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The Experimental Determination of the Moment of Inertia of a Model Airplane

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THE EXPERIMENTAL DETERMINATION OF A MODEL AIRPLANE

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Honors Project

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CHAPTER 1: INTRODUCTION

1.1 Defining the Moment of Inertia

The moment of inertia is a quantity that expresses a body's tendency to resist angular acceleration from torque about a specified axis. It is the sum of the mass of each particle in the body with the square of its distance from the axis of rotation.

1.2 The Importance of an Airplane's Moment of Inertia

In flight, the control surfaces of an aircraft produce aerodynamic forces. These forces are applied at the center of pressure of the control surfaces which are some distance from the aircraft center of gravity and produce torques (or moments) about the principal axes. The torques cause the aircraft to rotate. The ability to vary the amount of the force and the moment allows the pilot to maneuver or to trim the aircraft.

1.3 Analytically Determining an Object's Moment of inertia

The moment of inertia of any object having a shape that can be described by a mathematical formula such as a disk or solid rectangle can be easily calculated. It is when an object's shape or the shapes that make an object become irregular in which it is difficult to analytically determine said object's moment of inertia.

1.4 Experimentally Determining an Objects Moment of Inertia

For a body with a mathematically indescribable shape, the moment of inertia can be obtained via experiment using a pendulum which employs the relation between the period of oscillation and the moment of inertia of the suspended mass.

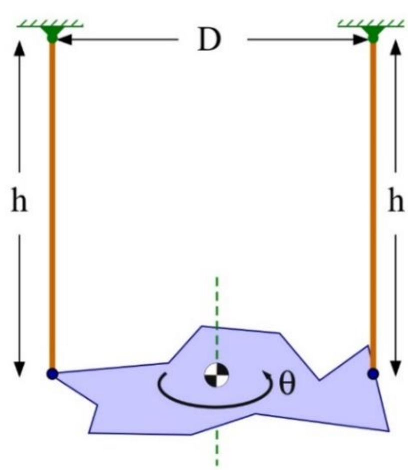


Figure 1: Pendulum configuration

To experimentally determine the moment of inertia, the apparatus must first be tested as a simple pendulum to ensure it is setup properly. To do this, gravity is experimentally estimated by swinging the object as a simple pendulum, measuring the period, and then using the equations below:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\omega = \sqrt{\frac{g}{h}} \rightarrow g = \omega^2 h = \frac{4\pi^2 h}{T^2}$$

If the experimental calculation for gravity is close enough to the known value, then the moment of inertia can be estimated using the equation below. To determine this, the period in the equation below results from rotating the object about the vertical axis rather than in a simple pendulum motion:

$$I = \frac{mgD^2T^2}{16\pi^2 h}$$

1.5 Project description

The objective of this project is to suggest that the moment of inertia for irregular shaped objects can easily be determined experimentally. The objects considered in this experiment were a solid wooden block and a model airplane. To ensure that the method and configuration is sufficient a wooden block with known moments of inertia was first tested, and the results from the experiment were compared to the known and accepted values. For this particular experiment, the periods were determined by timing how long 10 oscillations lasted for 10 different trials, and then taking the average of those 10 trials to best account for any outliers and human error. Following the wooden block test, the moment of inertia could then be determined of the model airplane as well as any other object using the same method.

Figure 3: Wooden block used for the experiment

Figure 2: Model airplane used for the experiment

CHAPTER 2: EQUIPMENT

- Two strings or wires of equal length
- Block of wood
- Model airplane
- iPhone for stopwatch and camera
- Ruler and measuring tape
- Electrical tape
- Screwdriver
- Screw eyes
- Scissors
- Binder clips

CHAPTER 3: PROCEDURE

1. Measure the mass, *m*, length, *a*, height, *b*, and depth, *c*, of the wooden block. Analytically calculate the moment of inertia of the wooden about the x, y, and z axes using known equations.
2. Configure pendulum with two strings or wires and suspend the block using the strings ensuring that the strings are parallel and of equal distance as shown in Figure 4. *Note: the block must be suspended about each axis.*

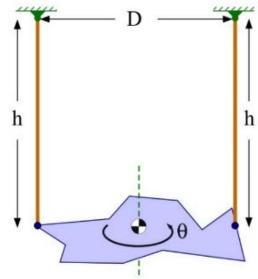


Figure 4: Pendulum configuration

3. Record measurements for *D*, the distance between wires, and *h*, the length of the wires.
4. To ensure that the pendulum is set up correctly a known value must be tested. For the purpose of this experiment, that value will be gravity. To test for gravity, swing the object in a pendulum motion starting from a small angle ($\theta \approx 5°$) as seen in Figure 5 and measure the period, *T*, the time it takes for the object to return to the starting position. One full back and forth swing is the period. To minimize human error 10 oscillations were measured for ten separate trials and the average period was calculated.

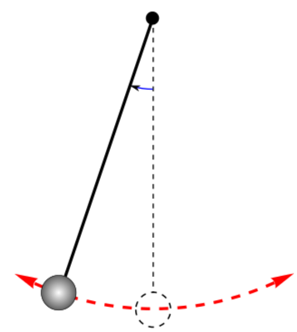


Figure 5: Swinging motion of the pendulum

5. To calculate gravity use the equation below:
$$g = \omega^2 h = \frac{4\pi^2 h}{T^2}$$
In this experiment inches were the primary unit used, therefore, the experimental gravity value should be near 386.40 in/s^2.
6. Once gravity is correctly calculated, the moment of inertia can be experimentally determined. Repeat step 3 to measure the period of the oscillations except rotate the object about the vertical axis as shown in Figure 6.

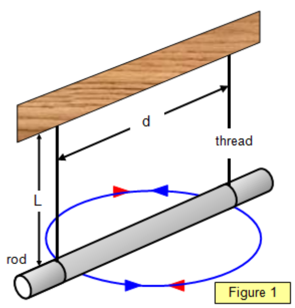


Figure 6: Rotation about the object's vertical axis

7. To determine the experimental values for moment of inertia use the equation below:
$$I = \frac{mgD^2T^2}{16\pi^2 h}$$
8. Compare the results from the experimental values to the accepted, analytical values. *Note: If deviation > 10%, something was likely done wrong.*
    a. Extremely well (less than 0.1% deviation)
    b. Very well (less than 1% deviation)
    c. Good (less than 10% deviation)
    d. Sort of (less than 50% deviation)
    e. Poor (less than 100% deviation)
9. Now that the comparison between the analytical and experimental values for the block's moments of inertia have confirmed that the configuration works, repeat steps 2-7, replacing the wooden block with the model airplane.

CHAPTER 4: DETERMINATION OF THE MOMENT OF INERTIA OF A SOLID WOODEN BLOCK

4.1 Analytical Determination

To calculate the moments of inertia about the solid rectangular block, the known equations below were used:

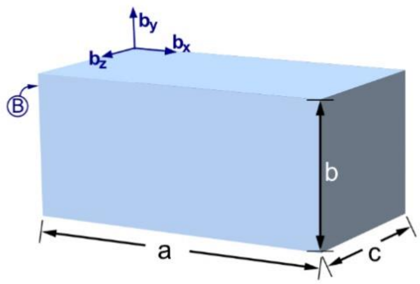


Figure 7: Axes and variables used for block

$$I_x = \frac{m(b^2 + c^2)}{12}$$

$$I_y = \frac{m(a^2 + c^2)}{12}$$

$$I_z = \frac{m(a^2 + b^2)}{12}$$

Table 1: Physical properties of wooden block

a (in)	b (in)	c (in)	m (g)	I_x (g*in^2)	I_y (g*in^2)	I_z (g*in^2)
4.875	1.5	3.5	178.8	216.05	536.63	387.63

4.2 Experimental Determination

The configuration, measurements, and calculations for the test to determine the block's moment of inertia can be found below. Additional data and figures for the experiment can be found in Appendix A.

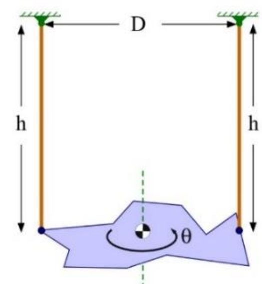


Table 2: Measurements of pendulum configuration per axis

Axis	m (g)	h (in)	D (in)	T, swinging pendulum (s)	T, rotation about said axis (s)	w (rad)
x	178.8	23.5625	4.25	1.5518	0.81	4.05
y	178.8	26.625	3.9375	1.65	1.45	3.80
z	178.8	23.875	5.8125	1.5621	0.80	4.02

Table 3: Experimental vs. known measurements of gravity

	Experimental (in/s^2)	Known (in/s^2)	Deviation
x	386.29	386.40	0.03%
y	385.01	386.40	0.36%
z	386.27	386.40	0.03%

Table 4: Experimental vs. analytical measurements of the block's moments of inertia

	Experimental (g*in^2)	Analytical (g*in^2)	Deviation
I_x	219.72	216.05	1.70%
I_y	534.38	536.63	0.42%
I_z	393.86	387.63	1.61%

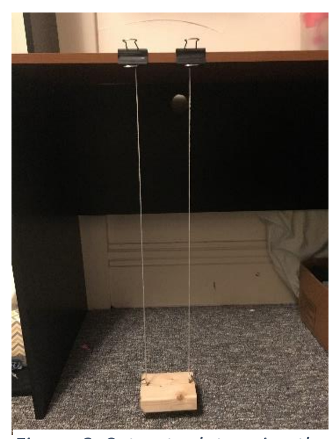
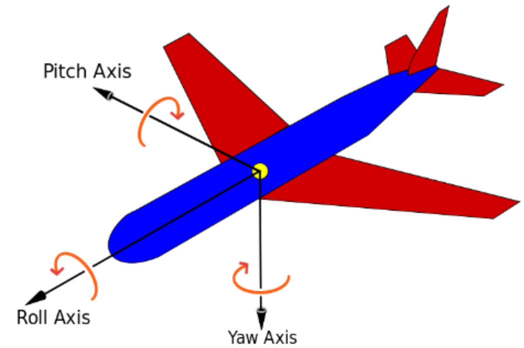


Figure 8: Setup to determine the moment of inertia about the y-axis of the wooden block

CHAPTER 5: DETERMINATION OF THE MOMENT OF INERTIA OF A MODEL AIRPLANE

5.1 Yaw, Roll, and Pitch Axes

It is necessary to control the attitude or orientation of a flying aircraft in all 3 dimensions. In flight, any aircraft will rotate about its center of gravity, a point which is the average location of the mass of the aircraft. We can define a 3-dimensional coordinate system through the center of gravity with each axis of this coordinate system perpendicular to the other two axes. The orientation of the aircraft can then be defined by the amount of rotation of the parts of the aircraft along these principal axes.



The yaw axis is defined to be perpendicular to the plane of the wings with its origin at the center of gravity and directed towards the bottom of the aircraft. A yaw motion is a movement of the nose of the aircraft from side to side. The pitch axis perpendicular to the yaw axis and is parallel to the plane of the wings with its origin at the center of gravity and directed towards the right-wing tip. A pitch motion is an up or down movement of the nose of the aircraft. The roll axis is perpendicular to the other two axes with its origin at the center of gravity, and is directed towards the nose of the aircraft. A rolling motion is an up and down movement of the wing tips of the aircraft

5.2 Experimental Determination

The configuration, measurements, and calculations for the test to determine the block's moment of inertia can be found below. Additional data and figures for the experiment can be found in Appendix B.

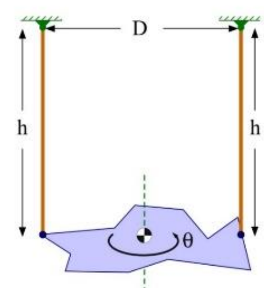


Table 5: Measurements of pendulum configuration for the airplane per axis:

	m (g)	h (in)	d (in)	T, swinging pendulum (s)	T, rotation about said axis (s)	w (rad)
Yaw	77.0	25.5	1.625	1.6147	2.3058	3.89
Roll	77.0	25.1875	2.4375	1.6097	0.9517	3.90
Pitch	77.0	25.1875	2.375	1.6051	1.5669	3.91

Table 6:Experimental vs. known measurements of gravity

	Experimental (in/s^2)	Known (in/s^2)	Deviation
Yaw	386.11	386.40	0.07%
Roll	383.76	386.40	0.68%
Pitch	385.96	386.40	0.11%

Table 7: Experimental vs. analytical measurements of the airplane's moments of

	Experimental ($g*in^2$)
Yaw	103.73
Roll	40.25
Pitch	103.59

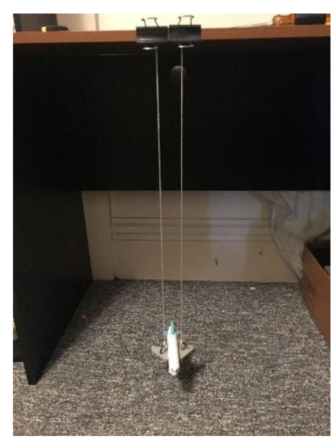


Figure 9: Pendulum configuration

CHAPTER 6: DISCUSSION OF RESULTS

6.1 Wooden Block Experiment

Prior to credibly being able to experimentally determine the moment of inertia of an irregularly shaped object, a model airplane, an object with a known moment of inertia, a wooden block, had to be tested first. To ensure that the pendulum was accurate, gravity had to be tested by measuring the period of the block swinging in a pendulum motion, and then using the equation below:

$$g_y = \omega^2 h = \frac{4\pi^2 h}{T^2} = \frac{4\pi^2 * 26.625in}{(1.65s)^2} = 385.01\, in/s^2 \approx 386.40\, in/s^2$$

Now that the pendulum's results have been confirmed, the moment of inertia can be determined by measuring the period of the block rotating it about it's vertical axis rather than swinging it like a pendulum, and then using the equation below:

$$I_y = \frac{mgD^2T^2}{16\pi^2 h} = \frac{178.8g * 386.40in/s^2 * (3.9375in)^2 * (1.45s)^2}{16\pi^2 * 26.625in} = 534.38g * in^2 \approx 536.63g * in^2$$

The results for the experimentally determined moments of inertia of the wooden block were rather successful as shown in Table 4. The deviations between the experimental and analytical values for I_x, I_y, and I_z were 1.70%, 0.42%, and 1.61% respectively which are very good.

6.2 Model Airplane Experiment

Now that the experimental method used in this experiment for determining the moments of inertia of an object have been confirmed, the moments of inertia of the model airplane can be determined. Similar to the test for the wooden block, gravity must first be confirmed to ensure a credible configuration:

$$g_{roll} = \omega^2 h = \frac{4\pi^2 h}{T^2} = \frac{4\pi^2 * 25.1875in}{(1.6097s)^2} = 383.76\, in/s^2 \approx 386.40\, in/s^2$$

As stated in the previous section, the moment of inertia can then be calculated about the same axis:

$$I_{roll} = \frac{mgD^2T^2}{16\pi^2 h} = \frac{77.0g * 386.40in/s^2 * (2.4375in)^2 * (0.9517s)^2}{16\pi^2 * 25.1875in} = 40.25g * in^2$$

The remaining results for the gravity test and moment of inertia calculations for the model airplane can be found in Table 6 and Table 7. These results are confirmed by the experiment using the wooden block which resulted very accurately.

CHAPTER 7: CONCLUSION

Based on the wooden block part of this experiment the test and results may be considered sufficient. Deviations between the experimentally determined and known values for gravity ranged from 0.03% to 0.68% for both the block and the airplane, and the deviations between the experimentally determined moments of inertia for the block ranged from 0.42% to 1.70%. The results from the airplane experiment make sense because a plane is most likely to rotate about its roll axis versus its yaw and pitch axes based on its weight distribution. The most likely source of error was likely human error while timing the periods of the block. To minimize this as much as possible 10 trials of 10 oscillations were done for each average period needed throughout the experiment. These results can be seen in Table 8 and Table 9 in Appendix A and Table 10 and Table 11 in Appendix B. Overall, this experiment proves that the moment of inertia can be determined experimentally not only of an airplane, but of any irregular shaped object.

APPENDIX A: TEST DATA AND FIGURES FROM THE WOODEN BLOCK EXPERIMENT

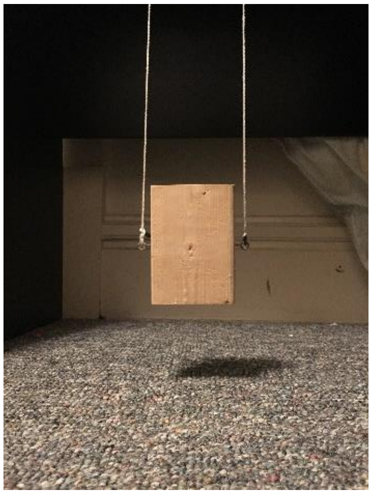


Figure 10: Configuration to determine the moment of inertia about the x-axis

Figure 11: Configuration to determine the moment of inertia about the y-axis

Figure 12: Configuration to determine the moment of inertia about the z-axis

Table 8: Time for the block to complete 10 oscillations in a pendulum motion to confirm gravity for each axis

	x	y	z
Trial	Times (s)		
1	15.53	16.52	15.56
2	15.50	16.53	15.62
3	15.51	16.52	15.71
4	15.50	16.48	15.63
5	15.49	16.53	15.59
6	15.53	16.53	15.55
7	15.46	16.55	15.68
8	15.60	16.50	15.63
9	15.56	16.54	15.59
10	15.50	16.53	15.65
Average	**15.52**	**16.52**	**15.62**

Table 9: Time for the block to complete 10 oscillations rotating about said axis to determine the moment of inertia

	x	y	z
Trial	Times (s)		
1	8.11	14.57	8.01
2	8.13	14.41	7.95
3	8.13	14.48	8.00
4	8.07	14.60	8.01
5	8.14	14.29	7.95
6	8.10	14.50	7.96
7	7.98	14.43	7.98
8	8.16	14.52	7.95
9	8.03	14.43	7.95
10	8.09	14.60	8.00
Average	**8.09**	**14.48**	**7.98**

APPENDIX B: TEST DATA AND FIGURES FROM THE AIRPLANE EXPERIMENT

Figure 13: Configuration to determine the moment of inertia about the yaw axis

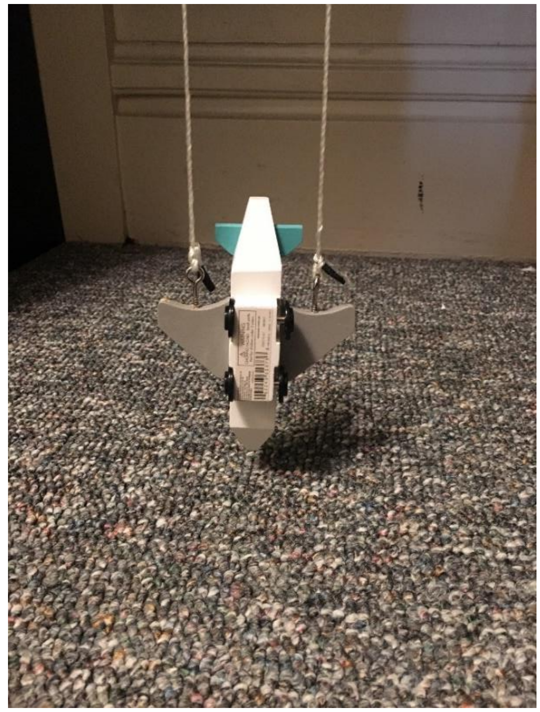


Figure 14:Configuration to determine the moment of inertia about the roll axis

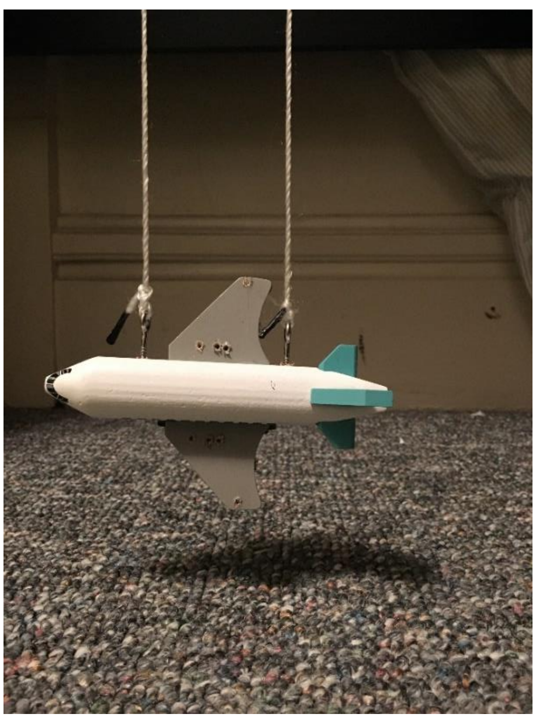


Figure 15: Configuration to determine the moment of inertia about the pitch axis

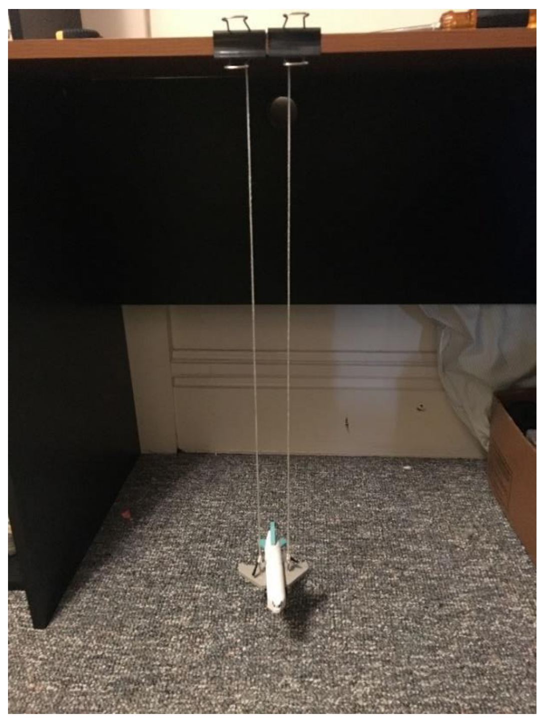


Figure 16: Overall configuration to determine the moment of inertia about the yaw axis

Table 10: Time for the block to complete 10 oscillations in a pendulum motion to confirm gravity for each axis

	Yaw	Roll	Pitch
Trial	Times (s)		
1	16.16	16.10	16.06
2	16.16	16.15	16.10
3	16.17	16.10	16.00
4	16.17	16.02	16.06
5	16.13	16.14	16.08
6	16.16	16.06	16.04
7	16.13	16.06	16.06
8	16.16	16.10	16.08
9	16.20	16.10	16.03
10	16.03	16.14	16.00
Average	**16.15**	**16.10**	**16.05**

Table 11: Time for the block to complete 10 oscillations rotating about said axis to determine the moment of inertia

	Yaw	Roll	Pitch
Trial	Times (s)		
1	23.02	9.53	15.83
2	23.00	9.40	15.65
3	22.89	9.56	15.61
4	23.07	9.55	15.67
5	23.13	9.60	15.51
6	23.10	9.48	15.68
7	23.04	9.61	15.77
8	23.19	9.41	15.70
9	23.05	9.45	15.67
10	23.09	9.58	15.60
Average	**23.06**	**9.52**	**15.67**

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