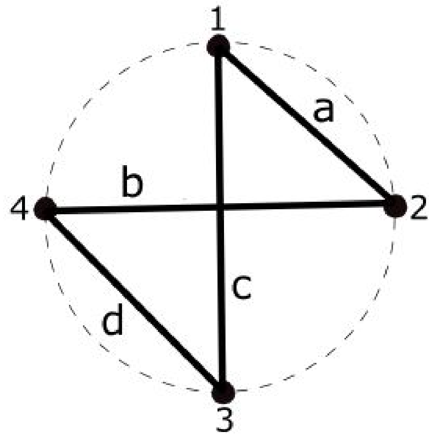

Figure 3.2: $N_{2,2}$ where edges are assigned variable conductances.

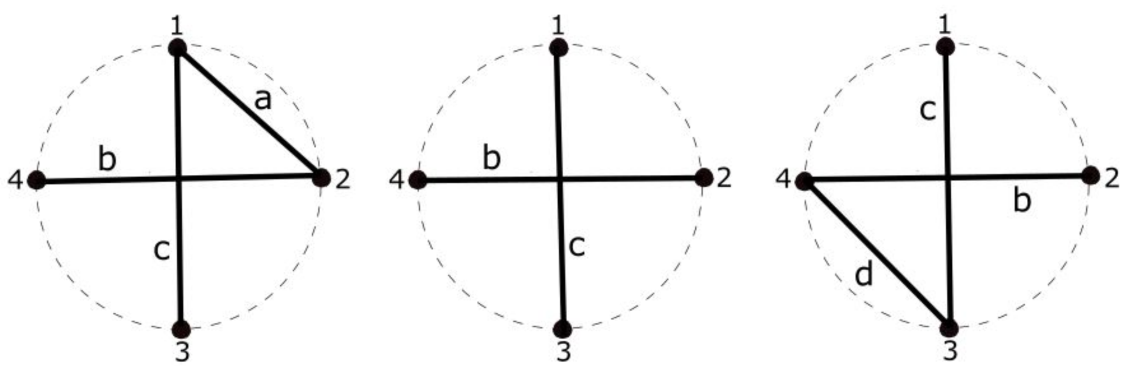

Figure 3.3: The three obstructive networks with circular pair of size k=2.

From this definition and the preceding discussion, we can conclude the following theorem.

Theorem 3.2.2 *These 3 obstructive networks in Figure 3.3 are the only subnetworks of $N_{2,2}$ that are not circular planar for all positive conductance values of their edges. In fact, for any size network N, if there exists a pair-induced subnetwork of N (with nodes renumbered to match the pair $(2,3;1,4)$) that is equivalent to one of these 3 obstructive networks, then N is not circular planar.*

3.3 Circular Pairs of Size k=3

Let us now consider circular pairs containing sets of three nodes. As in the case of circular pairs of size $k = 2$, we will limit our consideration to a complete bipartite circular network, this time with six nodes which we will label as 1 through 6. Let us call this network $N_{3,3}$ and label the nodes as shown in Figure 3.4. We arbitrarily select a circular pair to be the two sets of three nodes that are the partition of the nodes, i.e., $(2,3,4;1,6,5)$. We seek to find subnetworks of $N_{3,3}$ such that the circular minor is negative for all conductances. We refer to these as "obstructive networks".

To begin, we assign variables a through i for the conductances of the nine edges, as shown in Figure 3.5.

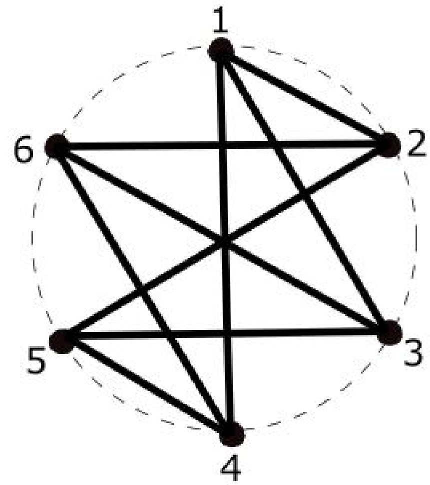

Figure 3.4: $N_{3,3}$ is a complete bipartite circular network with six nodes.

Then the resulting circular submatrix will be:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

The circular minor is equal to the determinant of this matrix, and we seek to make it negative. This can be expressed as:

$$aei + bfg + cdh - (afh + bdi + ceg) < 0$$

We seek to make this expression true for all conductance values. Thus, each variable a through i could either equal 0 (in which case, the wire is omitted), or be any positive value. Careful calculation by hand reveals 78 unique solutions. By first

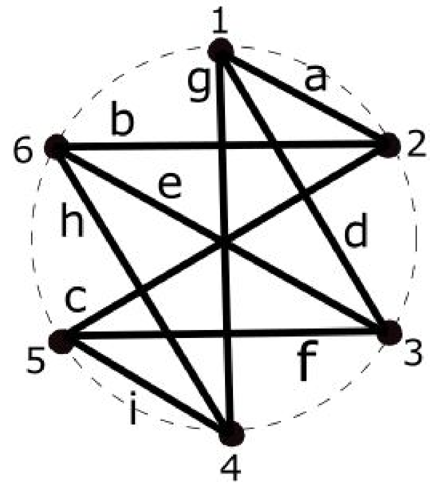

Figure 3.5: $N_{3,3}$ where edges are assigned variable conductances.

selecting subsets of $a...i$ with three variables as the only positives- $[a, f, h]$, $[b, d, i]$, and $[c, e, g]$- and setting all other variables equal to 0, we find the three subnetworks with the fewest edges. We can strategically add to each of these subsets to find additional subnetworks.

To verify that no additional subsets were missed, we note that if the determinant is negative for all positive values of the conductances in one such subset, then it is negative when those same conductances are all 1 (and the others still 0). Thus, let us assume that the conductance of a wire can only have a value of 1, and otherwise the variable will be equal to 0. This can be calculated using Microsoft Excel, where all 512 possible combinations of zeros and ones for $a...i$ are listed and the determinant is calculated for each.

Appendix A displays these results, where 87 cases are found to have a negative

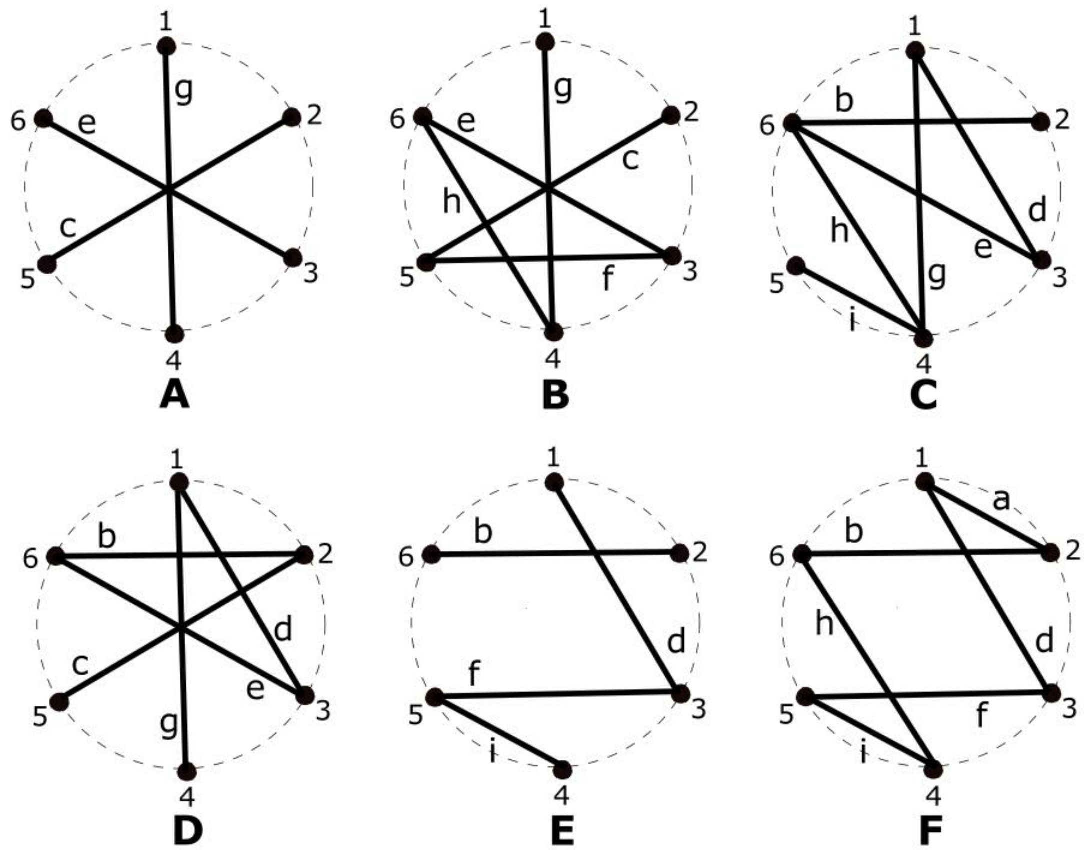

Figure 3.6: Some of the 78 possible obstructive networks which are subgraphs of $N_{3,3}$.

determinant. Of these 87 cases, 9 do not guarantee the determinant is negative for all positive values. Removing these cases, we are left with our 78 cases. We can then draw the obstructive network corresponding to each case. Several of these are shown in Figure 3.6.

It is notable that each of the 78 obstructive networks contains at least one of the obstructive networks in Figure 3.3. For example, obstructive network D in Figure 3.6 contains two of these obstructive networks. This is illustrated in Figure 3.7.

It is also important to note that some of these 78 networks are reflections of

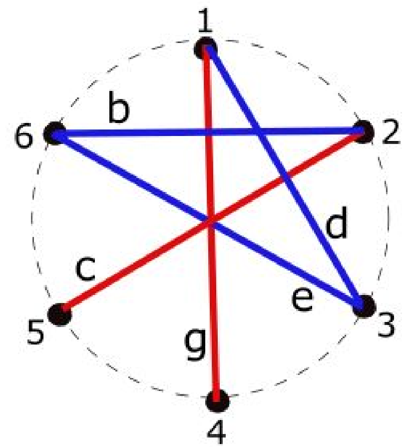

Figure 3.7: An obstructive network containing two smaller obstructive subnetworks.

one another. We consider only the reflections of a network through the line which divides the nodes into the two parts $2, 3, 4$ and $1, 6, 5$, or the line which passes through nodes 3 and 6. Algebraically, this occurs by reflecting the elements of the associated matrix across the main diagonal of the matrix, or by interchanging rows 1 and 3 and interchanging columns 1 and 3. Both of these algebraic methods preserve the determinant of the matrix.

In considering these particular reflections, we find that each network contains either zero, one, or two symmetries. For example, obstructive networks B and D in Figure 3.6 are reflections of one another across the line that passes through nodes 3 and 6. On the other hand, networks A and F already contain two symmetries.

When classifying the obstructive networks based on their number of symmetries, we find that there are two that contain two symmetries, twelve that contain one symmetry, and thirteen that contain no symmetries. Therefore, there are 27 unique networks up to reflection. These are displayed in Appendix B. Each of the thirteen networks with no symmetries can be reflected twice to generate four total networks, and each of the twelve networks with one symmetry can be reflected once to generate two total networks. Adding to this the two networks with two symmetries, we can obtain all 78 obstructive networks from these 27.

Our findings can be summarized as follows in Theorem 3.3.1.

Theorem 3.3.1 *These 78 obstructive networks are the only subnetworks of $N_{3,3}$ that are not circular planar for all positive conductance values of their edges. In fact, for any size network N, if there exists a pair-induced subnetwork of N (with nodes renumbered to match the pair $(2,3,4;1,6,5)$) that is equivalent to one of these 78 obstructive networks, then N is not circular planar.*

Theorem 3.3.2 *Let N be a network of any size. If a pair-induced subnetwork of N (with nodes renumbered to match the pair $(2,3,4;1,6,5)$) is equivalent to one of these 78 obstructive networks, then there is a size $k=2$ circular pair $(R;S)$ such that the pair-induced subnetwork of N (with nodes renumbered to match the pair $(2,3;1,4)$) is equivalent to one of the three obstructive networks in Theorem 3.2.2.*

This theorem can be verified by inspecting the pictures in Appendix B. The converse of this theorem is not true, as demonstrated in the following example.

3.3.1 Example

Let us consider obstructive network F from Figure 3.6. Notice that this network contains at least one of the obstructive networks from Theorem 3.2.2. Now

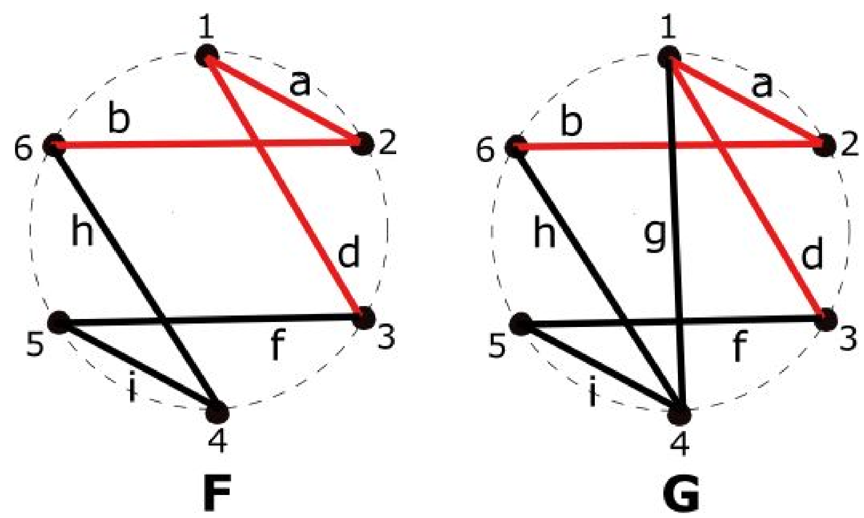

Figure 3.8: Two networks each containing a $k = 2$ obstructive network from Theorem 3.2.2.

let us consider another subnetwork of $N_{3,3}$ that is very similar to F and call this new network G. These two networks are shown in Figure 3.8.

Observe that the only difference between the two networks is the presence of the edge with conductance g in network G. Notice that, like F, G also contains at least one of the obstructive network patterns in Theorem 3.2.2. However, if we select the circular pair $(2, 3, 4; 1, 6, 5)$ and find the determinant for the associated circular minor, we have:

$$\begin{vmatrix} a & b & 0 \\ d & 0 & f \\ g & h & i \end{vmatrix} = bfg - (afh + bdi)$$

This determinant is not guaranteed to be negative for all conductance values. Thus, it is not equivalent to one of the 78 networks from Theorem 3.3.1.

CHAPTER IV

AREAS FOR FURTHER EXPLORATION

The results presented in the previous chapter raise additional questions for further research. While we identified obstructive networks by using circular pairs of size $k = 1$, $k = 2$, and $k = 3$, this process could be extended to circular pairs of larger sizes to find additional obstructive networks with more boundary nodes.

Also, Theorem 3.3.2 leads to the question of whether the 78 obstructive networks from Theorem 3.3.1 are also induced subgraphs of obstructive networks found using circular pairs of size $k = 4$. Do obstructive networks with more nodes always include obstructive networks with fewer nodes as induced subgraphs, as specified in Theorem 3.3.2? What similarities can we observe across all obstructive networks?

A third topic to consider is that of the counts of the obstructive networks for each circular pair size k. From our results in Chapter 3, we have the sequence $(0, 3, 78, ...)$. This sequence is not found in the Online Encyclopedia of Integer Sequences (OEIS). Perhaps by calculating additional numbers in this sequence, a formula could be obtained to find the count for any circular pair of size n. It is also interesting to consider finding additional values for the sequence of unique obstructive networks up to reflection, for which we have $(0, 2, 27)$ so far. Further terms would need to be calculated to determine if this sequence is found in the OEIS.

Also notable is that the count of $(0, 1)$ $n \times n$ matrices with nonzero determinants is a hard unsolved problem. From Appendix A, this count is 174 for 3×3 matrices, with half being positive and half being negative. In the OEIS, Entry A055165 gives the number of invertible $n \times n$ matrices with entries equal to 0 or 1 [4]. If a formula was found for the sequence $(0, 3, 78, ...)$, it would give a lower bound for the the hard unsolved problem.

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APPENDICES

APPENDIX A

EXCEL CALCULATIONS

The following consists of 512 rows of all possible combinations of zeros and ones for nine variables, a through i. The combinations were obtained by converting the number in the leftmost column to binary form. Then, each digit was extracted into a separate column corresponding to a variable. Finally, the determinant was calculated and negative determinants were flagged when substituting the values into the following matrix:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Number	Binary	a	b	c	d	e	f	g	h	i	Determinant	Negative Determinants
0	000000000	0	0	0	0	0	0	0	0	0	0	
1	000000001	0	0	0	0	0	0	0	0	1	0	
2	000000010	0	0	0	0	0	0	0	1	0	0	
3	000000011	0	0	0	0	0	0	0	1	1	0	
4	000000100	0	0	0	0	0	0	1	0	0	0	
5	000000101	0	0	0	0	0	0	1	0	1	0	
6	000000110	0	0	0	0	0	0	1	1	0	0	
7	000000111	0	0	0	0	0	0	1	1	1	0	
8	000001000	0	0	0	0	0	1	0	0	0	0	
9	000001001	0	0	0	0	0	1	0	0	1	0	
10	000001010	0	0	0	0	0	1	0	1	0	0	
11	000001011	0	0	0	0	0	1	0	1	1	0	
12	000001100	0	0	0	0	0	1	1	0	0	0	
13	000001101	0	0	0	0	0	1	1	0	1	0	
14	000001110	0	0	0	0	0	1	1	1	0	0	
15	000001111	0	0	0	0	0	1	1	1	1	0	
16	000010000	0	0	0	0	1	0	0	0	0	0	
17	000010001	0	0	0	0	1	0	0	0	1	0	
18	000010010	0	0	0	0	1	0	0	1	0	0	
19	000010011	0	0	0	0	1	0	0	1	1	0	
20	000010100	0	0	0	0	1	0	1	0	0	0	
21	000010101	0	0	0	0	1	0	1	0	1	0	
22	000010110	0	0	0	0	1	0	1	1	0	0	
23	000010111	0	0	0	0	1	0	1	1	1	0	
24	000011000	0	0	0	0	1	1	0	0	0	0	
25	000011001	0	0	0	0	1	1	0	0	1	0	
26	000011010	0	0	0	0	1	1	0	1	0	0	
27	000011011	0	0	0	0	1	1	0	1	1	0	
28	000011100	0	0	0	0	1	1	1	0	0	0	
29	000011101	0	0	0	0	1	1	1	0	1	0	
30	000011110	0	0	0	0	1	1	1	1	0	0	
31	000011111	0	0	0	0	1	1	1	1	1	0	
32	000100000	0	0	0	1	0	0	0	0	0	0	
33	000100001	0	0	0	1	0	0	0	0	1	0	
34	000100010	0	0	0	1	0	0	0	1	0	0	
35	000100011	0	0	0	1	0	0	0	1	1	0	
36	000100100	0	0	0	1	0	0	1	0	0	0	
37	000100101	0	0	0	1	0	0	1	0	1	0	
38	000100110	0	0	0	1	0	0	1	1	0	0	
39	000100111	0	0	0	1	0	0	1	1	1	0	
40	000101000	0	0	0	1	0	1	0	0	0	0	
41	000101001	0	0	0	1	0	1	0	0	1	0	
42	000101010	0	0	0	1	0	1	0	1	0	0	
43	000101011	0	0	0	1	0	1	0	1	1	0	
44	000101100	0	0	0	1	0	1	1	0	0	0	
45	000101101	0	0	0	1	0	1	1	0	1	0	
46	000101110	0	0	0	1	0	1	1	1	0	0	
47	000101111	0	0	0	1	0	1	1	1	1	0	

48	000110000	0	0	0	1	1	0	0	0	0	0	
49	000110001	0	0	0	1	1	0	0	0	1	0	
50	000110010	0	0	0	1	1	0	0	1	0	0	
51	000110011	0	0	0	1	1	0	0	1	1	0	
52	000110100	0	0	0	1	1	0	1	0	0	0	
53	000110101	0	0	0	1	1	0	1	0	1	0	
54	000110110	0	0	0	1	1	0	1	1	0	0	
55	000110111	0	0	0	1	1	0	1	1	1	0	
56	000111000	0	0	0	1	1	1	0	0	0	0	
57	000111001	0	0	0	1	1	1	0	0	1	0	
58	000111010	0	0	0	1	1	1	0	1	0	0	
59	000111011	0	0	0	1	1	1	0	1	1	0	
60	000111100	0	0	0	1	1	1	1	0	0	0	
61	000111101	0	0	0	1	1	1	1	0	1	0	
62	000111110	0	0	0	1	1	1	1	1	0	0	
63	000111111	0	0	0	1	1	1	1	1	1	0	
64	001000000	0	0	1	0	0	0	0	0	0	0	
65	001000001	0	0	1	0	0	0	0	0	1	0	
66	001000010	0	0	1	0	0	0	0	1	0	0	
67	001000011	0	0	1	0	0	0	0	1	1	0	
68	001000100	0	0	1	0	0	0	1	0	0	0	
69	001000101	0	0	1	0	0	0	1	0	1	0	
70	001000110	0	0	1	0	0	0	1	1	0	0	
71	001000111	0	0	1	0	0	0	1	1	1	0	
72	001001000	0	0	1	0	0	1	0	0	0	0	
73	001001001	0	0	1	0	0	1	0	0	1	0	
74	001001010	0	0	1	0	0	1	0	1	0	0	
75	001001011	0	0	1	0	0	1	0	1	1	0	
76	001001100	0	0	1	0	0	1	1	0	0	0	
77	001001101	0	0	1	0	0	1	1	0	1	0	
78	001001110	0	0	1	0	0	1	1	1	0	0	
79	001001111	0	0	1	0	0	1	1	1	1	0	
80	001010000	0	0	1	0	1	0	0	0	0	0	
81	001010001	0	0	1	0	1	0	0	0	1	0	
82	001010010	0	0	1	0	1	0	0	1	0	0	
83	001010011	0	0	1	0	1	0	0	1	1	0	
84	001010100	0	0	1	0	1	0	1	0	0	-1	Neg
85	001010101	0	0	1	0	1	0	1	0	1	-1	Neg
86	001010110	0	0	1	0	1	0	1	1	0	-1	Neg
87	001010111	0	0	1	0	1	0	1	1	1	-1	Neg
88	001011000	0	0	1	0	1	1	0	0	0	0	
89	001011001	0	0	1	0	1	1	0	0	1	0	
90	001011010	0	0	1	0	1	1	0	1	0	0	
91	001011011	0	0	1	0	1	1	0	1	1	0	
92	001011100	0	0	1	0	1	1	1	0	0	-1	Neg
93	001011101	0	0	1	0	1	1	1	0	1	-1	Neg
94	001011110	0	0	1	0	1	1	1	1	0	-1	Neg
95	001011111	0	0	1	0	1	1	1	1	1	-1	Neg
96	001100000	0	0	1	1	0	0	0	0	0	0	

97	001100001	0	0	1	1	0	0	0	0	1	0	
98	001100010	0	0	1	1	0	0	0	1	0	1	
99	001100011	0	0	1	1	0	0	0	1	1	1	
100	001100100	0	0	1	1	0	0	1	0	0	0	
101	001100101	0	0	1	1	0	0	1	0	1	0	
102	001100110	0	0	1	1	0	0	1	1	0	1	
103	001100111	0	0	1	1	0	0	1	1	1	1	
104	001101000	0	0	1	1	0	1	0	0	0	0	
105	001101001	0	0	1	1	0	1	0	0	1	0	
106	001101010	0	0	1	1	0	1	0	1	0	1	
107	001101011	0	0	1	1	0	1	0	1	1	1	
108	001101100	0	0	1	1	0	1	1	0	0	0	
109	001101101	0	0	1	1	0	1	1	0	1	0	
110	001101110	0	0	1	1	0	1	1	1	0	1	
111	001101111	0	0	1	1	0	1	1	1	1	1	
112	001110000	0	0	1	1	1	0	0	0	0	0	
113	001110001	0	0	1	1	1	0	0	0	1	0	
114	001110010	0	0	1	1	1	0	0	1	0	1	
115	001110011	0	0	1	1	1	0	0	1	1	1	
116	001110100	0	0	1	1	1	0	1	0	0	-1	Neg
117	001110101	0	0	1	1	1	0	1	0	1	-1	Neg
118	001110110	0	0	1	1	1	0	1	1	0	0	
119	001110111	0	0	1	1	1	0	1	1	1	0	
120	001111000	0	0	1	1	1	1	0	0	0	0	
121	001111001	0	0	1	1	1	1	0	0	1	0	
122	001111010	0	0	1	1	1	1	0	1	0	1	
123	001111011	0	0	1	1	1	1	0	1	1	1	
124	001111100	0	0	1	1	1	1	1	0	0	-1	Neg
125	001111101	0	0	1	1	1	1	1	0	1	-1	Neg
126	001111110	0	0	1	1	1	1	1	1	0	0	
127	001111111	0	0	1	1	1	1	1	1	1	0	
128	010000000	0	1	0	0	0	0	0	0	0	0	
129	010000001	0	1	0	0	0	0	0	0	1	0	
130	010000010	0	1	0	0	0	0	0	1	0	0	
131	010000011	0	1	0	0	0	0	0	1	1	0	
132	010000100	0	1	0	0	0	0	1	0	0	0	
133	010000101	0	1	0	0	0	0	1	0	1	0	
134	010000110	0	1	0	0	0	0	1	1	0	0	
135	010000111	0	1	0	0	0	0	1	1	1	0	
136	010001000	0	1	0	0	0	1	0	0	0	0	
137	010001001	0	1	0	0	0	1	0	0	1	0	
138	010001010	0	1	0	0	0	1	0	1	0	0	
139	010001011	0	1	0	0	0	1	0	1	1	0	
140	010001100	0	1	0	0	0	1	1	0	0	1	
141	010001101	0	1	0	0	0	1	1	0	1	1	
142	010001110	0	1	0	0	0	1	1	1	0	1	
143	010001111	0	1	0	0	0	1	1	1	1	1	
144	010010000	0	1	0	0	1	0	0	0	0	0	
145	010010001	0	1	0	0	1	0	0	0	1	0	

146	010010010	0	1	0	0	1	0	0	1	0	0	
147	010010011	0	1	0	0	1	0	0	1	1	0	
148	010010100	0	1	0	0	1	0	1	0	0	0	
149	010010101	0	1	0	0	1	0	1	0	1	0	
150	010010110	0	1	0	0	1	0	1	1	0	0	
151	010010111	0	1	0	0	1	0	1	1	1	0	
152	010011000	0	1	0	0	1	1	0	0	0	0	
153	010011001	0	1	0	0	1	1	0	0	1	0	
154	010011010	0	1	0	0	1	1	0	1	0	0	
155	010011011	0	1	0	0	1	1	0	1	1	0	
156	010011100	0	1	0	0	1	1	1	0	0	1	
157	010011101	0	1	0	0	1	1	1	0	1	1	
158	010011110	0	1	0	0	1	1	1	1	0	1	
159	010011111	0	1	0	0	1	1	1	1	1	1	
160	010100000	0	1	0	1	0	0	0	0	0	0	
161	010100001	0	1	0	1	0	0	0	0	1	-1	Neg
162	010100010	0	1	0	1	0	0	0	1	0	0	
163	010100011	0	1	0	1	0	0	0	1	1	-1	Neg
164	010100100	0	1	0	1	0	0	1	0	0	0	
165	010100101	0	1	0	1	0	0	1	0	1	-1	Neg
166	010100110	0	1	0	1	0	0	1	1	0	0	
167	010100111	0	1	0	1	0	0	1	1	1	-1	Neg
168	010101000	0	1	0	1	0	1	0	0	0	0	
169	010101001	0	1	0	1	0	1	0	0	1	-1	Neg
170	010101010	0	1	0	1	0	1	0	1	0	0	
171	010101011	0	1	0	1	0	1	0	1	1	-1	Neg
172	010101100	0	1	0	1	0	1	1	0	0	1	
173	010101101	0	1	0	1	0	1	1	0	1	0	
174	010101110	0	1	0	1	0	1	1	1	0	1	
175	010101111	0	1	0	1	0	1	1	1	1	0	
176	010110000	0	1	0	1	1	0	0	0	0	0	
177	010110001	0	1	0	1	1	0	0	0	1	-1	Neg
178	010110010	0	1	0	1	1	0	0	1	0	0	
179	010110011	0	1	0	1	1	0	0	1	1	-1	Neg
180	010110100	0	1	0	1	1	0	1	0	0	0	
181	010110101	0	1	0	1	1	0	1	0	1	-1	Neg
182	010110110	0	1	0	1	1	0	1	1	0	0	
183	010110111	0	1	0	1	1	0	1	1	1	-1	Neg
184	010111000	0	1	0	1	1	1	0	0	0	0	
185	010111001	0	1	0	1	1	1	0	0	1	-1	Neg
186	010111010	0	1	0	1	1	1	0	1	0	0	
187	010111011	0	1	0	1	1	1	0	1	1	-1	Neg
188	010111100	0	1	0	1	1	1	1	0	0	1	
189	010111101	0	1	0	1	1	1	1	0	1	0	
190	010111110	0	1	0	1	1	1	1	1	0	1	
191	010111111	0	1	0	1	1	1	1	1	1	0	
192	011000000	0	1	1	0	0	0	0	0	0	0	
193	011000001	0	1	1	0	0	0	0	0	1	0	
194	011000010	0	1	1	0	0	0	0	1	0	0	

195	011000011	0	1	1	0	0	0	0	1	1	0	
196	011000100	0	1	1	0	0	0	1	0	0	0	
197	011000101	0	1	1	0	0	0	1	0	1	0	
198	011000110	0	1	1	0	0	0	1	1	0	0	
199	011000111	0	1	1	0	0	0	1	1	1	0	
200	011001000	0	1	1	0	0	1	0	0	0	0	
201	011001001	0	1	1	0	0	1	0	0	1	0	
202	011001010	0	1	1	0	0	1	0	1	0	0	
203	011001011	0	1	1	0	0	1	0	1	1	0	
204	011001100	0	1	1	0	0	1	1	0	0	1	
205	011001101	0	1	1	0	0	1	1	0	1	1	
206	011001110	0	1	1	0	0	1	1	1	0	1	
207	011001111	0	1	1	0	0	1	1	1	1	1	
208	011010000	0	1	1	0	1	0	0	0	0	0	
209	011010001	0	1	1	0	1	0	0	0	1	0	
210	011010010	0	1	1	0	1	0	0	1	0	0	
211	011010011	0	1	1	0	1	0	0	1	1	0	
212	011010100	0	1	1	0	1	0	1	0	0	-1	Neg
213	011010101	0	1	1	0	1	0	1	0	1	-1	Neg
214	011010110	0	1	1	0	1	0	1	1	0	-1	Neg
215	011010111	0	1	1	0	1	0	1	1	1	-1	Neg
216	011011000	0	1	1	0	1	1	0	0	0	0	
217	011011001	0	1	1	0	1	1	0	0	1	0	
218	011011010	0	1	1	0	1	1	0	1	0	0	
219	011011011	0	1	1	0	1	1	0	1	1	0	
220	011011100	0	1	1	0	1	1	1	0	0	0	
221	011011101	0	1	1	0	1	1	1	0	1	0	
222	011011110	0	1	1	0	1	1	1	1	0	0	
223	011011111	0	1	1	0	1	1	1	1	1	0	
224	011100000	0	1	1	1	0	0	0	0	0	0	
225	011100001	0	1	1	1	0	0	0	0	1	-1	Neg
226	011100010	0	1	1	1	0	0	0	1	0	1	
227	011100011	0	1	1	1	0	0	0	1	1	0	
228	011100100	0	1	1	1	0	0	1	0	0	0	
229	011100101	0	1	1	1	0	0	1	0	1	-1	Neg
230	011100110	0	1	1	1	0	0	1	1	0	1	
231	011100111	0	1	1	1	0	0	1	1	1	0	
232	011101000	0	1	1	1	0	1	0	0	0	0	
233	011101001	0	1	1	1	0	1	0	0	1	-1	Neg
234	011101010	0	1	1	1	0	1	0	1	0	1	
235	011101011	0	1	1	1	0	1	0	1	1	0	
236	011101100	0	1	1	1	0	1	1	0	0	1	
237	011101101	0	1	1	1	0	1	1	0	1	0	
238	011101110	0	1	1	1	0	1	1	1	0	2	
239	011101111	0	1	1	1	0	1	1	1	1	1	
240	011110000	0	1	1	1	1	0	0	0	0	0	
241	011110001	0	1	1	1	1	0	0	0	1	-1	Neg
242	011110010	0	1	1	1	1	0	0	1	0	1	
243	011110011	0	1	1	1	1	0	0	1	1	0	

244	011110100	0	1	1	1	1	0	1	0	0	-1	Neg
245	011110101	0	1	1	1	1	0	1	0	1	-2	Neg
246	011110110	0	1	1	1	1	0	1	1	0	0	
247	011110111	0	1	1	1	1	0	1	1	1	-1	Neg
248	011111000	0	1	1	1	1	1	0	0	0	0	
249	011111001	0	1	1	1	1	1	0	0	1	-1	Neg
250	011111010	0	1	1	1	1	1	0	1	0	1	
251	011111011	0	1	1	1	1	1	0	1	1	0	
252	011111100	0	1	1	1	1	1	1	0	0	0	
253	011111101	0	1	1	1	1	1	1	0	1	-1	Neg
254	011111110	0	1	1	1	1	1	1	1	0	1	
255	011111111	0	1	1	1	1	1	1	1	1	0	
256	100000000	1	0	0	0	0	0	0	0	0	0	
257	100000001	1	0	0	0	0	0	0	0	1	0	
258	100000010	1	0	0	0	0	0	0	1	0	0	
259	100000011	1	0	0	0	0	0	0	1	1	0	
260	100000100	1	0	0	0	0	0	1	0	0	0	
261	100000101	1	0	0	0	0	0	1	0	1	0	
262	100000110	1	0	0	0	0	0	1	1	0	0	
263	100000111	1	0	0	0	0	0	1	1	1	0	
264	100001000	1	0	0	0	0	1	0	0	0	0	
265	100001001	1	0	0	0	0	1	0	0	1	0	
266	100001010	1	0	0	0	0	1	0	1	0	-1	Neg
267	100001011	1	0	0	0	0	1	0	1	1	-1	Neg
268	100001100	1	0	0	0	0	1	1	0	0	0	
269	100001101	1	0	0	0	0	1	1	0	1	0	
270	100001110	1	0	0	0	0	1	1	1	0	-1	Neg
271	100001111	1	0	0	0	0	1	1	1	1	-1	Neg
272	100010000	1	0	0	0	1	0	0	0	0	0	
273	100010001	1	0	0	0	1	0	0	0	1	1	
274	100010010	1	0	0	0	1	0	0	1	0	0	
275	100010011	1	0	0	0	1	0	0	1	1	1	
276	100010100	1	0	0	0	1	0	1	0	0	0	
277	100010101	1	0	0	0	1	0	1	0	1	1	
278	100010110	1	0	0	0	1	0	1	1	0	0	
279	100010111	1	0	0	0	1	0	1	1	1	1	
280	100011000	1	0	0	0	1	1	0	0	0	0	
281	100011001	1	0	0	0	1	1	0	0	1	1	
282	100011010	1	0	0	0	1	1	0	1	0	-1	Neg
283	100011011	1	0	0	0	1	1	0	1	1	0	
284	100011100	1	0	0	0	1	1	1	0	0	0	
285	100011101	1	0	0	0	1	1	1	0	1	1	
286	100011110	1	0	0	0	1	1	1	1	0	-1	Neg
287	100011111	1	0	0	0	1	1	1	1	1	0	
288	100100000	1	0	0	1	0	0	0	0	0	0	
289	100100001	1	0	0	1	0	0	0	0	1	0	
290	100100010	1	0	0	1	0	0	0	1	0	0	
291	100100011	1	0	0	1	0	0	0	1	1	0	
292	100100100	1	0	0	1	0	0	1	0	0	0	

293	100100101	1	0	0	1	0	0	1	0	1	0	
294	100100110	1	0	0	1	0	0	1	1	0	0	
295	100100111	1	0	0	1	0	0	1	1	1	0	
296	100101000	1	0	0	1	0	1	0	0	0	0	
297	100101001	1	0	0	1	0	1	0	0	1	0	
298	100101010	1	0	0	1	0	1	0	1	0	-1	Neg
299	100101011	1	0	0	1	0	1	0	1	1	-1	Neg
300	100101100	1	0	0	1	0	1	1	0	0	0	
301	100101101	1	0	0	1	0	1	1	0	1	0	
302	100101110	1	0	0	1	0	1	1	1	0	-1	Neg
303	100101111	1	0	0	1	0	1	1	1	1	-1	Neg
304	100110000	1	0	0	1	1	0	0	0	0	0	
305	100110001	1	0	0	1	1	0	0	0	1	1	
306	100110010	1	0	0	1	1	0	0	1	0	0	
307	100110011	1	0	0	1	1	0	0	1	1	1	
308	100110100	1	0	0	1	1	0	1	0	0	0	
309	100110101	1	0	0	1	1	0	1	0	1	1	
310	100110110	1	0	0	1	1	0	1	1	0	0	
311	100110111	1	0	0	1	1	0	1	1	1	1	
312	100111000	1	0	0	1	1	1	0	0	0	0	
313	100111001	1	0	0	1	1	1	0	0	1	1	
314	100111010	1	0	0	1	1	1	0	1	0	-1	Neg
315	100111011	1	0	0	1	1	1	0	1	1	0	
316	100111100	1	0	0	1	1	1	1	0	0	0	
317	100111101	1	0	0	1	1	1	1	0	1	1	
318	100111110	1	0	0	1	1	1	1	1	0	-1	Neg
319	100111111	1	0	0	1	1	1	1	1	1	0	
320	101000000	1	0	1	0	0	0	0	0	0	0	
321	101000001	1	0	1	0	0	0	0	0	1	0	
322	101000010	1	0	1	0	0	0	0	1	0	0	
323	101000011	1	0	1	0	0	0	0	1	1	0	
324	101000100	1	0	1	0	0	0	1	0	0	0	
325	101000101	1	0	1	0	0	0	1	0	1	0	
326	101000110	1	0	1	0	0	0	1	1	0	0	
327	101000111	1	0	1	0	0	0	1	1	1	0	
328	101001000	1	0	1	0	0	1	0	0	0	0	
329	101001001	1	0	1	0	0	1	0	0	1	0	
330	101001010	1	0	1	0	0	1	0	1	0	-1	Neg
331	101001011	1	0	1	0	0	1	0	1	1	-1	Neg
332	101001100	1	0	1	0	0	1	1	0	0	0	
333	101001101	1	0	1	0	0	1	1	0	1	0	
334	101001110	1	0	1	0	0	1	1	1	0	-1	Neg
335	101001111	1	0	1	0	0	1	1	1	1	-1	Neg
336	101010000	1	0	1	0	1	0	0	0	0	0	
337	101010001	1	0	1	0	1	0	0	0	1	1	
338	101010010	1	0	1	0	1	0	0	1	0	0	
339	101010011	1	0	1	0	1	0	0	1	1	1	
340	101010100	1	0	1	0	1	0	1	0	0	-1	Neg
341	101010101	1	0	1	0	1	0	1	0	1	0	

342	101010110	1	0	1	0	1	0	1	1	0	-1	Neg
343	101010111	1	0	1	0	1	0	1	1	1	0	
344	101011000	1	0	1	0	1	1	0	0	0	0	
345	101011001	1	0	1	0	1	1	0	0	1	1	
346	101011010	1	0	1	0	1	1	0	1	0	-1	Neg
347	101011011	1	0	1	0	1	1	0	1	1	0	
348	101011100	1	0	1	0	1	1	1	0	0	-1	Neg
349	101011101	1	0	1	0	1	1	1	0	1	0	
350	101011110	1	0	1	0	1	1	1	1	0	-2	Neg
351	101011111	1	0	1	0	1	1	1	1	1	-1	Neg
352	101100000	1	0	1	1	0	0	0	0	0	0	
353	101100001	1	0	1	1	0	0	0	0	1	0	
354	101100010	1	0	1	1	0	0	0	1	0	1	
355	101100011	1	0	1	1	0	0	0	1	1	1	
356	101100100	1	0	1	1	0	0	1	0	0	0	
357	101100101	1	0	1	1	0	0	1	0	1	0	
358	101100110	1	0	1	1	0	0	1	1	0	1	
359	101100111	1	0	1	1	0	0	1	1	1	1	
360	101101000	1	0	1	1	0	1	0	0	0	0	
361	101101001	1	0	1	1	0	1	0	0	1	0	
362	101101010	1	0	1	1	0	1	0	1	0	0	
363	101101011	1	0	1	1	0	1	0	1	1	0	
364	101101100	1	0	1	1	0	1	1	0	0	0	
365	101101101	1	0	1	1	0	1	1	0	1	0	
366	101101110	1	0	1	1	0	1	1	1	0	0	
367	101101111	1	0	1	1	0	1	1	1	1	0	
368	101110000	1	0	1	1	1	0	0	0	0	0	
369	101110001	1	0	1	1	1	0	0	0	1	1	
370	101110010	1	0	1	1	1	0	0	1	0	1	
371	101110011	1	0	1	1	1	0	0	1	1	2	
372	101110100	1	0	1	1	1	0	1	0	0	-1	Neg
373	101110101	1	0	1	1	1	0	1	0	1	0	
374	101110110	1	0	1	1	1	0	1	1	0	0	
375	101110111	1	0	1	1	1	0	1	1	1	1	
376	101111000	1	0	1	1	1	1	0	0	0	0	
377	101111001	1	0	1	1	1	1	0	0	1	1	
378	101111010	1	0	1	1	1	1	0	1	0	0	
379	101111011	1	0	1	1	1	1	0	1	1	1	
380	101111100	1	0	1	1	1	1	1	0	0	-1	Neg
381	101111101	1	0	1	1	1	1	1	0	1	0	
382	101111110	1	0	1	1	1	1	1	1	0	-1	Neg
383	101111111	1	0	1	1	1	1	1	1	1	0	
384	110000000	1	1	0	0	0	0	0	0	0	0	
385	110000001	1	1	0	0	0	0	0	0	1	0	
386	110000010	1	1	0	0	0	0	0	1	0	0	
387	110000011	1	1	0	0	0	0	0	1	1	0	
388	110000100	1	1	0	0	0	0	1	0	0	0	
389	110000101	1	1	0	0	0	0	1	0	1	0	
390	110000110	1	1	0	0	0	0	1	1	0	0	

391	110000111	1	1	0	0	0	0	1	1	1	0	
392	110001000	1	1	0	0	0	1	0	0	0	0	
393	110001001	1	1	0	0	0	1	0	0	1	0	
394	110001010	1	1	0	0	0	1	0	1	0	-1	Neg
395	110001011	1	1	0	0	0	1	0	1	1	-1	Neg
396	110001100	1	1	0	0	0	1	1	0	0	1	
397	110001101	1	1	0	0	0	1	1	0	1	1	
398	110001110	1	1	0	0	0	1	1	1	0	0	
399	110001111	1	1	0	0	0	1	1	1	1	0	
400	110010000	1	1	0	0	1	0	0	0	0	0	
401	110010001	1	1	0	0	1	0	0	0	1	1	
402	110010010	1	1	0	0	1	0	0	1	0	0	
403	110010011	1	1	0	0	1	0	0	1	1	1	
404	110010100	1	1	0	0	1	0	1	0	0	0	
405	110010101	1	1	0	0	1	0	1	0	1	1	
406	110010110	1	1	0	0	1	0	1	1	0	0	
407	110010111	1	1	0	0	1	0	1	1	1	1	
408	110011000	1	1	0	0	1	1	0	0	0	0	
409	110011001	1	1	0	0	1	1	0	0	1	1	
410	110011010	1	1	0	0	1	1	0	1	0	-1	Neg
411	110011011	1	1	0	0	1	1	0	1	1	0	
412	110011100	1	1	0	0	1	1	1	0	0	1	
413	110011101	1	1	0	0	1	1	1	0	1	2	
414	110011110	1	1	0	0	1	1	1	1	0	0	
415	110011111	1	1	0	0	1	1	1	1	1	1	
416	110100000	1	1	0	1	0	0	0	0	0	0	
417	110100001	1	1	0	1	0	0	0	0	1	-1	Neg
418	110100010	1	1	0	1	0	0	0	1	0	0	
419	110100011	1	1	0	1	0	0	0	1	1	-1	Neg
420	110100100	1	1	0	1	0	0	1	0	0	0	
421	110100101	1	1	0	1	0	0	1	0	1	-1	Neg
422	110100110	1	1	0	1	0	0	1	1	0	0	
423	110100111	1	1	0	1	0	0	1	1	1	-1	Neg
424	110101000	1	1	0	1	0	1	0	0	0	0	
425	110101001	1	1	0	1	0	1	0	0	1	-1	Neg
426	110101010	1	1	0	1	0	1	0	1	0	-1	Neg
427	110101011	1	1	0	1	0	1	0	1	1	-2	Neg
428	110101100	1	1	0	1	0	1	1	0	0	1	
429	110101101	1	1	0	1	0	1	1	0	1	0	
430	110101110	1	1	0	1	0	1	1	1	0	0	
431	110101111	1	1	0	1	0	1	1	1	1	-1	Neg
432	110110000	1	1	0	1	1	0	0	0	0	0	
433	110110001	1	1	0	1	1	0	0	0	1	0	
434	110110010	1	1	0	1	1	0	0	1	0	0	
435	110110011	1	1	0	1	1	0	0	1	1	0	
436	110110100	1	1	0	1	1	0	1	0	0	0	
437	110110101	1	1	0	1	1	0	1	0	1	0	
438	110110110	1	1	0	1	1	0	1	1	0	0	
439	110110111	1	1	0	1	1	0	1	1	1	0	

440	110111000	1	1	0	1	1	1	0	0	0	0	
441	110111001	1	1	0	1	1	1	0	0	1	0	
442	110111010	1	1	0	1	1	1	0	1	0	-1	Neg
443	110111011	1	1	0	1	1	1	0	1	1	-1	Neg
444	110111100	1	1	0	1	1	1	1	0	0	1	
445	110111101	1	1	0	1	1	1	1	0	1	1	
446	110111110	1	1	0	1	1	1	1	1	0	0	
447	110111111	1	1	0	1	1	1	1	1	1	0	
448	111000000	1	1	1	0	0	0	0	0	0	0	
449	111000001	1	1	1	0	0	0	0	0	1	0	
450	111000010	1	1	1	0	0	0	0	1	0	0	
451	111000011	1	1	1	0	0	0	0	1	1	0	
452	111000100	1	1	1	0	0	0	1	0	0	0	
453	111000101	1	1	1	0	0	0	1	0	1	0	
454	111000110	1	1	1	0	0	0	1	1	0	0	
455	111000111	1	1	1	0	0	0	1	1	1	0	
456	111001000	1	1	1	0	0	1	0	0	0	0	
457	111001001	1	1	1	0	0	1	0	0	1	0	
458	111001010	1	1	1	0	0	1	0	1	0	-1	Neg
459	111001011	1	1	1	0	0	1	0	1	1	-1	Neg
460	111001100	1	1	1	0	0	1	1	0	0	1	
461	111001101	1	1	1	0	0	1	1	0	1	1	
462	111001110	1	1	1	0	0	1	1	1	0	0	
463	111001111	1	1	1	0	0	1	1	1	1	0	
464	111010000	1	1	1	0	1	0	0	0	0	0	
465	111010001	1	1	1	0	1	0	0	0	1	1	
466	111010010	1	1	1	0	1	0	0	1	0	0	
467	111010011	1	1	1	0	1	0	0	1	1	1	
468	111010100	1	1	1	0	1	0	1	0	0	-1	Neg
469	111010101	1	1	1	0	1	0	1	0	1	0	
470	111010110	1	1	1	0	1	0	1	1	0	-1	Neg
471	111010111	1	1	1	0	1	0	1	1	1	0	
472	111011000	1	1	1	0	1	1	0	0	0	0	
473	111011001	1	1	1	0	1	1	0	0	1	1	
474	111011010	1	1	1	0	1	1	0	1	0	-1	Neg
475	111011011	1	1	1	0	1	1	0	1	1	0	
476	111011100	1	1	1	0	1	1	1	0	0	0	
477	111011101	1	1	1	0	1	1	1	0	1	1	
478	111011110	1	1	1	0	1	1	1	1	0	-1	Neg
479	111011111	1	1	1	0	1	1	1	1	1	0	
480	111100000	1	1	1	1	0	0	0	0	0	0	
481	111100001	1	1	1	1	0	0	0	0	1	-1	Neg
482	111100010	1	1	1	1	0	0	0	1	0	1	
483	111100011	1	1	1	1	0	0	0	1	1	0	
484	111100100	1	1	1	1	0	0	1	0	0	0	
485	111100101	1	1	1	1	0	0	1	0	1	-1	Neg
486	111100110	1	1	1	1	0	0	1	1	0	1	
487	111100111	1	1	1	1	0	0	1	1	1	0	
488	111101000	1	1	1	1	0	1	0	0	0	0	

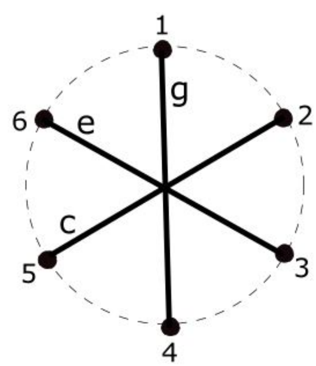
489	111101001	1	1	1	1	0	1	0	0	1	-1	Neg
490	111101010	1	1	1	1	0	1	0	1	0	0	
491	111101011	1	1	1	1	0	1	0	1	1	-1	Neg
492	111101100	1	1	1	1	0	1	1	0	0	1	
493	111101101	1	1	1	1	0	1	1	0	1	0	
494	111101110	1	1	1	1	0	1	1	1	0	1	
495	111101111	1	1	1	1	0	1	1	1	1	0	
496	111110000	1	1	1	1	1	0	0	0	0	0	
497	111110001	1	1	1	1	1	0	0	0	1	0	
498	111110010	1	1	1	1	1	0	0	1	0	1	
499	111110011	1	1	1	1	1	0	0	1	1	1	
500	111110100	1	1	1	1	1	0	1	0	0	-1	Neg
501	111110101	1	1	1	1	1	0	1	0	1	-1	Neg
502	111110110	1	1	1	1	1	0	1	1	0	0	
503	111110111	1	1	1	1	1	0	1	1	1	0	
504	111111000	1	1	1	1	1	1	0	0	0	0	
505	111111001	1	1	1	1	1	1	0	0	1	0	
506	111111010	1	1	1	1	1	1	0	1	0	0	
507	111111011	1	1	1	1	1	1	0	1	1	0	
508	111111100	1	1	1	1	1	1	1	0	0	0	
509	111111101	1	1	1	1	1	1	1	0	1	0	
510	111111110	1	1	1	1	1	1	1	1	0	0	
511	111111111	1	1	1	1	1	1	1	1	1	0	

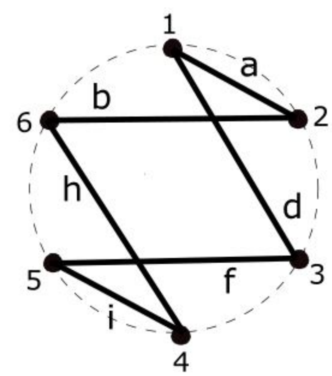
APPENDIX B

OBSTRUCTIVE NETWORKS UP TO REFLECTION FOR SECTION 3.3

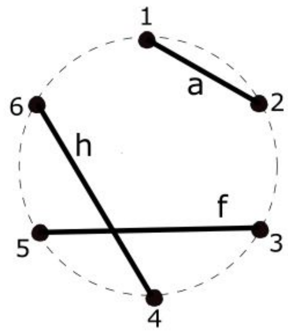
This appendix provides pictures for each of the 27 unique obstructive networks, up to reflection, that were found in Section 3.3. They are classified by their number of symmetries.

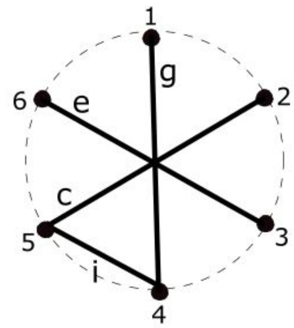
Obstructive Networks Containing Two Symmetries

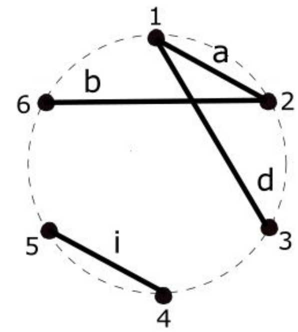


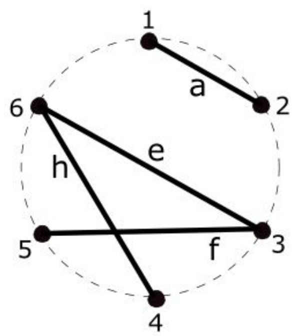


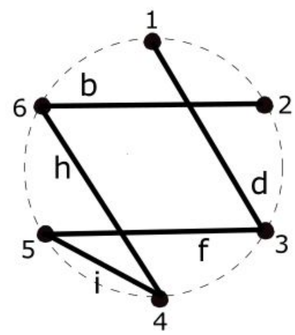
Obstructive Networks Containing One Symmetry

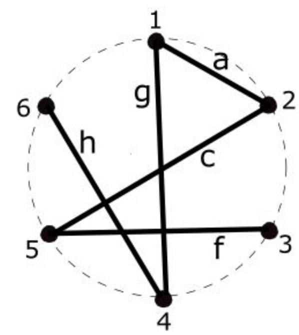


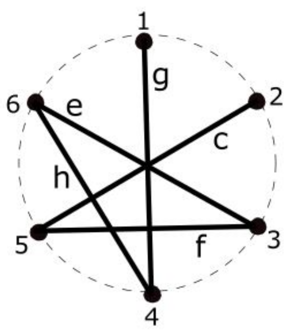


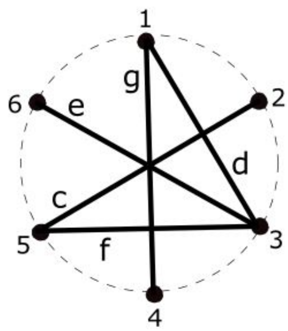


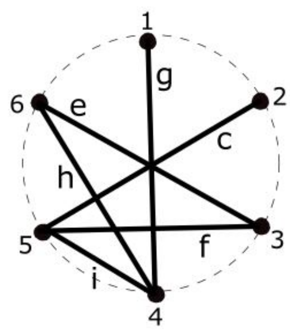


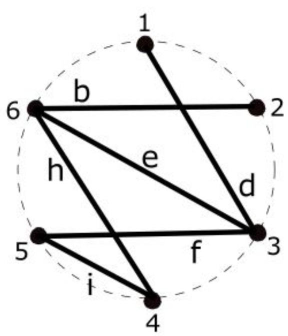


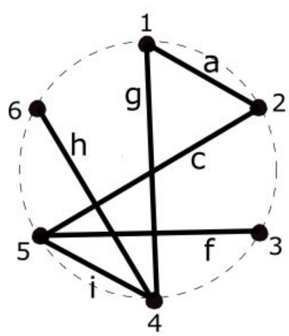


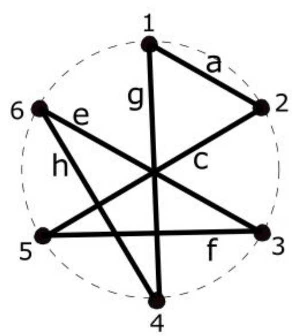












Obstructive Networks Containing No Symmetries