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Relating Modified Ergun's Equation to Hindered Settling Correlations

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Executive Summary

Often, students are burdened with memorizing multiple correlations to describe the behaviors of particles. However, these equations may be able to be simplified in terms of packed and fluidized beds of particles. The modified Ergun's equation and hindered settling correlations are two of the main methods for calculating particle and fluid velocities, as well as pressure drop, within beds. These values are influenced by several factors, including diameter and length of the bed, medium in which the particles are suspended, physical properties of the fluid and particle, and most importantly the particle size and porosity of the bed. It is the purpose of the research described in this report to determine if there are values of the porosity in which both the modified Ergun's equation and hindered settling correlations produce similar trends for the value of the particle velocity at varying particle sizes. If the hypothesis is true, then it will simplify equations used to describe packed and fluidized beds, as well as reduce the strain on students to learn multiple equations for similar situations.

After performing the required derivations to solve for the particle velocity using both the modified Ergun's equation and hindered settling correlations, the results were evaluated and compared. The fluid used for the evaluation of both the theoretical and physical models of the system was water at ambient conditions (60^{o}F and 1 atm), and the particle material was assumed to be poly-lactic acid (PLA) with an initial particle diameter of 0.108 inches. Under these conditions, the porosity was manipulated to represent values between 0.4242 and 0.89. The porosity could not be tested outside of this range, as dense packing of the particles was required to satisfy the requirement that the fluidized bed could act as a packed bed. The results of the analysis showed that both sets of equations produced a velocity value with 0.3% of each other at a porosity value of 0.52, and may satisfy the hypothesis. The percent difference for the range of

porosities at the constant particle size is shown below in **Table 1**. The porosity of 0.52 was then held constant and the particle size was then varied to determine if either correlation could be used to describe the particle velocity at the given porosity. However, as particle diameter varied, the modified Ergun's equation exhibited an inverse relationship with particle size and the hindered settling correlations exhibited a direct relationship. The trends observed can be seen in **Figure 1**, where if the hypothesis were true the slope of each line and magnitude of the velocity values would be similar. A physical model of the packed bed was then constructed, with three varying particle porosities. The arrays of particles were 3-D printed using a PLA material to porosities of 0.4242, 0.52, and 0.69. After measuring the settling times in the bed and calculating the velocities, the average value was compared to the average theoretical value. The measured velocity varied from theory by a value of 29.9%, -3.03% and -31.2% for porosity values of 0.4242, 0.52, and 0.69 respectively.

As a result of the above analysis, it was determined that the hypothesis was invalid and there is not a single correlation that can be used to describe both fluid flow in a packed bed and the settling velocity of particles in a fluidized bed. Although the trends for both equations vary, there is still a point of intersection for most values of porosity where both correlations produce a particle velocity that is within the acceptable range of error. This allows for the possibility of finding a relationship where one may be able to predict based on the particle size and porosity of the bed whether both equations will equate to a single particle velocity.

There have been a number of lessons that I have learned as a result of completing this research. Historically, I have thrived in team settings where several engineers could collaborate to solve difficult problems. However, this project has helped to teach me to be a more independent thinker and to rely on my own problem-solving skills to work through challenging parts of projects. It has been easy in the past to ask for help when I have been stuck on certain parts of a problem, but now I feel that I have the confidence to use my own reason and prior knowledge to work through problems from start to finish. In addition, this project heavily relied on my ability to both derive new equations and manipulate old equations to account for new variables or factors. This hasn't been a strength of mine over my college career, but this project gave me the opportunity to develop and refine these skills. Although the results resulted in an invalid initial hypothesis, it has opened up the door for future research in the field of hindered settling and fluid flow through packed beds.

Building on the research already performed and explained in this report, further calculations can be performed to determine a relationship for what values of porosity and particle size will result in similar velocities according to the hindered settling and modified Ergun's equations. As it has been shown, there are certain values for the porosity and particle size in a bed of particles in which both correlations calculate the same velocity. This intersection point can be calculated for a wide variety of conditions and plotted along a curve. With this curve, an equation can be constructed to determine when an intersection between the theoretical curves would occur. In terms of recommendations for future students undergoing the honors project, it is important to trust your own ability to research and problem solve to answer complex engineering questions.

Introduction/Background

For the purposes of the research assignment, the relationship between the modified Ergun's equation and hindered settling correlation was evaluated under high particle density conditions. According to the text in *Unit Operations of Chemical Engineering,* The modified Ergun's equation is used to describe the relationship between pressure drop in a packed bed of particles and the velocity of the fluid past the particles. Conversely, the hindered settling correlations are used to determine the bulk particle velocity in fluidized beds using the physical properties of both the particles and the fluid in which they are suspended. The modified Ergun's equation was manipulated to solve for the velocity of the particles in the bed, and the results were evaluated under porosity conditions ranging from 0.46 to 0.89 (high particle density). High particle density was required to satisfy the requirement that a fluidized bed could act under the conditions of a packed bed. Both packed beds and fluidized beds have applications in the chemical industry to be used as catalyzed reactors. However, a packed bed has a layer of catalyst particles in which the fluid flows through, as described by the modified Ergun's equation, and a fluidized bed allows for the particles to be suspended within the medium. The fluid velocity in a fluidized bed must be between the minimum fluidization velocity and the maximum velocity before the particles elutriate from the bed. These were important considerations to take into account when analyzing the results of the performed experiments.

The intent of the research was to define conditions within packed and fluidized beds in which either of the previously mentioned equations would be able to accurately predict the velocity and forces exhibited on the particles within the bed. This serves a greater academic purpose to simplify and reduce the quantity of equations required to describe flow within beds for students, as well as expand the scope of possibility previously defined for hindered settling and the modified Ergun's equation.

To examine the hypothesis that there are certain porosities where both sets of equations produce similar results, new derivations of the modified Ergun's equation and hindered settling correlations were performed by examining force balances on the particles in relation to pressure drop within the bed, defining expressions for the number of particles per unit volume of bed under varying porosities, and finally manipulating the correlations to both account for particle velocity since the modified Ergun's equation describes the fluid flow through the bed. For hindered settling correlations, the particle velocity was calculated using bulk properties described by CJ Geankoplis in his work *Transport Processes and Unit Operations*. Here, the particle size and porosity of the bed determine which equation most accurately describes the system and is further discussed in the experimental methods section of the report. The accuracy of the theoretical velocity versus particle diameter curves at a constant porosity could then be determined by designing and performing an experiment to measure the settling velocity of particles within a constant volume of water. Testing under a wide range of porosities would allow for the most accurate conclusions to be drawn from the physical experiments. The hypothesis was valid if both sets of equations produced similar trends for curves relating particle velocity to the particle diameter under a constant bed porosity. In this situation, either of the correlations derived would create an accurate depiction for particle velocity within the bed.

Experimental Methods

To prove the existence of a correlation between the modified Ergun's equation and hindered settling, a relationship between the fluid velocity in a packed bed and the settling velocity of particles in a densely packed slurry was to be determined. From here, the model was used with several different conditions, including varying porosity and particle diameter, to determine the conditions which produced similar results between the modified Ergun's equation and hindered settling relations. Before the velocity correlations can be derived, a relationship between the volume of the bed being investigated and the number of particles required to satisfy the porosity constraints of a packed bed was determined. The equation is shown below in **Equation 1**, with the full derivation shown in the sample calculations section of **Appendix B**.

$$\boldsymbol{N_p = \frac{6AL(1-\varepsilon)}{d_p^3 * \pi}} \qquad \textbf{(1)}$$

In the above equation, N_p refers to the total number of particles, A is the cross-sectional area of the bed, L is the height of the bed, ε is the porosity, and d_p is the particle diameter. This correlation was later used in calculating the total pressure drop experienced in a packed bed. To further determine the pressure-drop experienced by a single particle in the bed, a relationship between the force on an individual particle and the force of gravity was derived. A force balance was performed on the particle, relating the total force on the plate to the gravitational, buoyancy, and drag forces according to the information provided in *Unit Operations of Chemical Engineering*. Assuming the force on the plate was zero in a neutrally buoyant state, the drag force was equivalent to the difference between the gravitational and buoyancy forces. By solving this equation for the change in pressure and multiplying by the total number of particles in the bed, **Equation 2** was created:

$$\Delta \boldsymbol{P} = \left(\boldsymbol{\rho}_{\boldsymbol{p}} - \boldsymbol{\rho}\right)(\mathbf{1} - \boldsymbol{\varepsilon})\boldsymbol{L} * \left[\frac{\boldsymbol{6AL}(\mathbf{1}-\boldsymbol{\varepsilon})}{\boldsymbol{d}_{\boldsymbol{p}}^{\mathbf{3}}\boldsymbol{\pi}}\right] \qquad \textbf{(2)}$$

This change in pressure could then be used to determine the velocity utilizing the modified Ergun's equation. The velocity term in the Ergun's equation is defined as the difference between the fluid and particle velocities. Therefore, the equation was solved for the velocity term, and the fluid velocity was subtracted to obtain a correlation for the particle velocity in the bed according to the modified Ergun's equation. The original modified Ergun's equation is shown below in **Equation 3** (McCabe 166):

$$\frac{\Delta \boldsymbol{P}}{\boldsymbol{L}} = \left[\frac{\mathbf{150}\boldsymbol{V}_{\boldsymbol{o}}\mu}{\boldsymbol{d}_{\boldsymbol{p}}^{\mathbf{2}}\boldsymbol{\Phi}^{\mathbf{2}}}\right]\left[\frac{(\mathbf{1}-\boldsymbol{\varepsilon})^{\mathbf{2}}}{\boldsymbol{\varepsilon}^{\mathbf{3}}}\right] + \left[\frac{\mathbf{1.75}\boldsymbol{\rho g}\boldsymbol{V}_{\boldsymbol{o}}^{\mathbf{2}}}{\boldsymbol{d}_{\boldsymbol{p}}\boldsymbol{\Phi}}\right]\left[\frac{(\mathbf{1}-\boldsymbol{\varepsilon})}{\boldsymbol{\varepsilon}^{\mathbf{3}}}\right] \qquad \textbf{(3)}$$

Here, μ refers to the fluid viscosity and Φ is the sphericity of the particles. For the purposes of the analysis, the particles were assumed to be perfect spheres giving the sphericity a value of one. The V_o term was then replaced with the difference between the particle and fluid velocities, and the quadratic equation was then applied to **Equation 3** to solve for the particle velocity. The final equation used for further calculations is described by **Equation 4:**

$$\boldsymbol{U} = \boldsymbol{V} + \frac{\mathbf{42.9}\mu(\mathbf{1}-\boldsymbol{\varepsilon})}{\boldsymbol{d}_{\boldsymbol{p}}\boldsymbol{\Phi}\boldsymbol{\rho g}} + \frac{\boldsymbol{d}_{\boldsymbol{p}}\boldsymbol{\varepsilon}^{\mathbf{3}}\boldsymbol{\Phi}\sqrt{\left(\frac{\mathbf{150}\mu(\mathbf{1}-\boldsymbol{\varepsilon})^{\mathbf{2}}}{\boldsymbol{d}_{\boldsymbol{p}}^{\mathbf{2}}\boldsymbol{\Phi}^{\mathbf{2}}\boldsymbol{\varepsilon}^{\mathbf{3}}}\right)^{\mathbf{2}} + \left(\frac{\mathbf{7}\boldsymbol{\rho g}(\mathbf{1}-\boldsymbol{\varepsilon})}{\boldsymbol{d}_{\boldsymbol{p}}\boldsymbol{\varepsilon}^{\mathbf{3}}\boldsymbol{\Phi}}\right)\left(\frac{\Delta \boldsymbol{P}}{\boldsymbol{L}}\right)}}{\mathbf{3.5}\boldsymbol{pg}(\mathbf{1}-\boldsymbol{\varepsilon})} \qquad \textbf{(4)}$$

Above, U is the velocity of the particle and V is the velocity of the fluid. **Equation 2** was then combined with **Equation 4** to determine the particle velocity at varying porosities. Similarly, the relationship between the settling velocity in a densely packed fluidized bed and gravity was determined and compared to the results of the above equations.

The equations used to determine the settling velocity of particles in a bed according to hindered settling correlations change based on which of the three ranges the conditions apply to and utilize the bulk fluid and particle properties. The correlations used were first described by CJ Geankoplis, who defined the ranges as the Stoke's Law range, Intermediate range, and Newton's Law range, and is determined based on the value of a constant "k" which is independent of the terminal velocity of the particles. The value of k is described by **Equation 5** (McCabe 172):

$$k = d_p\left[\frac{g\rho(\rho_p - p^o)}{\mu^2}\right]^{1/3} \qquad (5)$$

Here, p^o is the bulk density of the slurry which can be calculated using both the density of the fluid and the particle along with the porosity, as shown in **Equation 6** (Transport Processes and Unit Operations):

$$\rho^o = (1 - \varepsilon)\rho_p + \varepsilon\rho \qquad (6)$$

For values of k less than 2.6, the terminal velocity of the particles would be calculated using Stoke's Law. Here, the velocity is calculated following **Equation 7** (McCabe 171):

$$u_t^o = \frac{g d_p^2(\rho_p - \rho)}{18\mu^o} \qquad (7)$$

The modified Ergun's equation is used to describe packed beds of particles where there is high particle density. To be able to compare hindered settling to the modified Ergun's equation, a dense packing porosity was assumed for the calculation ($0.46 < \varepsilon < 0.89$). According to the dense packing criteria, the bulk viscosity is defined by **Equation 8** (Transport Processes and Unit Operations):

$$\mu^o = \mu\left[1 - \frac{(1-\varepsilon)}{(1-\varepsilon_c)}\right]^{-2.5} \qquad (8)$$

The critical porosity for the system follows Brown's correlation (Brown et al.), and for a sphericity equivalent to 1 is equal to 0.4242. For the next range, where $2.6 < k < 68.9$, the flow is considered to be in the intermediate range. There is no definite equation to describe this type of flow, and is not used in the comparison between the above correlations. When $68.9 < k < 2360$, the flow follows the Newton's Law range. The same calculations for the bulk viscosity and density are used, and the terminal velocity is calculated according to **Equation 9:**

$$\boldsymbol{u_t^o = 1.75\sqrt{\frac{g d_p(\rho_p - \rho^o)}{\rho^o}}} \qquad \textbf{(9)}$$

The above equations were used to determine the theoretical curves for the velocity versus diameter of the particle holding porosity constant. To test the validity of the theory, an experiment was designed to measure the settling velocity of three arrays of spheres all with different values of porosity. A particle diameter of 0.108 inches was used to 3-D print an array of particles in a cubic shape with porosities of 0.4242, 0.52, and 0.69. Next, a glass cylinder was chosen and the dimensions of the container recorded to determine the liquid volume. Water was used as the medium, and the temperature was recorded to accurately predict its physical properties. For the experiment, one array of spheres was placed in the water and released. A stopwatch was then used to record the time taken for the sphere to settle to the bottom of the glass, and the velocity could be determined. This process was repeated 5 times for each of the porosity values. The results from the physical experiment could then be compared to the theoretical values predicted.

Data and Results

The theoretical values of the particle velocity were calculated keeping the particle diameter constant at 0.108 inches and varying the porosity for each of the two correlations previously defined. The velocities were the compared to determine a porosity in which both correlations produced similar results. **Table 1** shows the results of the comparison of the results.

Table 1: *shows the velocity difference between the hindered settling correlations and the modified Ergun's equation at varying porosities and a constant particle size. The two correlations produced similar results at a porosity of 0.52.*

Porosity e	Velocity Difference (ft/s)	Percent Difference
0.46	0.017	6.0%
0.47	0.014	4.9%
0.48	0.011	3.9%
0.49	0.009	2.9%
0.5	0.006	2.0%
0.51	0.004	1.1%
0.52	0.001	0.3%
0.53	0.001	0.4%
0.54	0.003	1.1%
0.55	0.006	1.7%
0.56	0.007	2.2%
0.57	0.009	2.7%
0.58	0.011	3.2%
0.59	0.012	3.6%
0.6	0.014	3.9%
0.61	0.015	4.2%
0.62	0.016	4.4%
0.63	0.016	4.5%
0.64	0.017	4.6%
0.65	0.017	4.6%
0.66	0.017	4.6%
0.67	0.017	4.5%
0.68	0.016	4.3%
0.69	0.015	4.1%

Under the defined conditions, a porosity of 0.52 produced velocities from both correlations that was within 0.3% of each other. This porosity was then held constant, and the particle size was changed to determine if the hypothesis was true and both correlations produced similar results with a constant porosity at differing particle sizes. The results are shown below in **Figure 1**:

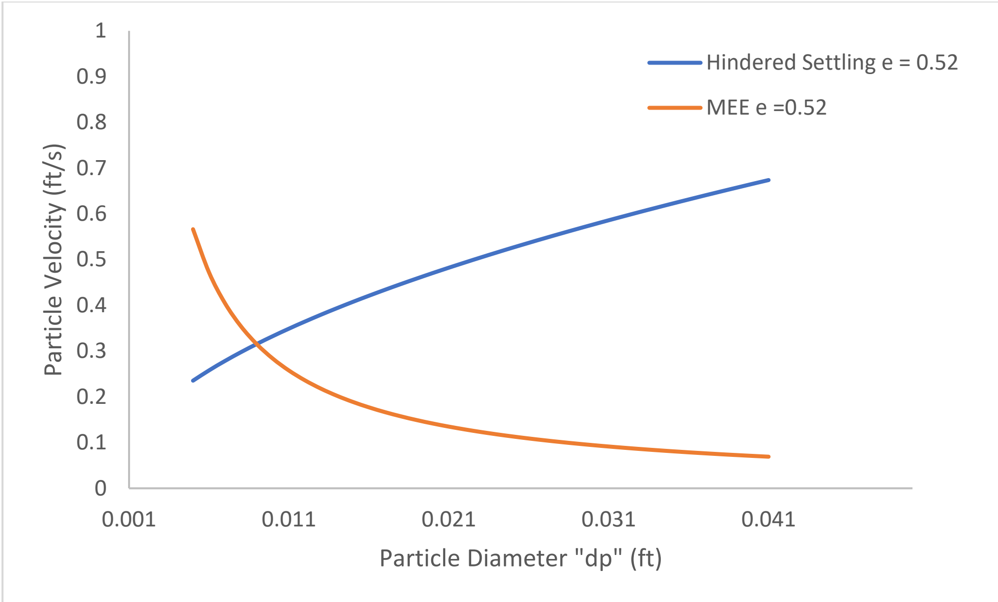

***Figure 1**: shows the theoretical relationship between the hindered settling and modified Ergun's equations (MEE) for the velocity at varying particle sizes and a constant porosity of 0.52. The upward shift in the hindered settling data occurs when the conditions change from Stoke's Law to Newton's law ranges.*

As observed in **Figure 1**, the theoretical curves for the hindered settling and modified Ergun's equations exhibit differing trends as particle size is increased. However, they intersect at the previously defined particle size of 0.108 inches in the Stoke's Law range of hindered settling. It was observed that the two correlations would only intersect in the Stokes Law range, as the larger particle sizes required to be modeled by Newton's Law produced very large velocity deviations. This correlation seems to disprove the stated hypothesis, as a single equation cannot be used to determine the particle velocity at a given porosity.

An experiment was designed to test the validity of the derived equations by 3-D printing arrays of spheres with differing porosities out of poly-lactic acid (PLA) and determining their settling time in a bed with a predetermined volume. The porosities chosen to evaluate were 0.4242, which is the critical porosity, 0.52, which was the porosity in which the velocity of a particle of size of 0.108 inches was calculated to be the same for both hindered settling and the modified Ergun's equation, and finally 0.69. The velocity was determined by measuring the settling time of the particles and dividing by the length of the bed. The experiment was repeated five times for each porosity, and the velocities were averaged and compared to the average theoretical velocity predicted between the modified Ergun's equation and hindered settling correlations. The figures below show the results from the trials of the experiments, and further data can be found in **Appendix A**.

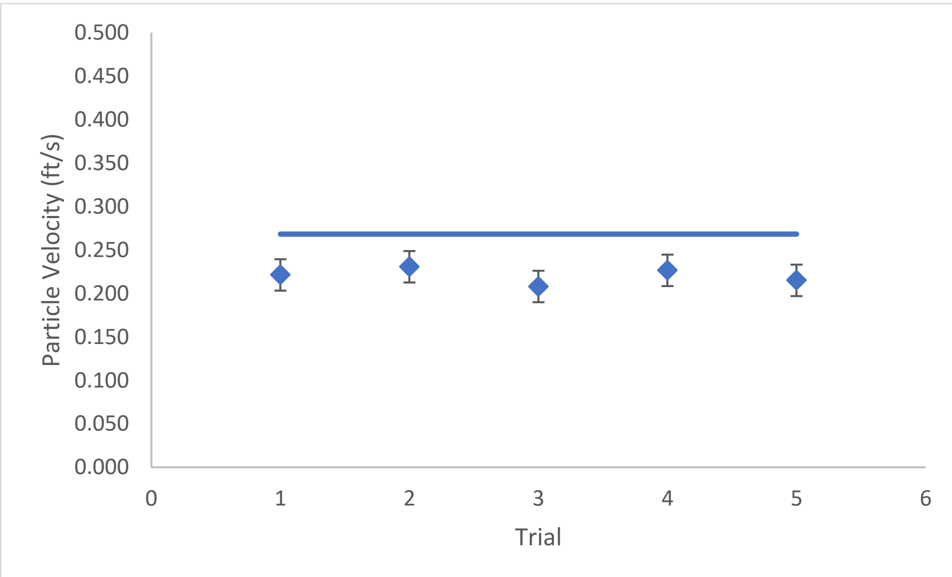

Figure 2: *shows the results of the first experiment in which the particles were arranged with a porosity of 0.4242. The points represent measured values of the velocity, and the solid curve represents the average theoretical velocity. Error bars were fixed to the points to show two standard deviations from the mean.*

The data obtained from using a porosity of 0.4242 showed the measured velocity was 19.6% lower than the theoretical velocity predicted. This result was expected, as previous analysis showed that at the constant particle size a porosity of 0.52 was the only value which produced consistent values between the two correlations.

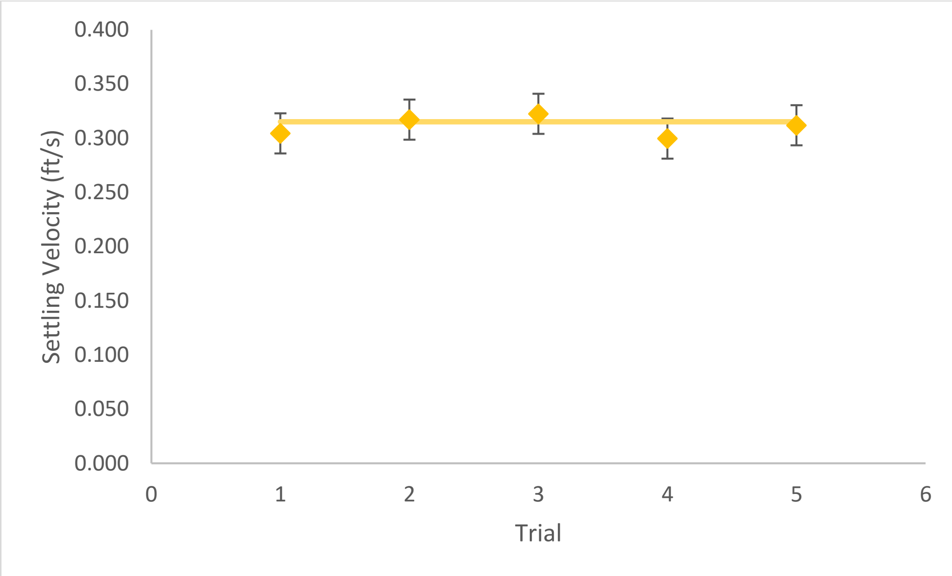

Figure 3: *shows the results of the second experiment in which the particles were arranged with a porosity of 0.52. The points represent measured values of the velocity, and the solid curve represents the average theoretical velocity. Error bars were fixed to the points to show two standard deviations from the mean.*

In the second experiment, a porosity of 0.52 was used and the resulting velocities were observed. As predicted from the theoretical models, the measured values of the velocity closely matched the predicted velocities varying by approximately 1.28%. There was a higher degree of standard deviation for the measurements in this experiment, which is shown by the larger error bars above and below each point. However, the theoretical curve still falls within the two standard deviations of the measured point at each of the velocities recorded.

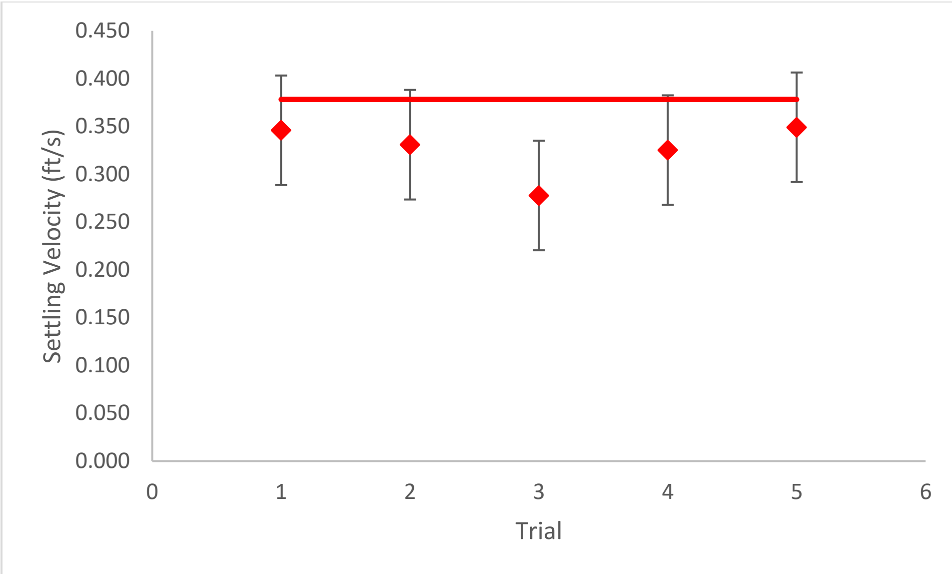

Figure 4: *shows the results of the second experiment in which the particles were arranged with a porosity of 0.52. The points represent measured values of the velocity, and the solid curve represents the average theoretical velocity. Error bars were fixed to the points to show two standard deviations from the mean.*

Finally, the experiment was performed using a porosity of 0.69. Again, the measured values deviated from the theoretical values significantly with an average deviation of 14.9% lower than predicted. The experimental results largely matched what the theoretical models had predicted, where only the porosity of 0.52 produced experimental values that were consistent with what the modified Ergun's equation and hindered settling correlations had calculated.

Discussion/Analysis

For the hypothesis to be considered true, both the hindered settling correlations and modified Ergun's equation would need to produce similar curves and values for the particle velocity at varying particle sizes at a given porosity. However, as seen by the trends produced by the equations, velocity and particle size had a direct relationship for hindered settling and an inverse relationship for the modified Ergun's equation no matter the porosity that was observed. The two theoretical curves did intersect at a single particle size for each porosity observed, showing that there are conditions that produce equivalent results between the expressions. However, there is no way to predict which particle size will lead to this intersection point for a given porosity. The Ergun's correlation produces an inverse relationship with particle size primarily because of the d_p^3 term in the denominator of the equation. This term has the largest effect on the magnitude of the velocity calculated, and results in lower velocities as the particle size increases. Also, the basis for the Ergun's equation measures the change in pressure as it relates to the velocity of the fluid. As particle size increases with a constant porosity, there is less space between particles for the fluid to flow and therefore decreases the observed velocity. Contrarily, the terminal velocity produced from hindered settling exhibits a d_p term in the numerator which produces the direct relationship between particle size and velocity.

The experimental results largely resembled what was predicted by the theoretical models. The measured data only correlated to the predicted values for the porosity of 0.52, and significantly varied for the porosity values of 0.4242 and 0.69. Although the theoretical models did not provide a valid estimation of the settling velocities, the measured values did follow the predicted trends based on the effects of porosity on particle size. As the porosity increased (distance between particles became greater), the settling velocity measured from the physical experiment also increased. This is primarily due to the increased flow allowed between the particles, which reduces the pressure drop and drag force experienced by the particles.

The main assumption evaluated throughout the research in the report was that under certain ranges of porosity, the modified Ergun's equation and hindered settling correlations would predict similar values for the particle's setting velocity. Based on the trends developed from the derivation of the theoretical models and the results obtained from the physical experiment, it can be concluded that the hypothesis is invalid. The two correlations produced opposing trends, where the modified Ergun's equation had an indirect relationship with the particle size at a constant porosity and the hindered settling correlations had a direct relationship with the particle size. If the hypothesis were true, both sets of equations would exhibit the same relationship with a changing particle size. Although the trends varied, the two equations intersected with a particle size of 0.108 inches and porosity of 0.52. Here, either correlation could be used to predict the particle velocity, and the physical model produced an average velocity which varied only 1.28% from the predicted value. Further research can be conducted to formulate a correlation to determine which porosities and particle sizes would result in an intersection between the curves for the modified Ergun's equation and hindered settling correlations.

References

Brown, G. G., et al. *Unit Operations*. Cbs Publishers & Distribution, 2005.

Geankoplis, Christie J. *Transport Processes and Unit Operations*. Prentice Hall, 2003.

McCabe, Warren L., et al. *Unit Operations of Chemical Engineering*. 7th ed., McGraw-Hill Education, 2005.

Appendix A

***Table 2**: shows the physical properties of the medium in the bed, in this case water, at ambient conditions. These properties were used to calculate the theoretical velocity of the settling particles, as well as the actual velocity observed in the performed experiments.*

Water Properties		
Pressure	1	atm
Temperature	60	F
Density	62.37	lb/ft^3
Viscosity	0.000759	lb/ft*s

***Table 3**: shows the physical properties of the medium in the bed, in this case water, at ambient conditions. These properties were used to calculate the theoretical velocity of the settling particles, as well as the actual velocity observed in the performed experiments.*

PLA Particle Properties		
dp	0.009	ft
Density	77.41	lb/ft^3

***Table 4:** shows the other values required to solve the hindered settling and modified Ergun's equations. The dimensions of the bed shown below were measured to accurately represent the physical experiment that was performed.*

Miscellaneous Values		
g	32.174	ft/s^2
g_c	32.174	$lbm*ft/lbf/s^2$
sphericity	1.000	
ε_c	0.424	
Length of Bed "L"	0.381	ft
Diameter of Bed	0.220	ft
Area of Bed	0.038	ft^2

***Table 5:** shows the experimental values for the velocity obtained from 3-D printed particles. For experiment 1, the particles were printed to have a porosity of 0.4242. The theoretical values of the velocity for the specific particle size and porosity are also shown, and the standard deviation for error in the measurements was calculated.*

Experimental Results			
Porosity	0.4242		
Length of Bed (ft)	0.3806		
Particle Size (ft)	0.0090		
Trial Number	Settling Time (s)	Velocity (Calculated)	Velocity (Theoretical)
1	1.72	0.221	0.268
2	1.65	0.231	0.268
3	1.83	0.208	0.268
4	1.68	0.227	0.268
5	1.77	0.215	0.268
Average	1.73	0.220	0.268
Standard Deviation	0.0718	0.0181	-

***Table 6:** shows the experimental values for the velocity obtained from 3-D printed particles. For experiment 1, the particles were printed to have a porosity of 0.52. The theoretical values of the velocity for the specific particle size and porosity are also shown, and the standard deviation for error in the measurements was calculated.*

Experimental Results			
Porosity	0.52		
Length of Bed (ft)	0.3806		
Particle Size (ft)	0.0090		
Trial Number	Settling Time (s)	Velocity (Calculated)	Velocity (Theoretical)
1	1.25	0.304	0.315
2	1.2	0.317	0.315
3	1.18	0.323	0.315
4	1.27	0.300	0.315
5	1.22	0.312	0.315
Average	1.22	0.311	0.315
Standard Deviation	0.036	0.019	-

Table 7: *shows the experimental values for the velocity obtained from 3-D printed particles. For experiment 1, the particles were printed to have a porosity of 0.69 The theoretical values of the velocity for the specific particle size and porosity are also shown, and the standard deviation for error in the measurements was calculated.*

Experimental Results			
Porosity	0.69		
Length of Bed (ft)	0.3806		
Particle Size (ft)	0.0090		
Trial Number	Settling Time (s)	Velocity (Calculated)	Velocity (Theoretical)
1	1.1	0.346	0.3782
2	1.15	0.331	0.3782
3	1.37	0.278	0.3782
4	1.17	0.325	0.3782
5	1.09	0.349	0.3782
Average	1.18	0.326	0.3782
Standard Deviation	0.113	0.057	-

Appendix B

Sample Calculations

Deriving the Number of Particles per Volume of Bed

Volume of one particle: $$\boldsymbol{V_p = \frac{4}{3}\pi\left(\frac{d_p}{2}\right)^3}$$

$$\boldsymbol{V_p = \frac{d_p^3\pi}{6}}$$

Where: d_p = particle diameter

V_p = particle volume

Total Volume of particles in the bed: $$\boldsymbol{V_{bed} = N_p\frac{[\frac{d_p^3\pi}{6}]}{(1-\varepsilon)}}$$

Where: V_{bed} = total volume of the bed

ε = porosity (empty volume fraction)

Total Number of particles in the bed: $$\boldsymbol{N_p = \frac{6V_{bed}(1-\varepsilon)}{d_p^3\pi}}$$

Deriving the Pressure Drop for a Single Particle in a bed for Bed for Modified Ergun's Equation

Force Balance: $$\boldsymbol{F_{plate} = F_g - F_B - F_D}$$

Where: F_{plate} = total force on plate

F_g = force on particle due to gravity

F_B = buoyancy force acting on particle

F_D = drag force acting on particle due to motion through the medium

Assuming: *$F_{plate} = 0$*

$$\boldsymbol{F_g - F_B = (\rho_p - \rho)(1-\varepsilon)AL(\frac{g}{g_c})}$$

$F_D = (P_1 - P_2)A$

Where: ρ_p *= density of the particle*

ρ *= density of the medium*

g = 32.174 ft/s^2

g_c *= 32.174* lb_m**ft /*lb_f */s*^{2}

$P_1 - P_2$ *= change in pressure in bed*

A = cross sectional area of bed

L = length of the bed

$$A(P_1 - P_2) = (\rho_p - \rho)(1 - \varepsilon)AL\left(\frac{g}{g_c}\right) - F_{plate}$$

$$(P_1 - P_2) = (\rho_p - \rho)(1 - \varepsilon)L(\frac{g}{g_c})$$

Deriving the Total Pressure Drop Experienced by the Bed due to Particle Motion

$$(P_1 - P_2)_{total} = (\rho_p - \rho)(1 - \varepsilon)L(\frac{g}{g_c})Np$$

$$(P_1 - P_2)_{total} = (\rho_p - \rho)(1 - \varepsilon)L\left(\frac{g}{g_c}\right)[\frac{6AL(1-\varepsilon)}{d_p^3\pi}]$$

Deriving the Equation for the Particle Velocity Through a Bed According to the Modified Ergun's Equation

General Modified Ergun's Equation: $$\frac{\Delta P}{L} = \left[\frac{150V_o\mu}{d_p^2\Phi^2}\right]\left[\frac{(1-\varepsilon)^2}{\varepsilon^3}\right] + \left[\frac{1.75\rho g V_o^2}{d_p\Phi}\right]\left[\frac{(1-\varepsilon)}{\varepsilon^3}\right]$$

Where: μ *= viscosity of the fluid*

V_o *= {V-U}*

V = velocity of the fluid

U = velocity of the particle

Φ *= sphericity of the particle (assumed to be 1)*

Replacing V_o with {V-U}

$$\frac{\Delta P}{L} = \left[\frac{150\{V-U\}\mu}{d_p^2\Phi^2}\right]\left[\frac{(1-\varepsilon)^2}{\varepsilon^3}\right] + \left[\frac{1.75\rho g\{V-U\}^2}{d_p\Phi}\right]\left[\frac{(1-\varepsilon)}{\varepsilon^3}\right]$$

Applying the Quadratic Equation to Solve for U

$$\{V-U\} = \frac{-b \pm\sqrt{b^2-4ac}}{2a}$$

Where: $a = \left[\frac{1.75\rho g}{d_p\Phi}\right]\left[\frac{(1-\varepsilon)}{\varepsilon^3}\right]$

$b = \left[\frac{150\mu}{d_p^2\Phi^2}\right]\left[\frac{(1-\varepsilon)^2}{\varepsilon^3}\right]$

$c = -\frac{\Delta P}{L}$

$$U = V + \frac{42.9\mu(1-\varepsilon)}{d_p\Phi\rho g} + \frac{d_p\varepsilon^3\Phi\sqrt{\left(\frac{150\mu(1-\varepsilon)^2}{d_p^2\Phi^2\varepsilon^3}\right)^2 + \left(\frac{7\rho g(1-\varepsilon)}{d_p\varepsilon^3\Phi}\right)\left(\frac{\Delta P}{L}\right)}}{3.5pg(1-\varepsilon)}$$

<u>*Solving for the Settling Velocity of a Particle According to Hindered Settling Correlations*</u>

Stoke's Law Range (K < 2.6)

$$U_t^o = \frac{d_p^2 g(\rho_p-\rho)}{18\mu^o}$$

Where: $U_t^{\,o}$ *= bulk terminal velocity of the particles*

$\varepsilon_c = \varepsilon_{loose} = 0.4242$ *assuming a sphericity of 1*

Such that: $\mu^o = \mu[1-\left(\frac{(1-\varepsilon)}{(1-\varepsilon_c)}\right)]^{-2.5}$ *for dense-packed slurries*

Newton's Law Range (68.9<K<2360)

Where: $U_t^o = 1.75\sqrt{\frac{g d_p(\rho_p-p^o)}{\rho^o}}$

Such That: $p^o = (1-\varepsilon)\rho_p + \varepsilon\rho$