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The Effects and Motion of an Air-Filled Cylinder being Submerged in Water in the Axial Direction

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The Effects and Motion of an Air-Filled Cylinder being Submerged in Water in the Axial Direction

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Executive Summary

Submarines operate on the simple principle of a difference the relative density of the submarine itself and the density of the fluid in which it operates in. Today's technologically advanced submarines operate by increasing their density by allowing seawater to rush into the ballast tanks of the vessel and fill until the desired buoyancy is achieved and the submarine will either rise or sink. If the submarine is desired to stay at a certain depth under water, a neutral buoyancy is required where the vessel will neither sink nor rise. To rise, a submarine will supply compressed air to the same ballast tanks forcing water out, resulting in the loss volume being replaced. By doing so, the relative density of the sub is decreased until its magnitude is less than that of the fluid surrounding, causing it to rise. While the fundamental principles have been examined for submarines, the same principles have not been widely applied to other facets of motion in fluid, namely water. While the forces associated with buoyancy have been analyzed and derived in the past, few works focus on objects that change the magnitude of their buoyancy force as the depth at which they are submerged is increased. Many objects could be modeled for the purposes of this study, but a cylinder was chosen because it represents a common geometry that poses no new challenges to manufacture and can simplify derivations due to the fact that information in pertaining to drag coefficients is readily available. The purpose of this work is to describe a relationship between the depth at which a cylinder, filled with air and open end downward, is submerged and the direct forces acting upon it, primarily the forces of gravity, buoyancy, and drag (during motion). The study focuses on two scenarios of a cylinder being submerged; one being in a large body of water and rising and the second being in a tube that has a diameter closely in value to that of the cylinder to model the effects of wall forces acting on the

cylinder during motion as it travels deeper into water. The second of the scenarios aimed to demonstrate the effects of the viscous drag force between the wall and cylinder.

It was found that when modeling the effects of gas compression and how the depth of neutral buoyancy could be determined, that the geometry had a large effect on the neutral buoyancy depth, in particular, the diameter of the cylinder. As the diameter of the cylinder was increased, this had a large effect on the gravity term because the thickest part of the cylinder increases in size, but the volume of compressed air also increases and does so to the squared of the radius term. This large increase in moles of air proved to highly increase the buoyancy force since the gas being compressed was in much large volume and the displaced volume was larger over larger periods of time. It was observed that when the length over diameter (L/D) ratio was less than unity, this effect was substantial. When the L/D was greater than unity and the diameter was held constant while length increased, the effects of changing neutral buoyancy depth were minimal.

Once neutral buoyancy parameters were established, the velocity was modeled as a function of time and depth for cylinder released at a point where the buoyancy forces are greater than the gravity forces and the cylinder rises. Similar results were observed where L/D values less than one shows long time periods for the cylinder to reach the surface because the drag acting on the closed end of the cylinder dominates in the force balance. Large L/D values where able to reach increasingly larger velocities near the surface. This analysis was completed using incremental analysis in which velocity was assumed constant over very small time periods ($1/10^{th}$ seconds).

Lastly, if this was to be applied in reality, to keep the cylinder in its axial position, a tube of similar diameter would need to be placed in the body of water to ensure that the cylinder moves along its axial direction. When doing so, the dominant drag force is no longer by means of the perpendicular surface, but by means of the viscous drag occurring between the wall of the two cylinders. This drag force was modeled as a relationship with the constant linear shear force acting between the walls, within the fluid as a function of viscosity. This analysis resulted in near terminal velocity limits being reached at long time periods and large depths as the cylinder was allowed to free fall through the tube. By performing a fitting on the curves, quantitative relationships were made to model the velocity as a function of time and depth for these cases were obtained.

The work presented within this project allowed myself to deviation from the normal team-based work and learn to better adapt to individual work and enhance my personal problem-solving abilities. I feel as if by completing this work my understanding of compression of gases at depths has vastly increased and I feel comfortable solving equations not at steady state (such as deriving the velocity profiles for a rising cylinder).

Future work should focus on designing experiments to help prove the fundamental relationships demonstrated here. After initial geometries were determined, it was found that the calculated depths were too large to test, feasibly. Future work should decease cylinder geometry and test both predictions for the velocity as a function of time and depth. The drag coefficient should also be experimented as for the purposes of this study, the sides of the cylinder were not accounted for when in large body motion. As the length of the cylinder increases, even in large body motion, the friction force acting on the sides of the cylinder will be apparent and could effect the value of the drag coefficient.

Introduction

To model the effects of an air-filled, open-ended cylinder being submerged in water, the fundamental operating basis for submarines was analyzed to investigate the similarity. Submarines rely on the ability to displace water with compressed air to decrease the relative density of the vessel to allow it to raise to the surface. Submarines also rely on the ability to flush in large amounts of seawater into ballasts tanks to increase the relative density of the vessel in order to overcome the density of the surrounding fluid, in magnitude. Prior works, in the majority, do not analyze geometries that have open-ended faces where a specific amount of air can be trapped as it is submerged. This work is important because it focuses solely on an object that's buoyancy force is changing. While the scope of the study started with determining the point of neutral buoyancy depending on a cylinder's geometry, it also included determining the velocity of the cylinder as it rises as both a function of time and position (depth). Two separate circumstances were evaluated, the first being a cylinder in a large body of water with a liquid level that is not affected by the position of the cylinder and the primarily force acting against the motion is the drag force. The second configuration that was evaluated was the same cylinder placed in a tube that has a diameter very close to that of the cylinder itself. By using a tube with a similar diameter, the viscous drag force between the cylinder and tube's wall becomes the dominate acting force on the cylinder. The second configuration is important to consider because in applications, a tube will most likely be needed to stabilize the cylinder as it rises through water to ensure that it moves only in the direction parallel to its central axis. If the tube is used in a large body of water, water is free to flow in and out of the tube as the cylinder is rising through the tube. If the tube is closed at one end and is filled only with a defined amount of water, the net water flow in and out of the tube is zero, and height of the liquid in the tube will change as the cylinder is submerged in the tube. The former configuration is within the main scope of this study.

Background

Submarines operate on the principle of a difference in density between the vessel itself and the fluid that it operates in. A submarine, cylinder, or any object having a distinctive mass, will have a force due to gravity (F_g). The force due to gravity is defined as the mass of the object (kg) multiplied acceleration due to gravity (9.807 m/s^s). When a force balance is analyzed, this force acts downward on a free-body diagram. When an object is submerged in a fluid, it also exhibits a force called the buoyancy force ($F_{Buoyancy}$). The buoyancy force acts directly opposite of the force due to gravity. The simplified force balance for an object not in motion can be shown to be

$$F_{net} = Mass * Acceleration = F_g - F_{Buoyancy} = 0$$

To establish neutral buoyancy, the force due to gravity and the force due to buoyancy must be equal and act in opposite directions. When this condition is satisfied, the object being analyzed will neither sink nor rise. The buoyancy force is demonstrated by Archimedes principle that defines the buoyancy force as

$$F_{Buoyancy} = Volume\ Displaced * Density_{Fluid} * Acceleration_{Gravity}$$

In previous works, the volume displaced is normally a constant because the object has a defined shape and volume as it is submerged. A simple example of this would be a cylinder that is closed on both ends with a defined volume that is submerged in water or another fluid. This study differs in that the buoyancy force of a cylinder with one open end that is submerged in water will change as a function of depth as the air (gas) originally in the cylinder will compress as the pressure at greater depths increases. As the pressure of the gas inside the cylinder increases due to the increasing pressure at the water and air interface, the water level inside the cylinder rises, decreasing the volume of air in the cylinder. As the bottom of the cylinder fills with the operating fluid (water), the buoyancy force continues to decrease as the displaced volume decreases. The volume of the air within the cylinder as its depth increases can be modeled with the Ideal Gas Law as

$$V = \frac{nRT}{P}$$

Where P is the pressure within the cylinder, V is the volume, n is the number of moles of air within the cylinder, R is the gas constant, and T is the temperature of the air. The number of moles is considered a constant and is ultimately determined by the geometry of the cylinder and equates to the number of moles calculated using the ideal gas law at 1 atmosphere (atm) and in the determined volume of the cylinder at 298 Kelvin. At pressures greater than 5 bar absolute, a compressibility term is introduced into the ideal gas law equation to correct for compressibility at higher pressures. The pressure term shown in the above equation can be modeled at the monometer equation

$$P_2 - P_1 = pgh$$

where P_2 represents the pressure at the water and air inside the cylinder interface, p is the density of the fluid, g is the acceleration due to gravity, and h is the difference in height between points 1 and 2. Point 1 represents the surface of the large liquid where the pressure is equal to atmospheric. This principle is valid because it can be generally shown that the pressure at the interface is equal to the pressure of the surrounding fluid outside of the cylinder at identical depths. For the purposes of this study, the force due to gravity only considers the mass of the cylinder itself as the mass of the air inside the cylinder is negligible as the density of the air is vastly less than that of the cylinder itself (aluminum). Therefore, the force due to gravity is shown as a constant value below.

$$F_g = Mass\ (kg) * Acceleration_{Gravity}$$

Therefore, the buoyancy force can be calculated as a function of depth submerged and is done by relating the ideal gas law equation and the buoyancy force formula from Archimedes principle. The gravity force, as stated, is a constant that is based on geometry of the cylinder when using non-polymer cylinders (i.e. the weight of metallic constructed cylinders leads to the neglect of the initial mass of air).

<u>Experimental Methods</u>

It is the expectation that in future work, due to the magnitude of the calculated depths for normal cylinders from this work, that experiments will be constructed that can evaluate the accuracy of the model presented. Therefore, the experimental methods outlined within this section describe the steps and procedures used to determine the neutral buoyancy depths of each cylinder as well as the velocity as a function of depth/time and depth as a function of time. Detailed calculations that are described here can be found in Appendix A. The first objective was to solve for the depth of neutral buoyancy given a certain geometry for a cylinder. To accomplish this a cylinder of diameter of 0.083 meters and a length of 0.1 meters (L/D = 1.205) with a wall thickness of 0.003 meters (3 mm) was used that is constructed out of Aluminum (Density = 2700 kg/m^3). This was the initial size cylinder used, but other geometries (namely different L/D cylinders) were also evaluated. A few assumptions were used in the calculation of the neutral buoyancy depth that are important to discuss. The first being that the air inside the cylinder remains isothermal as it is submerged. In reality, it is expected that as the air in the cylinder is compressed, the temperature will increase, but after results were obtained, it was found that neutral buoyancy was not achieved at pressures large than 10 bar at the air/water interface. For the purposes of simplifying the calculations, the air temperature was assumed constant. Another assumption that was made was that the water temperature also remained constant at all depths, subsequently meaning that the density of the water remained constant over depth. In a controlled experiment, the water temperature can be maintained through heating or cooling sources to a specified temperature and therefore, the assumption of constant fluid density is applicable.

After fluid properties were obtained for water and air, an initial amount of air was determined to be inside the cylinder before it is submerged; using the ideal gas law at 298 K and pressure equal to atmospheric. Once the initial amount of moles of air was determined, this mass amount of air remained constant and as stated before, did not contribute to the gravity term in the force balance, considerably. Values for depth (defined as the submerged distance from the liquid surface to the water/air interface inside the cylinder) were estimated and based on these values, a pressure was determined inside the cylinder. Using the calculated pressure, the ideal gas law in combination with the effects of compressibility, "Z", were utilized and a new air volume was calculated (isothermal compression of an ideal gas). The total volume displaced by the cylinder itself is a known value and therefore, the calculated air volume can be added to the initial cylinder volume to determine the total displaced volume of the air/cylinder combination. This value is inputted into the buoyancy force equation and calculated at an arrangement of depths. The force due to gravity (acting downwards and constant) is then subtracted from the buoyancy force (acting upwards as a function of depth) until at a certain depth, the forces balance and the force balance is equal to zero. At this point, the system is at equilibrium and there is no acceleration in either direction, vertically. This neutral buoyancy depth was calculated for several cylinders (L/D = 1.205, 0.602, and 2.410), all constructed of aluminum (Density = 2700 kg/m^3).

For each of these instances, after a depth was determined, a point was chosen just before the neutral depth was reached for the evaluation of velocity.

The second objective of the study was to determine if the cylinder is released at a depth that allows the cylinder to rise (buoyancy force is greater than the force due to gravity), the velocity as both a function of time and depth. Also, for comparison, the depth as a function of time was also needed to see the effect of changing cylinder geometry. To determine these quantities, a more elaborate approach was needed. A time dependent force balance was required since the cylinder will rise as the buoyancy force (a function of depth) is increasing. The time dependent force balance used was modeled as

$$m\frac{dv}{dt} = F_{Buoyancy} - F_g - F_{Drag}$$

At this step, a force due to drag was introduced into the force balance because when the object is in motion, a force resulting from the viscous drag of the cylinder accelerating through the medium is not negligible. The drag force is defined as

$$F_{Drag} = \frac{1}{2}C_d p v^2 A$$

Where p is the fluid density, v is the velocity of the cylinder, A is the area that experiences the drag force, and C_d is the drag coefficient for the given geometry at a given Reynolds number. According to findings in *Unit Operation of Chemical Engineering* by Warren L. McCabe, it discusses and gives correlations for specific shapes as a function of the Reynolds number. It is shown in Figure 7.3 (shown below) that since the length of the cylinder is small, the drag on the sides of the cylinder is negligible (McCabe, 2005).

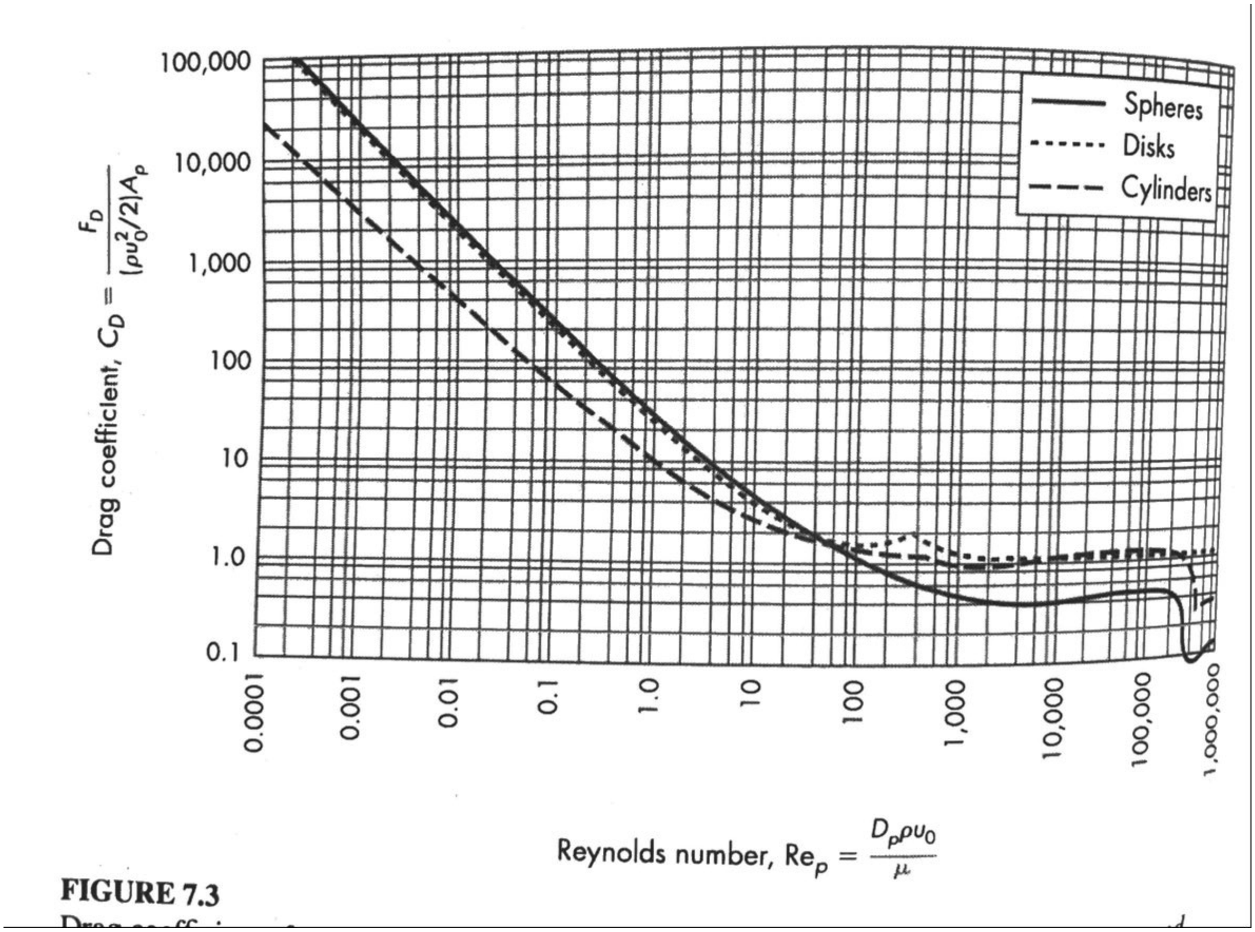

Figure 1: Shows the correlation for drag coefficient as a function of Reynolds number for spheres, disks, and cylinders.

This translates to the fact that the drag coefficient will be the same as for a disk with the flat surface of the disk as the projected area. The figure (7.3) shows the disk and cylinder correlations are almost the same. Hence, for low Re (Re<1), you can model the drag coefficient as Cd=24/Re. What was found in this study is that the Reynolds in nearly all circumstances was above 1000, leading to the assumption that at higher Reynolds numbers, the drag coefficient approaches a constant value of $C_d = 1.0$. This value of the drag coefficient was used for all proceeding calculations and was assumed constant as all Reynolds numbers exceeded 1000.

To solve for the time dependent force balance differential, the method of incremental analysis was used to model the velocity as it changed over time. It was determined that a reasonable approach to solving the objective was to assume constant velocity over very small time periods (1/10th of a second). By using guessed velocity values for each 0.1 second intervals, and then relating the velocity to the new depth at the next interval, iterations were completed to determine to accurate velocity. By equating both sides of the force balance differential and adjusting velocity until both sides equate was the method used to calculate velocity over each time period. Using this velocity, the change in depth over that timeframe of 0.1 seconds was also determined and that became the starting depth for the next interval. This process was repeated until a depth of 0 meters was reached. This analysis was completed for 4 different cylinder geometries (L/D = 1.205, 0.602, 2.410 and 4.819) to compare velocities.

The final objective of the study was to determine the effects do the viscous drag on a cylindrical object as it travels down a tube with a diameter close to the diameter of the cylinder. This was important because it most likely closely approximates to the applications used as it will allow for velocity in the axial direction and will not allow the cylinder to rotate. If the cylinder is to rotate, the drag coefficient will change and no longer be constant, further altering the velocity profile. From *Unit Operation of Chemical Engineering,* the drag force due to shear from the thin layer of liquid between the two-cylinder walls is expressed as (McCabe, 2005)

$$F_{Drag} = \tau A = \mu(\frac{v}{\Delta R})(2\pi r l)$$

Where the shear force is the dominant force for drag within the system and can be represented by multiplying the viscosity by the velocity divided by the difference in the radii of the two cylinders. And then multiplying this value by the area on which this shear takes place (the outer area of the moving cylinder). This takes into account a linear relationship of the shear rate and the wall of non-moving cylinder (tube). Using the same incremental analysis, the drag force is used for the friction term and all others are done as before to account for compression of the gas.

Data and Results

The detailed calculation results can be shown in Tables in Appendix B. The first objective, determining the neutral buoyancy depth for each cylinder, was completed and the results are shown in Figure 2.

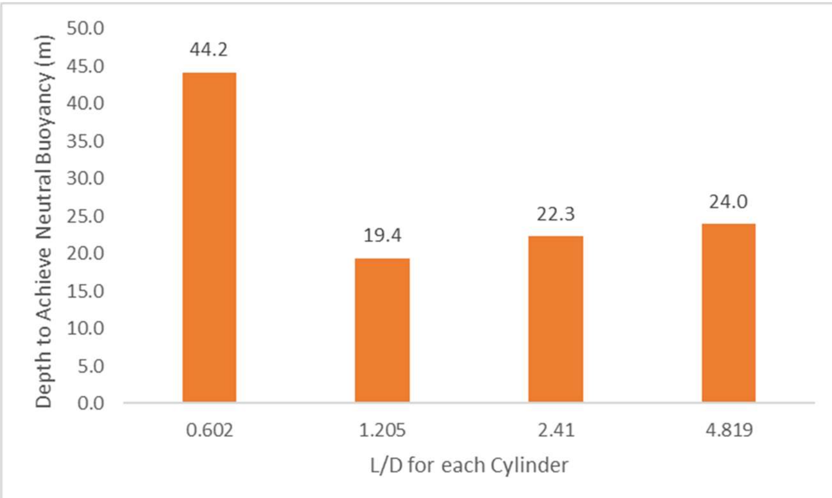

Figure 2: Shows the calculated neutral buoyancy depth for various cylinders constructed of aluminum with a density = 2700 kg/m³.

Another important characteristic that was considered was how the calculated volume changed as the depth of the cylinder changed. The calculated results from that analysis are shown in Figure 3.

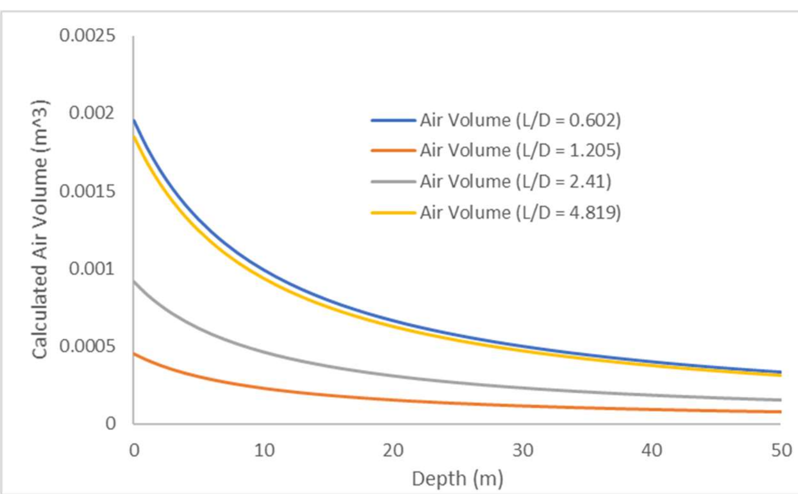

Figure 3: Shows the Isothermal compression of air as an Ideal Gas while taking compressibility into account at Pressures exceeding 5 bar absolute.

After neutral buoyancy was determined for each of the cylinder, the cylinder was raised in depth roughly 1 meter to a point in which the force of buoyancy is greater than the force due to gravity and forces the cylinder to rise. As the cylinder rises, the buoyancy force continues to increase since the depth is decreasing (pressure is decreasing). All four cylinder's velocities were plotted with respect to time, and the depth as a function of time was also plotted. The results of the incremental analysis used to compute these relationships are shown in Figure 4 below.

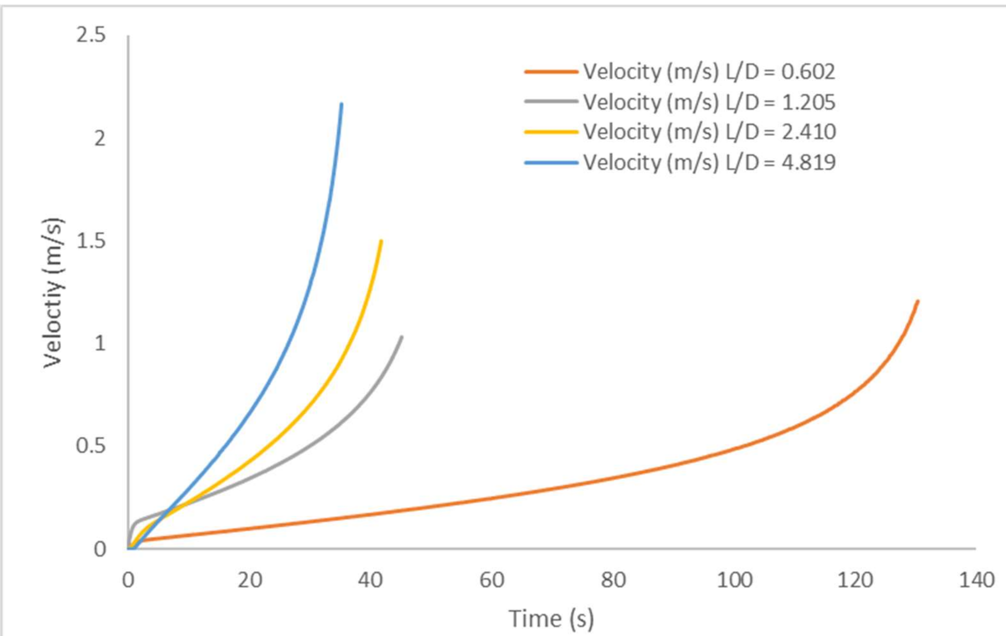

Figure 4: Shows the velocity versus time comparison for 4 cylinder geometries. Values were calculated using incremental analysis and a balance of the force over small time differences using constant velocity.

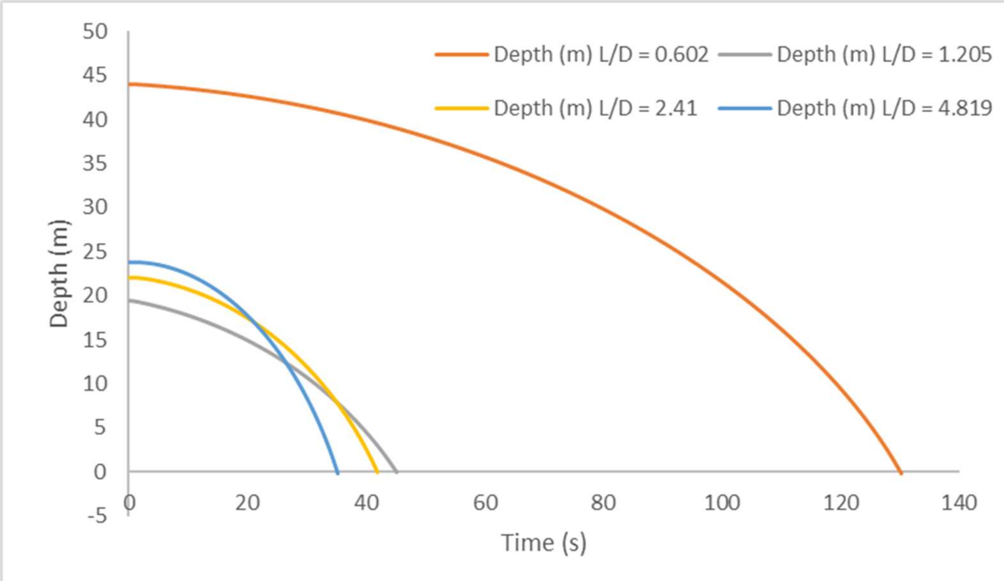

Figure 5: Shows the relationship for the cylinders rising condition as depth as a function of time.

Once the velocities as a function of time and depth as a function of time were determined and plotted, it was then important to shows the velocities of the cylinders as a function of depth itself. The results for the four cylinders are shown in Figure 6 below.

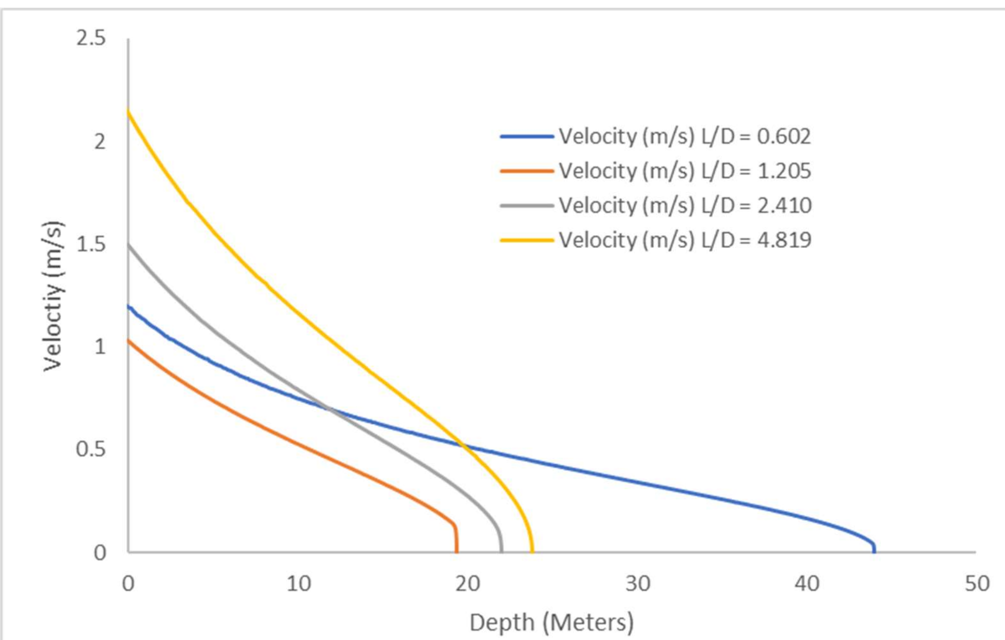

Figure 6: Shows the velocity of each cylinder as a function of depth. The calculated values shown here are the main objective of the project/study.

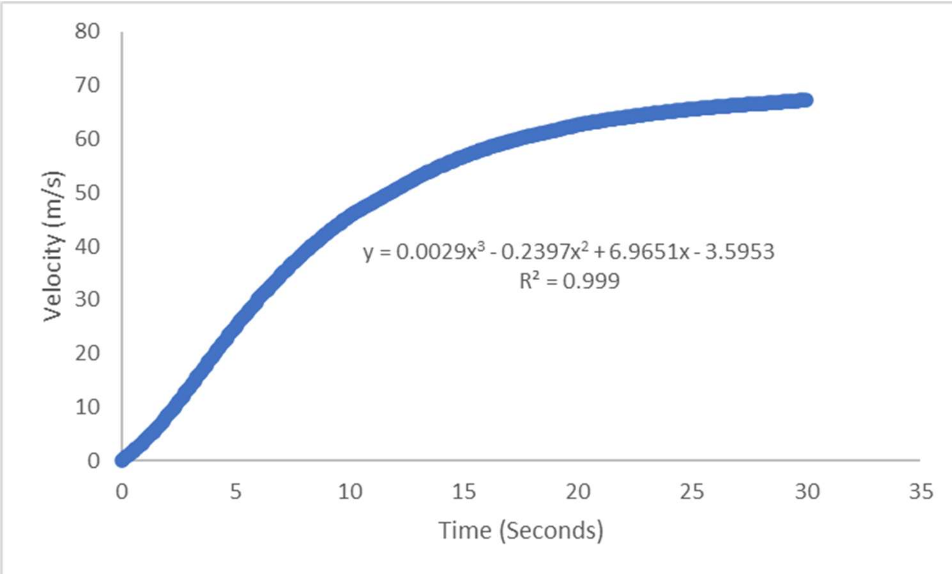

Figure 7 Shows the velocity as a function of time for a cylinder modeled by the experiment explained in the experimental methods section (glass cylinder). The fitted third order polynomial equation as well as the R^2 value for the fittings is shown as well.

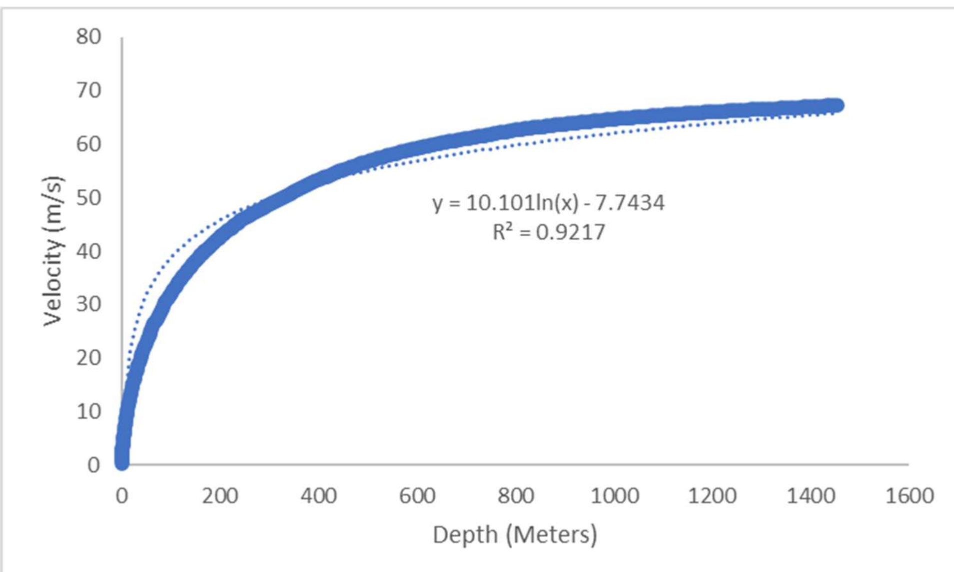

Figure 8 shows the velocity as a function of depth for the same cylinder system shown in Figure 7. A logarithmic fittings was completed that bets fit the data and is shown as well.

Discussion/Conclusions

The theoretical experimental results demonstrated a few significant findings about the buoyancy of open-end objects as well as about the velocity of the objects as they are released from a point in which they will rise. For the first objective (determining the depth of neutral buoyancy), the four different L/D cylinder values were analyzed and it was found that if the L/D value was significantly less than unity, than the neutral buoyancy depth was the greatest. This is due to the fact that as the L/D decreased, the diameter increased. Due to the fact that the volume of a cylinder takes into account the squared term of the radius, the doubling of the radius actually increased potential occupied space by a multiple of 4. This had the effect of quickly raising the volume of air inside the cylinder at atmospheric conditions before it was submerged. Because of this increased volume, the buoyancy force was dominant over the force due to gravity and therefore it required a larger depth to submerge. It is also worth noting that in Figure 2, that after the diameter was doubled, the subsequent trials (3 & 4) occurred with a length that was continually being increased. As diameter is held constant, and the length of the cylinder was increased, the depth to neutral buoyancy did increase. This is to be expected because the increased volume of air outweighs the effects of the increased mass that contributes to the force due to gravity. This alone demonstrates that the force of buoyancy is directly and primarily related the ratio of length of diameter for values greater than unity. It is also shown that the changing of the diameter will have the largest impact on the neutral buoyancy depth.

The second objective of the study was to determine the velocity profiles of the cylinders as they were released from a point in which they would rise, not sink. The main challenges that existed with this calculation where the lack of constant values during the analysis. To simplify the calculations, it was required to perform incremental analysis at very small-time ranges with the assumption that during these very small-time ranges, that velocity was constant (acceleration equal to zero). By doing so, a guess value was used and compared to the calculated value to determine the correct velocity that minimized the squared error between the two sides of the force balance differential. A solver model in Microsoft Excel was used to minimize the differences between each side of the equation by manipulating velocities. Upon the analysis of the four geometries, it was found that at L/D values less than unity (namely the trial at 0.602), the velocity was highly impacted by the larger diameter cylinder because the drag force is directly proportional to the projected area that experiences the flow field. Figures 3 and 4 show prove that the velocity of the cylinder as it rises is high dependent upon the projected area that experiences drag (the drag force) and not as much so dependent upon the buoyancy force from the compressed gas. Figure 3 shows that for L/D = 0.602 and L/D = 4.819, the calculated air volume (proportional to the amount of buoyancy force acting on the object), were nearly identical. The buoyancy force is proportional to displaced volume and therefore, the effect of the drag due to the large area drastically decreases velocity over the range that the cylinder is rising. It is important to emphasize again that using incremental analysis, the Reynold’s number was evaluated over very small time period and it was possible to determine the Reynold’s number at very small amount of time after the cylinder was “released”. The Reynold’s number remained greater than 1000, indicating from literature, that the drag coefficient, for all velocities, could be estimated as unity. This was a general assumption that was made and further research into the affects of the open end of the cylinder should take place. If a better understanding of how the drag coefficient can be represented for a cylindrical object that has an open end, then the drag coefficient can be better approximated. Future work on the basis of the velocity of a cylinder as it rising should be completed and conduct an experiment to test the validity of the findings made here. If such finds follow the trends explained, correction factors or parameters of the governing equations should be adjusted to fit the experimental data (primarily taking further into account compressibility and non-isothermal compression).

The third and final objective of the study was to analyze the velocity of a cylinder in near steady motion that is falling due to the fact that the force due to gravity is out-weighing the force of the buoyancy in all circumstances. Figure 7 shows the velocity as a function time for the application when the cylinder is falling side another cylinder. It was found that for this model, a third order polynomial fit was most appropriate. A limit near 65 m/s was reached at long times of free falling through the tube, indicating a limit to where the terminal velocity begins to be reached and the gravity force and the force due to the viscous drag between the cylinder and the tube wall equate. The primarily plot was to be off velocity as a function of depth to indicate the acceleration over time throughout the sinking of the cylinder. This is shown in Figure 8 and nearly follows a logarithmic plot with a summed squared error (SSE) equaling 0.253, signifying an exceptional fit. Some error does manifest in the depth range from 20 – 220 meters, but its magnitude is small and, in this range, there are not many practical applications that can be demonstrated by means of robust experiments. Future work for this objective should include experimental procedures that test the derived logarithmic and polynomial relationships for the velocity as a function of depth and time, respectively.

By completing this study, the neutral buoyancy of open-ended cylinder was determined for multiple geometries and it was found the if the length is held constant, the increasing of the diameter has a large impact on the neutral buoyancy depth and for L/D values less than 1, as the diameter is further increased, the depth will increase rapidly. By holding diameter constant, and increasing length, it only increased depth by a small amount and shows that a long cylinder displaces less volume and the compressive forces are larger for the larger diameter. After establishing neutral buoyancy, the velocity of these cylinders was required to understand how the drag force effects a cylinder rising due to the difference between buoyancy and gravity forces. It was found that by using incremental analysis, even at very small time periods after the cylinder is released, the Reynolds number was larger than 1000, indicating that the drag coefficient could be modeled as a disk/sphere since its length is short and its projected area is identical and is the main factor for determining the coefficient of drag. It was found that for large Reynolds numbers the drag coefficient approaches unity and that was used for the calculations. Similar to the neutral buoyancy calculations, it was found that the diameter of the cylinder (small L/D values) resulted in longer times to reach the surface and for the majority of the time, lesser velocities. When the L/D was less than unity, it resulted in a near constant velocity for a long period of time over the majority of the motion. This shows that for L/D less than one, the acceleration will approach zero (the force balance will approach zero). For L/D values the velocity reaches a point as a function of time where the buoyancy force far outweighs the gravity force and the cylinder accelerates rapidly to the surface. It was found that as the L/D was increased (diameter held constant and length increased) that the maximum velocity that is reach near the surface increased drastically. This was due to the fact that the main area for drag was always considered the full closed face of the cylinder in all cases and by increasing length, the compression forces do not overcome the gravity forces drastically. Finally, the velocity as a function of time and depth of the cylinder when submerged inside a tube in a large body of water was analyzed. It was found that the velocity over time fit a cubic function nicely and a upper limit reached the terminal velocity was evident as the cylinder fell in the fluid. When the velocity was modeled with respect to depth, a logarithmic trend results and again, an upper limit was established.

Literature Cited

McCabe, W. L. (2005). *Unit Operations of Chemical Engineering.* New York: McGraw-Hill Education.

Appendix A

Detailed Calculations for Modeling Purposes

Calculating Moles Initially of Air in Cylinders

$$n = \frac{PV}{RT} = (1\ atm) * (0.0018487\ m^3) * \frac{1000 \frac{L}{1m^3}}{\left(0.08206 \frac{L * atm}{mol * K}\right) * 298K} = 0.07560\ moles\ air$$

Setting up Volume as a function of Depth

$$V_{cylinder} = \pi r^2 * Thickness + (2 * \pi * r) * (Length - Thickness) * (Thickness)$$

$$= 3.2679 * 10^{-4} m^3$$

$$V = \frac{nRT}{P}, \qquad P_2 - P_1 = \rho gh, \qquad P_2 = \rho gh + P_1, \qquad V = \frac{nRT}{\rho gh + P_1}$$

$$V = \frac{0.07560\ moles\ air * 0.08206\ \left(\frac{L * atm}{mol * K}\right) * 298K * \left(\frac{1m^3}{1000L}\right) * \left(101325 \frac{Pa}{1atm}\right)}{\left(997.96 \frac{kg}{m^3}\right) * 9.807 \frac{m}{s^2} * Height\ (m) + 101325\ Pa} + 3.2679 * 10^{-4} m^3$$

$$V = 3.2679 * 10^{-4} + \left(\frac{187.321}{9786.99 * h + 101325}\right)$$

Note: Compressibility added into the equation when pressures exceeded 5 bar.

Force Differential Used for Modeling Velocities

$$m * \left(\frac{dv}{dt}\right) = mg - \left(0.5 * \rho * v^2 * C_d * A_p\right) - (\rho * V * g) \qquad Used\ for\ Rising\ Cylinder$$

$$m * \left(\frac{dv}{dt}\right) = mg - \left(\mu * \left(\frac{v}{\Delta R}\right)\right) - (\rho * V * g) \qquad Used\ for\ Falling\ Cylinder\ Cylinder\ (Linear\ Shear\ for\ Friction\ term)$$

Appendix B

Data for presented results

Table 1: Neutral Buoyancy data for L/D =0.602.

Depth (m)	Balance (Al)
0	15.48022634
1	13.79798482
2	12.38810435
3	11.18939406
41	0.227048418
42	0.153597961
43	0.082900879
44	0.014805202
45	-0.050830062

Table 2: Shows the velocity as a function of time and depth.

Time (s)	Depth (m)	Velocity (m/s)
0	44	0
0.1	43.999668	0.003318206
0.2	43.99901	0.006586432
0.3	43.99803	0.009795556
0.4	43.996743	0.012865378
0.5	43.995162	0.01581239
0.6	43.993303	0.018592214
0.7	43.991182	0.021211552
0.8	43.988817	0.023646794
0.9	43.986225	0.025917442
1	43.983425	0.028001936
129.8	0.4745694	1.164251284
129.9	0.3572781	1.172913118
130	0.2386541	1.186240507
130.1	0.1198131	1.188409449
130.2	0.0002507	1.195624002
130.3	-0.120155	1.204053354

Table 3: Neutral Buoyancy data for L/D = 1.205.

Depth (m)	Balance (Al)
0	2.880986128
1	2.49179566
2	2.165616549
3	1.88829215
4	1.649611086
5	1.442022402
18	0.075895381
19	0.020930051
20	-0.030413542

Table 4: Shows velocity as a function of time and depth for L/D = 1.205.

Time (s)	Depth (m)	Velocity (m/s)
0	19.407647	0
0.1	19.405833	0.01813438
0.2	19.402299	0.035343604
0.3	19.397182	0.051174838
0.4	19.390647	0.065343393
0.5	19.382872	0.077753985
0.6	19.374029	0.0884267
0.7	19.364281	0.09748143
0.8	19.353772	0.105085189
0.9	19.342629	0.111434812
1	19.330957	0.116721849
44.7	0.3875844	1.002222219
44.8	0.2866576	1.009268431
44.9	0.1850129	1.016446996
45	0.0826351	1.023777877
45.1	-0.020475	1.031102726

Table 5: Neutral buoyancy data for L/D = 2.410.

Depth (m)	Balance (Al)
0	6.131651237
1	5.341124208
2	4.678586391
3	4.115282684
4	3.630471652
5	3.208815732
20	0.21799117
21	0.120354221
22	0.028752994
23	-0.057355404

Table 6: Shows velocity as a function of time and depth for L/D = 2.410.

Time (s)	Depth (m)	Velocity (m/s)
0	22	0
0.1	21.999719	0.002809916
0.2	21.999242	0.004771528
0.3	21.998625	0.006167795
0.4	21.997902	0.007232894
0.5	21.99708	0.008213283
0.6	21.996048	0.01032416
0.7	21.994528	0.015201544
0.8	21.992541	0.019871259
0.9	21.990076	0.02465174
1	21.987143	0.029328673
41.5	0.5815964	1.436655404
41.6	0.436493	1.451034322
41.7	0.2899122	1.465807325
41.8	0.1418221	1.480901052
41.9	-0.007853	1.496756063

Table 7: Neutral buoyancy data for L/D =4.819.

Depth (m)	Balance (Al)
0	12.63933741
1	11.04563492
2	9.709958681
3	8.574338406
4	7.596959366
5	6.74690103
22	0.335894552
23	0.162300022
24	-0.001188003

Table 8: Shows velocity as a function of time and depth for L/D = 4.819.

Time (s)	Depth (m)	Velocity (m/s)
0	23.8	0
0.1	23.799848	0.001519494
0.2	23.799662	0.001864795
0.3	23.799455	0.002067716
0.4	23.799239	0.002154343
0.5	23.799024	0.002150424
0.6	23.798816	0.002081582
0.7	23.798619	0.001973481
0.8	23.798434	0.001852127
0.9	23.798259	0.001744004
1	23.79782	0.004391372
34.8	0.6719093	2.045894929
34.9	0.4644649	2.074444631
35	0.2540442	2.104206278
35.1	0.0405789	2.134653133
35.2	-0.176152	2.167312983

Table 9: Shows data for when the drag force due to wall effects was introduced, changing the way the drag force was calculated and how it affected the velocity.

Time (s)	Depth (m)	Velocity (m/s)
0	0	0
0.1	0.034109	0.34109037
0.2	0.102258	0.68149455
0.3	0.204575	1.02316627
0.4	0.344383	1.39807553
0.5	0.518658	1.74275009
0.6	0.730414	2.1175618
0.7	0.98051	2.50096517
0.8	1.269062	2.88551509
0.9	1.597119	3.28057317
1	1.965631	3.68511718
22.1	933.1489	64.0700387
22.2	939.562	64.1301083
22.3	945.9809	64.1894332
22.4	952.4057	64.248026
22.5	958.8363	64.3058975
22.6	965.2726	64.3630626
22.7	971.7145	64.4195309
22.8	978.1621	64.4753157
22.9	984.6151	64.5304285
23	991.0736	64.5848779
23.1	997.5375	64.6386776
23.2	1004.007	64.6918348
23.3	1010.481	64.7443602
23.4	1016.961	64.796265
23.5	1023.445	64.8475523
23.6	1029.935	64.8982339