Spring 2015

The Importance of a Strong Mathematical Foundation

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Honors Research Project

Submitted to

The Honors College

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Senior Research Project

The Importance of a Strong Mathematical Foundation

The University of Akron Honors College

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Jasmine Wriston

Faculty Sponsor: Dr. Laurie Dunlap

Submitted To:

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Abstract

The mathematical skills students learn from kindergarten through eighth grade are the foundational skills upon which all higher level mathematics courses build. It is highly beneficial that students master previous mathematics concepts, applications, and skills, prior to learning algebra and other higher level mathematical courses. Mastering elementary and middle level mathematics before learning algebra increases students' chances for success when taking an algebra course. This study tested 39 ninth and tenth graders, from the college preparatory program Upward Bound, on the mathematical domains of fractions and ratios/proportions. Participants took one of two tests, each composed of three questions increasing in difficulty. Calculators were not permitted. The fractions test was composed of a third, fourth, and fifth grade question and only 5 of 20 participants were able to pass the assessment. The ratios/proportions assessment was composed of a sixth grade question and two seventh grade questions and only 2 participants out of 19 were able to pass the assessment. To better aid in the creation of strong mathematical foundations educators should strive to assess student understanding prior to instruction and teach students based off their current understanding and not their current grade level. Educators should also be sure to not only teach procedural knowledge but also conceptual understanding.

Purpose

While studying to become a middle school mathematics educator at a state university, I was required to take part in over one hundred hours of service learning. During service learning one can take on many different roles within the school setting such as simply being an observer, tutoring individual students or small groups of students, or even developing and implementing whole lesson plans for entire classes. Approximately seventy hours of my service learning was spent tutoring individual students or small groups of students in mathematics. A mathematics curriculum was presented to me that aligned to the current grade level of the students receiving tutoring. For example, if I was in a fifth grade classroom, I would be given a fifth grade mathematics curriculum so that I could instruct the students on such grade level content.

While tutoring students in the mathematical domain, I noted that many students lacked mastery of prior mathematical applications, processes, and knowledge, necessary to allow them to begin mastering their current grade level instruction. For example, I would be tutoring students on operations with fractions, a fifth grade standard denoted as CCSS.MATH.CONTENT.5.NF.B.3 in the Common Core State Standards (CCSS), when students had failed to master basic applications of base ten numbers, a fourth grade standard denoted as CCSS.MATH.CONTENT.4.NBT.B.4 by the national CCSS. Initially, when I would come across gaps in mathematical content understanding and connections, I would attempt to continue instructing on the provided grade level curriculum and attempt to aid students in making such overwhelming strides. Eventually however, it became evident that without the mastery of previous grade level mathematical standards, students struggled to reach current grade level mathematical expectations. I decided to assess student understanding to discover their level of mastery within the domain of number and operations in base ten. Many students failed to understand the basic concepts of multiplying or dividing numbers or lacked the conceptual understanding of positive and negative numbers. This hindered students from mastering current grade level applications that required the strong foundational understanding of base ten numbers.

In mathematics, conceptual understanding and applied knowledge greatly build upon each other. One must understand the base ten number system and aligning operations, such as multiplication and division, before one is able to manipulate fractions. One must master fractions before one can evaluate ratios and proportions. A complete and thorough understanding of mathematical applications should be mastered before instruction of algebra can begin (Brown & Quinn 2007). Due to mathematics building upon itself, with each new mastered domain opening the door for developmental understanding of another, students cannot afford to be taught at levels above their current understanding. Students must be met where they are cognitively within the subject of mathematics and not simply taught according to their current grade level.

Students learn at different paces and require a variety of individualized teaching methods and strategies to master content. It is generally accepted that students need engaging problem-based instruction to aid in their mastery of subject matter. But even the best teaching practices will fall short if students are not being met at their current cognitive level. Even though a student may be in sixth grade, if they have not met fifth grade mathematics standards, they should not be taught sixth grade material. Teachers need to implement differentiated instruction into classrooms to better meet all students at their current mathematical understandings. Mathematics builds on itself. If the foundation of mathematics is not mastered, the building blocks of mathematics not developed, students will struggle to make necessary connections within the content material or fully understand higher level mathematical concepts. If one never learns their multiplication tables, how can they ever independently do multiplication of two digit numbers, three digit numbers, or even long division? If students are unable to understand the basic concept of a fraction, how can they ever add two fractions together, or look at two fractions and know which one is larger?

Background

The learning of mathematics is in and of itself valuable to every student. Mathematics gives one the ability to understand daily temperatures and truly internalize the difference between 65° and -15°. Mathematics allows one to balance check books, estimate tips, compute their change from a transaction, calculate the price of an item for sale, and double the yield of a recipe. Everyone uses mathematical applications everyday within their everyday life. However, mastering the subject matter of mathematics is so much more important beyond that of its everyday use. The mastering of mathematics corresponds directly to each student's future and success in life, be it in the work force, college, or the military (Wang 2003). Mastering basic mathematics skills, such as fractions, better prepares one for higher level mathematics, which in turn develops students who are college and career ready upon graduation of high school, thus supporting the goal of creating global citizens in the 21st century.

Mastering basic mathematics skills, such as fractions, better prepares one for higher level mathematics such as algebra. According to Brown and Quinn, "students who fail to master the foundational conceptual understanding of fractions, such as operations with fractions, are often unable to conceptualize algebraic functions and commonly exhibit error patterns when learning algebra" (Brown & Quinn 2007 pg.1). When students fail to understand the algebraic shortcuts that are implemented during mathematical application they might fail to develop the conceptual understanding that will carry them into higher level mathematics. "Elementary algebra is built on a foundation of fundamental arithmetic concepts" (Brown & Quinn 2007 pg.1). If students don't fully understand basic arithmetic concepts, be it with simple base ten numbers or fractions, they likely will not be able to apply such concepts to equations with unknown variables. In order for students to be able to gain understanding from higher level mathematics courses, they must enter such courses with a strong foundational background. If such a foundation is never fully built the end result is algebra becoming an overwhelming conglomeration of unrelated facts and algorithms that students randomly use in a last ditch effort to solve problems (Brown and Quinn 2007). It is necessary that students receive instruction based on the mathematical knowledge they bring to class and not based on the grade level they currently are in to ensure that all students are fully prepared for higher level mathematics courses such as algebra. Not only does the ability to master and understand fractions predict a student's ability to master and understand algebra, but so too does the ability to master algebra predict success in college and or in life (Wang 2003).

Acquiring mathematics skills is not only important for those students planning to attend college but also for those students who are not seeking further education beyond high school. According to Jia Wang, "mathematics achievement is related positively to early labor market success" (Wang 2003, pg. 14). This statement relates that even the success of students who opt out of going to college is still directly correlated to their mathematical skills. Those who develop a strong mathematics foundation and who continue to build upon it in high school acquire such skills as problem solving, critical thinking, reasoning, and perseverance (Wang 2003). These skills and attributes are all highly sought after in both college and the work force yielding proactive students and/or employees. Therefore, mathematics not only provides students with everyday mathematical application knowledge, but also provides students with marketable skills and qualities that will aid in them securing a job or graduating college (Wang 2003). This is arguably a purpose of not only mathematics instruction but education in its entirety as well; to make students college and/or career ready.

To ensure that U.S students are college and/or career ready upon graduation from high school, and to measure the quality of mathematics instruction received by U.S. students, The

Program for International Student Assessment (PISA), compares the mathematic achievement of U.S. 15 year-olds to the 15 year-olds of 65 other education systems worldwide (NCES 2014-024 U.S. Department of Education). The increase of technology, global communication, and world economies, makes it important that schools not only ensure students are college and or career ready, but also that students are capable of becoming active citizens in a global world. With mastery of mathematics clearly corresponding to students' success in life, it is necessary that U.S. students are receiving a competitive mathematics education as compared to other countries.

The PISA assesses students on four different mathematical content categories and three mathematical process categories (NCES 2014-024 U.S. Department of Education). The four mathematical content categories are; change and relationship, space and shape, quantity, and uncertainty and data. The PISA assesses to see if 15 year-olds are capable of modeling change and relationships with the appropriate functions and equations, understanding perspective, engaging in mental calculation, and applying probability and statistics. The three process categories students are tested on are labeled as formulate, employ, and interpret. When assessed on the following content and processes categories only 9 percent of 15 year-old U.S. students scored at proficiency level 5 or above (NCES 2014-024 U.S. Department of Education). "The U.S. percentage was lower than 27 education systems, higher than 22 education systems, and not measurably different than 13 education systems" (NCES 2014-024 U.S. Department of Education pg. 9).

Based on the PISA's global testing, United States students are not mastering the content knowledge of mathematics. Since every country received the same assessment, and all test takers were the same age and from a variety of different schools within each country, perhaps the significant gap in achievement of U.S. students as compared to other countries, say Shanghai, China for example, is due to the mathematical instruction received by students (NCES 2014-024 U.S. Department of Education). Quite possibly, students' mathematical content knowledge is being assessed before instruction in other counties, while in the U.S the majority of students are simply being taught the mathematical curriculum aligning to their current grade level. Another possibility as to why students in other countries have stronger mathematical foundations is because the study of mathematics is valued higher than in the U.S. Perhaps these countries place higher importance on both conceptual and procedural understanding.

Both procedural and conceptual knowledge are important components of mathematical understanding; an issue only arises when students fail to ever grasp conceptual understanding that reveals to them why such procedural applications are appropriate and work (Lin C. 2013). Students cannot only learn the procedural application of turning an improper fraction into a mixed number or finding common denominators, but also acquire the basic conceptual understanding of fractions that reveals to them why such mathematical algorithms work. As seen in this study many participants failed to master the understanding of a fraction as made evident by their placement of fractions on a number line. These same students were then unable to work with fractions and apply their application to answer real world problems. "Conceptual knowledge is described as the relationships and interconnections of ideas that explain and give meaning to mathematical procedures" (Lin C. 2013 pg.2). Students should obtain this conceptual knowledge in order to master mathematical applications and create strong foundations.

In a similar study, a fraction assessment was given to 143 high school students currently enrolled in a basic algebra 1 class (Lin C. 2013). Nearly 48% of the students were unable to find the sum of 5/12 and 3/8. One common error was that students were adding numerators and denominators. And the students who knew they needed to obtain common denominators failed to remember how to do so (Lin C. 2013). This is the perfect example of students not mastering the conceptual understanding of fractions and only partially understanding procedural knowledge. Students must master both the conceptual understanding and the procedural application of all mathematics domains in order to truly obtain mastery of Ohio's New Learning Standards.

To better bring to light the gap in mathematical understanding currently held by students hindering them from fully grasping higher level mathematics such as algebra, I have designed a study to help aid in analyzing and evaluating a high school student's mathematical foundation. These foundations should have been developed in their entirety from the first day of kindergarten to the last day of eighth grade. If student's mathematical foundations have not been developed it is predicted that they will struggle in higher level mathematical courses.

Methodology

I constructed two mathematics assessments. These mathematic tests were designed to be completed within thirty minutes and the use of calculators was not permitted. The questions that made up these tests were designed in correlation to the Common Core State Standards (CCSS) for Mathematics for grades three through seven. Each test consisted of three questions increasing in grade level difficulty. One test focused on the mathematics domain of fractions and had a third, fourth, and fifth grade level questions. The other test focused on the domain of ratios/proportions and had a sixth and two seventh grade level question. The mathematical domains of fractions and ratios/proportions were selected due to their strong correlation to success in higher level mathematics courses (Brown and Quinn 2007).The questions making up my mathematics tests were sample questions pulled from the Partnership of Assessments for

Readiness of College and Careers (PARCC) assessments. These questions were designed to align closely with the Common Core Mathematic Standards, thus it was ideal to utilize them. Because the PARCC questions were designed to be given online, my advisor and I reformulated them for a paper-and-pencil format. Adjusting the assessment to be taken with paper and pencil allowed participants to conveniently take the assessments and I as the researcher to review them.

To validate the clarity of all questions on both assessments, sample assessments were given to nine pre-service mathematics teachers and 10 professors within the university's mathematics department. The university's mathematics department staff simply reviewed each assessment to ensure that all participants would understand what each question was asking them to do, solve, or manipulate. The pre-service teachers actually took the assessments to further reveal the clarity of each question on both assessments.

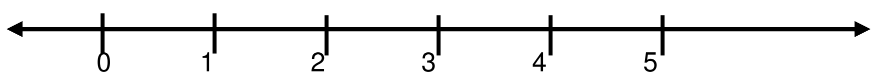
Students currently in ninth and tenth grade completed my assessments. The average student begins taking algebra in the ninth grade. Ideally, all students should therefore have mastered the mathematical applications that make up my two assessments. Students who are currently taking algebra classes or higher should have met the expectations of lower level mathematics courses, thus permitting them to continue on to advanced mathematics courses. Therefore, all participants should pass my assessments. Based on my experience, I assume that participants received instruction based on their grade level and not their mathematical content knowledge. This may have yielded in gaps and failed connections throughout their understanding of the mathematics content. To fully bring to light the rocky mathematical foundations so many U.S. students hold, students who should have mastered the concepts that make up my assessments were the students tested. Essentially, underclassmen high school students took tests composed of third through seventh grade mathematics questions. At the time, such students were currently in algebra or a higher level class, thus they should have proficiency in the content presented within my assessments.

Assessments

As previously mentioned, the fractions test was composed of three questions aligning to third, fourth, and fifth grade mathematics standards as determined by the national CCSS. The first question is shown below.

1. Locate each fraction on the number line. Mark the location with a dot and write the fraction underneath its location.

$\frac{1}{2}$ $\frac{3}{2}$ $\frac{6}{2}$



This third grade question requires students to evaluate that 1/2 is located exactly between zero and one, 3/2 is more than 1/2 and therefore must be further down the number line between one and two, and 6/2 is the largest fraction reducing to exactly three. This question purposefully supplies participants with three fractions having the same denominator because in the third grade students are just beginning to learn that a "fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts and understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*" *(CCSS* pg.4*)* as related by the standard denoted as *CCSS.MATH.CONTENT.3.NF.A.1*. Overall, in third grade students should master the conceptual understanding that a fraction represents a part/whole, and gain the understanding of comparing fractions with the same denominator by placing them on a number line.

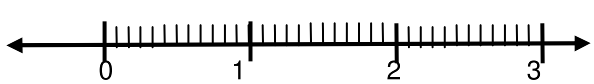
The second question on the fractions mathematics assessment supplies participants with the fractions 3/2 and 5/6. It is shown on the next page. Participants are yet again asked to place these fractions on two different number lines and then determine which fraction is larger. Participants are asked to explain how they know which fraction is larger. Lastly, participants are asked to supply a fraction that is between 3/2 and 5/6 and explain how they know their fraction is between 3/2 and 5/6. This question aligns to the fourth grade mathematics standard that states students should extend their understanding of fraction equivalence and ordering by comparing two different fractions with different numerators and denominators, denoted as CCSS.MATH.CONTENT.4.NF.A.2 (CCSS). In order for participants to be able to answer this question they have to have mastered their third grade fraction standards relating the basic conceptual understanding of fractions. That will allow participants to realize that 1/6 is smaller than 1/2, but still see the need to evaluate 5/6 as compared to 3/2. There are many ways that participants can evaluate the problem. For example, participants can either evaluate that 3/2 is larger than 5/6 by analyzing the placement of the fractions on the number line or if they did master fractions in previous years they can change 3/2 to 9/6 and compare 9/6 to 5/6.

Below is the second question of the fractions assessment.

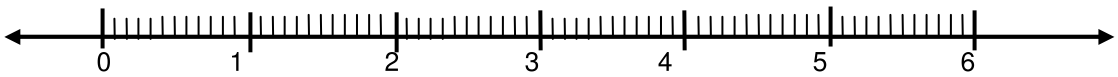
2. Ava and Mia are comparing the fractions $\frac{3}{2}$ and $\frac{5}{6}$.

Part A

Ava created this number line to graph $\frac{3}{2}$. Locate this fraction on the number line. Mark the location with a dot and write the fraction underneath its location.



Mia created this number line to graph $\frac{5}{6}$. Locate this fraction on the number line. Mark the location with a dot and write the fraction underneath its location.



Part B

Is $\frac{3}{2}$ greater than or less than $\frac{5}{6}$? Explain how you know.

Part C

Write a fraction that is between $\frac{3}{2}$ and $\frac{5}{6}$? Explain how you know your fraction is between $\frac{3}{2}$ and $\frac{5}{6}$.

The third question concluding the fractions assessment supplied participants with the statement that 12 pencils were shared among four people. One person received 1/3 of the pencils, another received 1/4, and the remaining pencils were shared between two other people with one person receiving one more pencil than the other. Participants were asked to create a number line to represent the total number of pencils combined that the two people who receive 1/4 and 1/3 obtained. Lastly, participants were required to evaluate how many pencils the remaining two people each received. This question is a fifth grade level question aligning to the fifth grade fraction standard that states students should be able to use equivalent fractions as a strategy to add and subtract fractions. The mathematics standard denoted as CCSS.MATH.CONTENT.5.NF.A.2 states that students should be able to, “solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators" (CCSS pg. 4). In order for participants to be able to accurately complete this problem, they must have mastered both the third grade fraction standards and the fourth grade fraction standards.

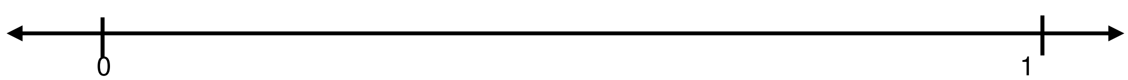
Below is the third question on the fractions assessment.

3. Mr. Edmunds shared 12 pencils among his four sons as follows:

- Alan received $\frac{1}{3}$ of the pencils.
- Bill received $\frac{1}{4}$ of the pencils.
- Carl received more than 1 pencil.
- David received more pencils than Carl.

Part A

On the number line, represent the fraction of the total number of pencils that was given to Alan and Bill combined. Note: You will need to break the number line into sections of equal size and then thicken/darken sections until you have enough to represent the fraction.



Part B

What fraction of the total number of pencils did Carl and David each receive? Justify your answer.

Similar to the fractions test, the ratios and proportional relationships test is also composed of three questions. The questions that make up this test are a sixth grade and two seventh grade questions. The first question supplies participants with data of three different bands concerning their number of brass players and percussion players. Participants are first asked to find the ratio between brass players and percussion players by utilizing the three band's data. Participants should discover that there are 3 brass players per every percussion player. Next, participants are given a scenario that states that of 210 students, a band director wishes to have 80% of them be brass and percussion players. Using the unit rate previously discovered, (3:1), determine how many students should play brass instruments. This sixth grade question aligns with the first Common Core standard relating ratios and proportional relationships starting in the sixth grade. Beginning in the sixth grade, students should be able to "Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities" as stated by the mathematics standard denoted as CCSS.MATH.CONTENT.6.RP.A.1 (CCSS pg. 7).

Below is the first question of the ratios and proportions assessment.

1. Mr. Ruiz is starting a marching band at his school. He first does research and finds the following data about other local marching bands.

	Band 1	**Band 2**	**Band 3**
Number of Brass Instrument Players	123	42	150
Number of Percussion Instrument Players	41	14	50

Part A

Write your answer in the blank space.

Mr. Ruiz realizes that there are _____________ brass instrument player(s) per percussion player.

Part B

Mr. Ruiz has 210 students who are interested in joining the marching band. He decides to have 80% of the band be made up of percussion and brass instruments. Use the unit rate you found in Part A to determine how many students should play brass instruments.

Show or explain all of your steps.

The second question on the ratios and proportions assessment supplies participants with three friends, a page numbered book they are currently each reading, the number of pages already read, and the number of days it took them to each read said pages. Participants are first asked to find each person's average reading rate and then arrange the friends in order from the fastest reading rate to the slowest. Next, participants are asked to determine which friend will finish reading their book first given that they continue reading at the same rate. Participants are lastly asked to order the friends from the one who will finish reading in the shortest time to the longest time. By the seventh grade, students should be able to analyze proportional relationships and use them to solve real-world mathematical problems. This is precisely what is being asked of participants in this assessment question. This question aligns perfectly with the math standard denoted as CCSS.MATH.CONTENT.7.RP.A.1, which states that students should be able to "compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units" (CCSS).

Below is the second question of the ratios and proportions assessment.

2. On Friday, three friends shared how much they read during the week.

- Barbara read the first 100 pages from a 320-page book in the last 4 days.
- Colleen read the first 54 pages from a 260-page book in the last 3 days.
- Nancy read the first 160 pages from a 480-page book in the last 5 days.

Part A

A person's average reading rate can be defined at the number of pages read divided by the number of days. Place the three friends reading rates in order from greatest to least by writing their <u>names</u> in the appropriate blank spaces.

Greatest Rate ________________________
(pages per day) (Put Name Above)

____
(Put Name Above)

Least Rate ________________________
(pages per day) (Put Name Above)

Part B

If the three friends continue to read every day at their rates, who will have read their entire book in the shortest time? Longest time?

Order the friends from the one who read her book in the shortest time to the one who her book in the fastest time.

Shortest time ________________________
(Put Name Above)

Middle time ________________________
(Put Name Above)

Longest time ________________________
(Put Name Above)

The last question on the ratios and proportions test supplies participants with the speed of four different objects related to them in time (seconds) by distance (meters). The speed of object A and object B are displayed on a line graph with which participants have to interpret to discover the speed. The speed of object C and object D are displayed on a chart with one column stating the time in seconds and the other column aligning to the distance covered per second value. Participants are also informed that objects C and D have constant speeds. Participants then have to determine the constant of proportionality for objects C and D which will relate their speed and then calculate the speed of objects A and B by utilizing their graphs. Lastly, participants are asked to order the objects from greatest speed to least speed. This will require participants to represent the speeds in similar ratio orientations such as fractions, in order to better compare the speeds. This question again requires students to analyze proportional relationships and use them to solve real-world and mathematical problems. This question more specifically aligns to the math standard denoted as CCSS.MATH.CONTENT.7.RP.A.2.B, which states that students should be able to "identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships" (CCSS pg. 7).

Below is the third question of the ratios and proportions assessment.

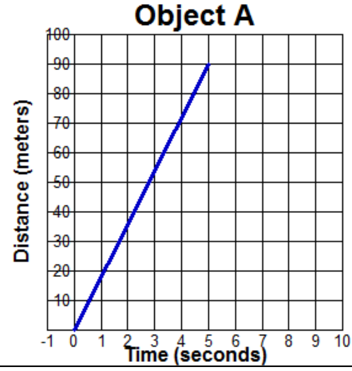
3. The speed of an object is defined as the change in distance divided by the change in time.

Information about objects **A**, **B**, **C** and **D** are shown below. Objects **C** and **D** both have constant speed.

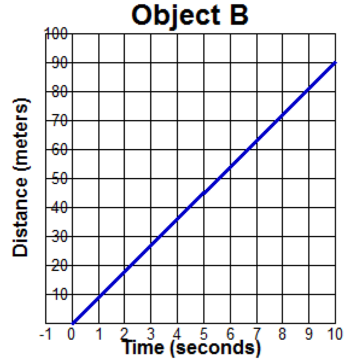
Based on the information given, list the objects in order from greatest speed to least speed in the table provided.

	Object
Greatest Speed	
Least Speed	

Object A



Object B



Object C

Time (seconds)	Distance (meters)
0	0
3	10
6	20
9	30

Object C moves at constant speed.

Object D

Time (seconds)	Distance (meters)
0	0
1.5	10
3	20
4.5	30

Object D moves at constant speed.

Mathematics Domains and Standards

Not only did the two assessments utilized within this study align to standards solely within the Fractions or Ratios & Proportional Relationships Common Core domains, but also within the Number & Operations in Base Ten domain and the Operations and Algebraic Thinking domain. No calculators were permitted while taking either assessment. Many standards within the Operations and Algebraic Thinking domain should have also been mastered in order to adequately complete either assessment. Students not only had to know how to multiply and divide numbers, but also be able to analyze a problem and distinguish which operation should be implemented to discover the solution. One standard in particular that students should have mastered in order to fully display their mathematical content knowledge is the standard denoted as CCSS.MATH.CONTENT.3.OA.C.7, which states that "by the end of grade 3, students should know from memory all products of two one-digit numbers" (CCSS pg. 2). If participants are unable to perform basic multiplication, it is highly unlikely they will complete either assessment in its entirety.

Appendixes 1A and 2A, attached at the end of this report, outline the standards by grade level and domain that each assessment is testing participants on. With the fractions assessment beginning at the third grade level and ending at the fifth grade level, it aligns to lower level standards than the ratios and proportions assessment which starts at the sixth grade level. The aligning standards being relayed within each chart came directly from the National Common Core State Standards website.

Study Design

A population of 39 ninth and tenth grade students completed my constructed assessments. The students I gave my assessments to belonged to the University of Akron's Upward Bound program. I had students from the traditional Upward Bound program and students from the specialized Math and Science Upward Bound Program. Upward Bound is a program designed to prepare students for a smooth transition into college and then success in college thereafter. The University of Akron's Upward Bound program was ideal for my research because it supplied me with a decent sample size, the students attended different middle schools all over Akron so my sample size experienced different mathematics instruction, and the students are desiring to attend college so it will be interesting to see if their mathematic knowledge is adequate and on grade level. My goal was to assess ninth and tenth graders in the hopes of discovering if they had mastered the middle level mathematics standards from third through seventh grade before submerging fully into high school.

Although participants belong to Upward Bound one cannot assume that students will be better prepared academically to successfully complete my assessments. Attending Akron Public Schools, an urban school district with a low socioeconomic status, mathematics achievement may not be up to state standards. Participants are not in Upward Bound because they have mastered all academic standards making them ready for college, but rather participants have the potential to attend college with extra assistance.

Hypothesis

Before administering my assessments to the ninth and tenth graders of Upward Bound, I hypothesized that the majority of students, that is over half of my total sample size, would not pass my assessments. I identified passing as answering 66% or more of either assessment correctly. Therefore, I estimated that over half of the students would answer two or more of the questions on their assessment incorrectly. Based on my experiences, I believe students are being passed along in the subject of mathematics and not meeting mastery of the common core content standard before receiving new instruction on new material. Due to this, I predicted less than half of participants tested will be able to pass my assessments.

Results

Fractions Assessment

The overall performance of participants on each question is displayed in the chart below. An answer was deemed correct if a student correctly answered more than half of the question.

Question #	Correct	Incorrect
1	10	10
2	9	11
3	0	20

Fraction Assessment *Chart 1*

Number of participants who correctly or incorrectly answered each part of each question

	Freshman				**Sophomore**			
	Male		Female		Male		Female	
	Correct	Incorrect	Correct	Incorrect	Correct	Incorrect	Correct	Incorrect
Question 1 Standard(s) CCSS.MATH.CONTENT.3.NF.A.1 CCSS.MATH.CONTENT.3.NF.A.2 CCSS.MATH.CONTENT.3.NF.A.3	2	3	3	3	1	2	4	2
Question 2 Part A Standard(s) CCSS.MATH.CONTENT.4.NF.A.1	2	3	2	2	2	3	0	6
Question 2 Part B Standard(s) CCSS.MATH.CONTENT.4.NF.A.2	1	3	3	3	2	2	6	0
Question 2 Part C Standard(s) CCSS.MATH.CONTENT.4.NF.A.1 CCSS.MATH.CONTENT.4.NF.A.2	1	3	4	2	2	2	3	3
Question 3 Part A Standard(s) CCSS.MATH.CONTENT.5.NF.A.1	1	3	1	5	0	4	0	6
Question 3 Part B Standard(s) CCSS.MATH.CONTENT.5.NF.A.2	0	4	0	6	1	3	1	5
Totals	7	19	13	21	8	16	14	22

Chart 2

Fractions Assessment	
Question #	Average Score
1	50.00%
2	46.67%
3	10.00%

The results displayed in chart 1 break down the results of the fraction assessment by participant grade level and gender. It identifies the number of individuals who correctly or incorrectly answered each question categorized by their current grade level and gender. Based on this chart there is no practical difference in performance based on either gender or grade level. The sophomores did not outperform the freshman or vice versa and neither did the male participants out perform the female or vice versa. Chart 1 also aligns the Common Core Standard that each question aligns with. Essentially, if a participant correctly answered the question, this was used as an indicator that they understood the fraction standards. However, if a participant answered a question incorrectly or simply did not attempt a question, this was used as an indicator that they did not understand the aligning standard corresponding to that question.

The results displayed in chart 2 were obtained by accessing each individual question and evaluating the average percent of each question that was answered correctly For example, question one only had one part and therefore participants had to answer question one in its entirety correctly. The average score of question one was 50% with half of the participants correctly answering the question and the other half answering incorrectly. Question 2 had three parts with each part being worth 1/3 of a point. The average score of question two, with one point being 100%, was 46.67 %. Only four participants were able to answer question two correctly in its entirety. Lastly, question 3 was composed of two parts each worth half of a point. The average score earned on question three was 10.00%. No participant answered question three on the fractions assessment correctly in its entirety. Over half of the participants had to correctly answer each question to yield an overall passing score of the question. Since the sample population was not able to correctly answer any question as a majority, the overall result of the fraction assessment per question was a failure.

The overall result of the entire fraction assessment was also a failure with only 5 out of 20 participants successfully passing the assessment. Passing the assessment was determined by participants earning at least a 66% on the assessment.

Chart 3 Fractions Assessment Overall Results Per. Student

Students	Question 1 1 Point		Question 2 1 Point		Question 3 1 Point		Total	Pass/Fail
		Part 1: ⅓ Point	Part 2: ⅓Point	Part 3: ⅓ Point	Part 1: ½ Point	Part 2: ½ Point		
Male 9	0	0	0	0	0	0	0	Fail
Male 9	1	0	0	0	0	0	1	Fail
Male 9	1	⅓	⅓	⅓	½	O	2 ½	**Pass**
Male 9	0	⅓	0	0	0	0	⅓	Fail
Female 9	0	0	0	0	½	0	½	Fail
Female 9	0	0	0	⅓	0	0	⅓	Fail
Female 9	0	0	0	0	0	0	0	Fail
Female 9	1	⅓	⅓	⅓	0	0	2	**Pass**
Female 9	1	⅓	⅓	⅓	0	0	2	**Pass**
Female 9	1	0	⅓	⅓	0	0	1⅔	Fail
Male 10	0	0	⅓	⅓	0	0	⅔	Fail
Male 10	1	⅓	⅓	⅓	0	½	2½	**Pass**
Male 10	0	⅓	0	0	0	0	⅓	Fail
Male 10	0	0	0	0	0	0	0	Fail
Female 10	1	0	⅓	⅓	0	0	1⅔	Fail
Female 10	1	0	⅓	⅓	0	0	1⅔	Fail
Female 10	0	0	⅓	0	0	0	⅓	Fail
Female 10	1	0	⅓	0	0	0	1⅓	Fail
Female 10	0	0	⅓	0	0	0	⅓	Fail
Female 10	1	0	⅓	⅓	0	½	2⅙	**Pass**

Chart 4 Assessment Demographics

Fractions Assessment Demographics				
	Freshman		**Sophomore**	
	Male	Female	Male	Female
Algebra	3	6	4	5
Geometry	2	3	4	6
Algebra 2	1	X	2	3
Trigonometry	1	X	2	3
Statistics	X	X	1	X

A total of 20 participants completed the fraction assessment. Ten of them were freshman and ten of them sophomores. 18 of the participants took or are currently taking algebra. The other two participants who stated they had not taken algebra revealed that they took or are taking geometry. Eight participants took mathematic courses higher than algebra 1 and geometry. However, regardless of the grade level and previous mathematics tests taken, no single participant completed the assessment in its entirety correctly. Five individual participants were able to pass the assessment by earning a 66% or higher on the assessment. Since only 5 participants passed the assessment, meaning that 15 participants received failing scores, the overall average of the fraction assessment is a failing score.

Ratios and Proportions Assessment

Ratios and Proportions Assessment *Chart 5*

	Freshman				**Sophomore**			
	Male		Female		Male		Female	
	Correct	Incorrect	Correct	Incorrect	Correct	Incorrect	Correct	Incorrect
Question 1 Part A Standard(s) CCSS.MATH.CONTENT.6.RP.A.1 CCSS.MATH.CONTENT.6.RP.A.2	3	2	1	3	2	2	3	3
Question 1 Part B Standard(s) CCSS.MATH.CONTENT.6.RP.A.3	0	5	0	4	0	4	0	6
Question 2 Part A Standard(s) CCSS.MATH.CONTENT.7.RP.A.1	2	3	3	1	2	2	1	5
Question 2 Part B Standard(s) CCSS.MATH.CONTENT.7.RP.A.2	1	4	0	4	0	4	1	5
Question 3 Standard(s) CCSS.MATH.CONTENT.7.RP.A.2.A CCSS.MATH.CONTENT.7.RP.A.2.B	0	5	0	4	2	2	1	5
Totals	6	19	4	16	6	14	6	24

The results displayed in chart 5 breaks down the results of the ratios and proportions assessment by participant grade level and gender. It yet again identifies the number of individuals who correctly or incorrectly answered each question categorized by their current grade level and gender. Based on this chart there is no practical difference in performance based on either gender or grade level. The sophomores did not out-perform the freshman or vice versa and neither did the male participants out-perform the female or vice versa. Chart 5 also identifies the Common Core Standard that each question corresponds to. Essentially, if a participant correctly answered the question, they should have mastered such ratios and proportions standards. However, if a participant answered a question incorrectly or simply did not attempt a question it was recorded as a zero.

It should also be noted that the questions that made up my assessment were arranged in increasing order of difficultly. This means that question two is harder than question one and question three harder than question two. Question one is actually a sixth grade question while question two and three are seventh grade questions covering different standards. It is interesting to note that when analyzing chart 6 the participants earned an overall average score on question two higher than on question one. Even though question two is a seventh grade question while question one was a sixth grade question.

Chart 6

Ratios and Proportions Assessment	
Question #	Average Score
1	23.68%
2	26.32%
3	15.79

The results displayed in chart 6 were obtained by accessing each individual question and evaluating the average percent of each question that was answered correctly. For example, question one had two parts each worth half of a point. No participant earned full credit on question one, nine participants earned half a point, and the remaining ten students earned zero points. This created the overall average percent score of 23.68% for question one. Question two yet again had two parts each worth half of a point. No participant earned full credit on question two, ten students each earned half a point, and the remaining nine students earned zero points. This created the overall average percent score of 26.32%. Lastly, question 3 was composed of a single one part question requiring participants to correctly answer all of question 3. Three students correctly answered question 3. This created the overall average percent score of 15.79%.

With a passing score of the overall assessment being determined as correctly answering 66% or more of the assessment, students had to correctly earn a minimum of two full points out of a total of three points. Individual student success on the test overall, based on student performance per question, is better examined and explained in Chart 7. Because the average score on each question was below 66%, the sample population did not have a passing average on the ratios and proportions assessment or on any of the individual assessments. However, 2 of the 19 participants did receive scores of 66% or higher on the assessment. This is displayed in greater detail in Chart 7.

Chart 7 Ratios and Proportions Assessment Overall Results Per. Student

Students	Question 1		Question 2		Question 3	Total	Pass/Fail
	Part 1: ½ Point	Part 2: ½ Point	Part 2: ½ Point	Part 3: ½ Point	1 Point		
Male 9	½	0	½	0	0	1	Fail
Male 9	½	0	½	0	0	1	Fail
Male 9	0	O	O	½	O	½	Fail
Male 9	0	0	0	0	0	0	Fail
Male 9	½	0	0	0	0	½	Fail
Female 9	½	0	½	0	0	1	Fail
Female 9	0	0	½	0	0	½	Fail
Female 9	0	0	0	0	0	0	Fail
Female 9	0	0	½	0	0	½	Fail
Male 10	0	0	0	0	0	0	Fail
Male 10	½	0	½	0	0	1	Fail
Male 10	½	0	½	0	1	2	**Pass**
Male 10	0	0	0	0	1	1	Fail
Female 10	½	0	0	0	0	½	Fail
Female 10	½	0	½	0	1	2	**Pass**
Female 10	0	0	0	½	0	½	Fail
Female 10	½	0	0	0	0	½	Fail
Female 10	0	0	0	0	0	0	Fail
Female 10	0	0	0	0	0	0	Fail

Ratios/Proportions Assessment Demographics *Chart 8*

	Freshman		**Sophomore**	
	Male	Female	Male	Female
Algebra	5	4	4	6
Geometry	3	1	4	5
Algebra 2	1	X	2	X
Trigonometry	2	X	2	X
Statistics	X	X	1	1

A total of 19 participants completed the ratio and proportions assessment. Nine of them were freshman and ten of them sophomores. 17 of the participants had taken or were currently taking Algebra 1. One of the other two participants who stated to have not taken algebra related that they took or are taking geometry while the other participant simply stated to have taken an "other" mathematics course. 3 participants have taken mathematics courses higher than algebra 1 and geometry. However, regardless of the grade level and previous mathematics courses taken, no single participant completed the assessment in its entirety correctly. Only two individual participants were able to pass the assessment by correctly answering 2/3 of the assessment, or two of the three questions. Since only two participants passed the assessment, meaning that 17 participants failed, the overall result of the ratio and proportions assessment as a whole is a failure.

Analysis of Student Responses

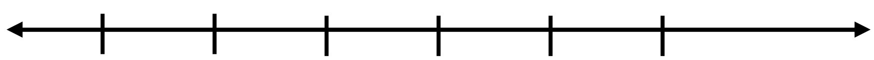
Question two on the fractions assessment asked participants to explain their reasoning, or to explain their answers. Many students simply left questions blank if they were unsure how to complete the problem or simply wrote "I don't know" or "I don't remember how to do this". Perhaps these students never understood the basic conceptual concept of fractions when fractions were first introduced to them. Other students however, who did complete each problem, but answered incorrectly, did offer explanations that shine a light onto their thinking process. Below are four different students' explanations labeled as Student A, B, C, and D. Following students' explanations are my analyses of the student's mathematical skill set based on their provided reasoning.

Student A: Fraction Assessment

Question 1:

Locate each fraction on the number line. Mark the location with a dot and write the fraction underneath its location.

$\frac{1}{2}$ $\frac{3}{2}$ $\frac{6}{2}$



Student A placed 1/2 in between one and two on the number line, 3/2 in between three and four, and 6/2 was placed after five. This student's placement of these fractions indicated that they knew 3/2 was larger than 1/2 and 6/2 larger than 3/2 but they failed to truly understand the part to whole conceptual concept of fractions. This hindered such student from correctly placing the fractions on the number line. This student's misunderstanding of fractions is further brought to light with their explanation of question 2.

Question 2: When supplied the fractions 3/2 and 5/6 Student A placed 3/2 in between two and three on the number line and 5/6 between five and six. Again this student was unable to demonstrate the basic third grade fraction standard which simply states that students understand that fractions represent a part/whole relationship. When asked which is greater, 3/2 or 5/6, the second part of question 2, Student A replied, “3/2 is less than 5/6 because 5/6 has a larger number up top. Student A’s response indicates that they think of the fraction as two separate numbers instead of the singular representation of a number. Student A also fails to understand that the larger the denominator the smaller the “pieces” that make up the numerator. The knowledge that 3/2 is greater than 1 and 5/6 is less than one, meaning 3/2 is the greater fraction was not held by Student A. This tenth grade student, currently taking algebra, was unable to correctly answer any question on the fraction assessment including the third grade question. This indicates that they may have never mastered the very first Common Core Fraction Standard starting in third grade that simply states students will understand the conceptual representation of a fraction.

Other students also answered the question incorrectly by placing 3/2 in-between two and three on a number line and 5/6 in between five and six as well. These students also all said that 5/6 was a greater fraction that 3/2 and explained their reasoning as the following:

Student B 9^{th} Grade:

“3/2 is the smaller fraction because it has smaller numbers than 5/6.”

Student C 9^{th} Grade:

“5/6 is greater because it is farther away from zero on the number line.”

Student D 10^{th} Grade:

“No it’s less than because 5/6 it’s further down the number line.”

These participants were unable to accurately complete any question on the fraction assessment perhaps due to lack of conceptual understanding of fractions. These four students, along with nine others, are currently taking or have already attempted to take algebra while they may have never understood the concept of fractions.

Conclusion

Prior to conducting this study, I had hypothesized that the majority of participants would not be able to pass either assessment, be it the fractions assessment or the ratios and proportions assessment. The goal was to bring to light the incomplete mathematical foundations held by current high school students. I felt that students were simply being taught mathematics concepts correlating to their current grade level and not based on the current mathematical knowledge they brought to class. Due to this, students were not mastering each grade level standard before continuing onto higher level instruction. This lack of mastery creates huge gaps in student understanding hindering students from making the necessary content connections and gaining conceptual understanding. This study did in fact reveal the incomplete mathematical foundations current high school underclassmen hold with only 5 out of 20 participants being able to pass the fractions assessment. No participant was able to correctly answer all three questions in their entirety. Again, the fractions assessment was composed of a third, fourth, and fifth grade question. Based on my results, indicators show current students taking algebra and geometry may have never mastered elementary mathematics standards.

Only two participants passed the ratios and proportions test. With only 9 of the 19 participants having the ability to determine a basic ratio of 3:1 given data sets, the sample population lacked mastery of even the most basic ratio standard hindering them from accurately completing questions 2 and 3 which aligned to higher level standards. Because the questions in this assessment did not ask participants to explain their answers it is hard to evaluate students' thought processes. With only three students correctly answering question 3, this assessment not only revealed that the majority of students may not understand basic ratios, but that they also may lack an understanding the concept of proportionality and unit rates. Again, the ratios and proportions assessment was composed of a sixth and two seventh grade questions. Students currently taking algebra and geometry possibly have yet to gain a full understanding of middle school mathematics standards.

Suggestions

To aid in the creation of strong mathematical foundations it is suggested that educators, assess student understanding prior to beginning instruction on new material, ensure conceptual knowledge and understanding is mastered and not simply procedural knowledge, and incorporate tangible learning aids and manipulatives to aid in student understanding. Students' background knowledge and prior understanding should be assessed before instructing them on new material to ensure that students are learning within their zone of proximal development. A student's zone of proximal development is the skill level with which they are capable of mastery with the assistance and scaffolding of an instructor (Poehner M. 2012). Students will struggle to learn if they are taught at their independent level or within their frustration level. Educators should assess student mathematical understanding prior to instruction and teach them from the discovered baseline of knowledge. If students continue to be taught mathematics simply based on their current grade level the faulty mathematical foundations discovered in this study may continue to be produced. Teaching based on grade level has the potential to create gaps in understanding and hinder students from assimilating new knowledge with prior knowledge

(Poehner M. 2012). Students should be met where they currently are and taught based on their individual skill level. “If not, students who are only capable of haphazard mathematical applications that encompass short cuts they are unable to explain and eventually apply incorrectly, will continue to be the product of the current mathematical curriculum.”(Poehner M. 2012 pg.5)

To aid in students obtaining both conceptual understanding and procedural application skills educators should implement tangible learning aids, such as manipulatives in early instruction. Manipulatives are defined as, “concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students (Swan P. 2010).” Common manipulatives are fraction tiles, base ten blocks, geoboards, algebra tiles, and Cuisenaire rods. When learning new mathematical concepts students can be presented with different aligning manipulatives that will supply them with a visual representation of the numerical mathematic application taking place. Manipulatives greatly aid in students obtaining the conceptual understanding of why different procedural applications work (Swan P. 2010).

As presented in this study, having a strong mathematical foundation is very important. Overall, obtaining the necessary mathematics skills and aligning perseverance and problem solving skills better prepares one for life after high school, ensuring that all students are college and career ready. However many students do not have the strong mathematical foundations that will better prepare them for higher level mathematics courses. To aid in the production of strong mathematical foundations educators should assess student understanding before beginning instruction, teach both conceptual and procedural knowledge, and incorporate manipulatives into the classroom to aid in such development of conceptual knowledge. By adjusting the current mathematics curriculum to include these suggestions, more students will be able to gain a deeper understanding of mathematics. If students failed to master prior grade level standards, they are unlikely to ever master such standards. However, if students are taught at their current level of mathematical understanding, I would expect them to have a better chance of deepening their mathematical understanding.

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http://www.corestandards.org/Math/

Appendix 1A

Fractions Assessment and Aligning Standards

	Numbers & Operations Base 10	Operations & Algebraic Thinking	Fractions
Kindergarten	Work with numbers 11-19 to gain foundations for place value. CCSS.MATH.CONTENT.K.NBT.A.1	Understand addition, and understand subtraction. CCSS.MATH.CONTENT.K.OA.A.1 CCSS.MATH.CONTENT.K.OA.A.2	N/A
Grade 1	Use place value understanding and properties of operations to add and subtract. CCSS.MATH.CONTENT.1.NBT.C.4 CCSS.MATH.CONTENT.1.NBT.C.6	Represent and solve problems involving addition and subtraction. CCSS.MATH.CONTENT.1.OA.A.1	N/A
Grade 2	Use place value understanding and properties of operations to add and subtract. CCSS.MATH.CONTENT.2.NBT.B.5 CCSS.MATH.CONTENT.2.NBT.B.6 CCSS.MATH.CONTENT.2.NBT.B.9	Work with equal groups of objects to gain foundations for multiplication. CCSS.MATH.CONTENT.2.OA.C.3 CCSS.MATH.CONTENT.2.OA.C.4	N/A
Grade 3	Use place value understanding and properties of operations to perform multi-digit arithmetic. CCSS.MATH.CONTENT.3.NBT.A.2 CCSS.MATH.CONTENT.3.NBT.A.3	Solve problems involving the four operations, and identify and explain patterns in arithmetic. CCSS.MATH.CONTENT.3.OA.D.8 CCSS.MATH.CONTENT.3.OA.D.9	Develop understanding of fractions as numbers. CCSS.MATH.CONTENT.3.NF.A.1 CCSS.MATH.CONTENT.3.NF.A.2 CCSS.MATH.CONTENT.3.NF.A.3
Grade 4	Use place value understanding and properties of operations to perform multi-digit arithmetic. CCSS.MATH.CONTENT.4.NBT.B.4 CCSS.MATH.CONTENT.4.NBT.B.5	Use the four operations with whole numbers to solve problems. CCSS.MATH.CONTENT.4.OA.A.1 CCSS.MATH.CONTENT.4.OA.A.3	Extend understanding of fraction equivalence and ordering. CCSS.MATH.CONTENT.4.NF.A.1 CCSS.MATH.CONTENT.4.NF.A.2
Grade 5	Understand the place value system. CCSS.MATH.CONTENT.5.NBT.A.1	N/A	Use equivalent fractions as a strategy to add and subtract fractions. CCSS.MATH.CONTENT.5.NF.A.1 CCSS.MATH.CONTENT.5.NF.A.2

* http://www.corestandards.org/Math/Content

Appendix 2A Ratios and Proportions Assessment and Aligning Standards

	Numbers & Operations Base 10	**Operations & Algebraic Thinking**	**Ratios & Proportional Relationships**
Kindergarten	Work with numbers 11-19 to gain foundations for place value. CCSS.MATH.CONTENT.K.NBT.A.1	Understand addition, and understand subtraction. CCSS.MATH.CONTENT.K.OA.A.1 CCSS.MATH.CONTENT.K.OA.A.2	N/A
Grade 1	Use place value understanding and properties of operations to add and subtract. CCSS.MATH.CONTENT.1.NBT.C.4 CCSS.MATH.CONTENT.1.NBT.C.6	Represent and solve problems involving addition and subtraction. CCSS.MATH.CONTENT.1.OA.A.1	N/A
Grade 2	Use place value understanding and properties of operations to add and subtract. CCSS.MATH.CONTENT.2.NBT.B.5 CCSS.MATH.CONTENT.2.NBT.B.6 CCSS.MATH.CONTENT.2.NBT.B.9	Work with equal groups of objects to gain foundations for multiplication. CCSS.MATH.CONTENT.2.OA.C.3 CCSS.MATH.CONTENT.2.OA.C.4	N/A
Grade 3	Use place value understanding and properties of operations to perform multi-digit arithmetic. CCSS.MATH.CONTENT.3.NBT.A.2 CCSS.MATH.CONTENT.3.NBT.A.3	Solve problems involving the four operations, and identify and explain patterns in arithmetic. CCSS.MATH.CONTENT.3.OA.D.8 CCSS.MATH.CONTENT.3.OA.D.9	N/A
Grade 4	Use place value understanding and properties of operations to perform multi-digit arithmetic. CCSS.MATH.CONTENT.4.NBT.B.4 CCSS.MATH.CONTENT.4.NBT.B.5	Use the four operations with whole numbers to solve problems. CCSS.MATH.CONTENT.4.OA.A.1 CCSS.MATH.CONTENT.4.OA.A.3	N/A
Grade 5	Understand the place value system. CCSS.MATH.CONTENT.5.NBT.A.1	N/A	N/A
Grade 6	N/A	N/A	Understand ratio concepts and use ratio reasoning to solve problems. CCSS.MATH.CONTENT.6.RP.A.1 CCSS.MATH.CONTENT.6.RP.A.2 CCSS.MATH.CONTENT.6.RP.A.3
Grade 7	N/A	N/A	Analyze proportional relationships and use them to solve real-world and mathematical problems. CCSS.MATH.CONTENT.7.RP.A.1 CCSS.MATH.CONTENT.7.RP.A.2 CCSS.MATH.CONTENT.7.RP.A.2.A

* http://www.corestandards.org/Math/Content