Spring 2015

Design of an SAE Baja Racing Off-Road Vehicle Powertrain

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Zips Baja Off-Road Racing

2015 Powertrain

Design, Fabrication, and Testing

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Executive Summary

This report will present a thorough look at the design, analysis, construction, and testing of the 2015 Zips Baja powertrain. The main goals of the season were the following:

1.) Decrease System Weight
2.) Increase Power Transmission Efficiency
3.) Develop a Functional Gearbox Design
4.) Reduce Center of Gravity Height
5.) Increase Vehicle Speed
6.) Decrease Maintenance of the Transmission
7.) Meet all gear ratio requirements with a single speed reduction following the rubber belt CVT as in the ZB14 racecar.
8.) Incorporate Inboard Braking on the vehicle

Below is a table containing goals of the 2015 car (ZB15) compared to actual specifications from the 2014 car (ZB14). Also presented are the preliminary estimations from ZB15. For efficiency information, please refer to page 10 of this report as well as references [1] and [2] at the end of the report.

Parameter	**2015 Vehicle Goal**	**2014 Vehicle Actual**	**2015 Vehicle Actual**
Total Weight w/o Driver	360 lbs.	470 lbs.	387 lbs
Front Track	52″	52.4″	52.75′′
Rear Track	48″-50″	43.3″	47.25′′
Wheel Base	60″	59.2″	63.5′′
Maximum Speed	35 mph	31 mph	35 mph
Transmission Weight	120 lbs	155 lbs	134 lbs
CG Height	18′′	23′′	18.5′′
Tire Diameter	22″	22″	22′′

Table 1: 2014 Actual Vehicle Specifications, 2015 Vehicle Goals, and 2015 Actual Specifications

We also saw great success in the events of the competition which were heavily about the low gear ratio performance of the vehicle. In 2014 we placed 10th in hill climb, and then in 2015 we placed 13th in hill climb and 7th in suspension/traction. This, coupled with the feedback from drivers from endurance, ensures that we developed one of the best performing cars on the track. The car has also successfully completed endurance in 2014 and 2015, indicating further success.

What is SAE Mini-Baja?

SAE Baja is a collegiate program where each team builds an off-road race vehicle to a strict set of rules, and competes across the nation with other teams. SAE Baja originated in 1976 at the University of South Carolina, where the first competition took place. Baja Racing began as a program to teach young engineering students about design, metalworking, fabrication, problem solving, critical thinking, and automotive technologies. Today, hundreds of universities from around the world compete in national and international competitions to test their cars and represent their schools.

SAE Baja competitions consists of both Static and Dynamic events. The static events are typically a cost report, design report, design presentations, and a sales presentation. The dynamic portion can consist of traction events such as a hill climb, tractor pull or rock crawl, a suspension (or traction) and maneuverability course, acceleration, and the four hour endurance race. Each of the dynamic events tests both the entire car, as well as one subsystem rigorously. For example, acceleration and hill climb test the drivetrain, while maneuverability and suspension test how the suspension and steering were designed. A well-designed Baja car must be able to perform well in all of the events. Most important of all things, the Baja car must not break. Component failure is the major cause of the lack of success that the team has seen over the past decade. There always must be a well-educated decision that balances strength and weight of every component on the car.

History

The Solid Rear Axle Swing Arm and Transition Years

Starting in the late 1990s, a student on the University of Akron's Baja team thought of an interesting concept to couple the Engine to the wheels. This concept was a swing-arm axle, pictured below in figure 1. This type of powertrain perfectly integrated the suspension and drivetrain systems for a system which was, overall, very light and maneuverable. From my own investigations, the gear ratios that were used on the earlier cars were very good for a 14 – 15 inch diameter tire, which was what the cars used at that time. To comment on the maneuverability of this suspension type, the roll center in the rear of the car was essentially on the ground, which allowed for a healthy amount of over-steer (assuming a double a-arm suspension in the front and simply comparing roll center heights from front to rear). As any baja-goer understands, a small degree of over-steer capability is essential to be competitive in maneuverability events at competition, and is also advantageous for endurance.

Figure 1: Picture of 1996 Zips Baja Car with Solid Rear Axle Swing Arm Suspension/Drivetrain

At some point along the line, the competitions changed from being held on predominantly flat dirt tracks to involving some number of obstacles with significant height. Some of these obstacles were large rocks, trees, railroad ties, etc. The increased number of obstacles which would normally come 6-10 inches off of the ground warranted rear tires that could handle such obstacles. As I am sure you may imagine, a 14 inch diameter tire is not going to take kindly to climbing over a 12-15 inch diameter log. Thus, Baja SAE saw a shift from a 14 inch diameter tire to a 22 inch diameter tire. However, my investigations showed that no change in transmission gear ratio accompanied such a modification for the University of Akron. This meant that a gear ratio which allowed optimal engine performance with a 14 diameter tire was now being used for a 22 inch diameter tire.

The Sequential Synchromesh Gearbox

During the 2012-2013 Baja Season, the University of Akron's powertrain leader designed a four speed sequential synchromesh gearbox. The term sequential implies that the transmission cannot be shifted out of order, meaning that the gears must progress in the following fashion: 1-2-3-4-3-2-1-2… and so on. This four speed gearbox was used in conjunction with a centrifugal clutch on the motor crankshaft to transfer power from the crank to the gearbox via a chain drive during high revs of the engine, yet leave the engine de-coupled from the gearbox at its idle speed. The four-speed sequential synchromesh gearbox, although elegant in design, proved to be a bit too heavy for a practical racing application. It also had a bit of trouble during operation due to binding associated to the centrifugal cam mechanism which changed the gears. We will now discuss a few aspects of the 2013 car which merited improvement, which will inform our discussion on the design of the transmission for the 2014 and 2015 cars

Weight

The weight of the 2013 car was just over 550 pounds, with the transmission and engine contributing to over 150 pounds of that weight. This weight was not advantageous for success in the dynamic events of the competition, as well as causing issues when passing braking and enjoying the overall ride of the vehicle.

Shifting

The shifting in the sequential synchromesh gearbox was accomplished using a cylindrical cam. This cam, when rotated, would push shifting collars in a linear fashion along the transmission shaft, these collars then would couple the gears to the shaft in a torsional sense for the transmission of power. However, binding occurred during this rotation which served to make certain gears unattainable. However, the team learned that the interface between any type of shifting collar and driver-actuated cam must be carefully designed and prototyped to avoid issues that may be difficult to see from a design standpoint.

Multiple Forward Drive Gears

The concept of having multiple forward drive gears makes sense because you can keep your engine operating in an rpm range where it has optimum performance (torque and horsepower), however, this concept is slightly overshadowed by the use of rubber belt continuously variable transmissions (CVTs) in Baja Racing. A properly tuned rubber belt CVT is just as effective at letting the engine operate in an rpm spectrum where it is most effective as multiple forward drive gears are.

Conclusions from Past Performance

From my observations of Akron Baja cars in the late 1990s and early 2000s when we experienced a great deal of success I learned that a single gear reduction following a CVT is the most lightweight solution to effective power transfer in this type of application. I also learned from my observations of the unchanged gear ratios on newer cars with 22 inch diameter tires that some re-calculation needed to be done in terms of finding the new optimum post-CVT reduction. Finally, I learned that multiple forward drive gears are heavy and cumbersome to performance in this type of racing application while a rubber belt CVT drive offers the best-rounded performance and is much simpler.

Transmission Selection

2014 Zips Baja Car Powertrain

I was fortunate enough to be able to lead powertrain design on the Baja team for two years in a row, which allowed me to not only test theories and modify them, but also to learn from mistakes. A brief review of the 2014 car powertrain design is in order, as it informs many of the design decisions made for the 2015 Baja car.

The 2014 Akron Baja Car (ZB14) uses a Comet 780 series CVT coupled to a two stage chain reduction to couple the Engine to the 22 inch diameter tires. The layout of this chain reduction was designed with two things in mind. First, there was a desire to see the acceleration benefits of raising the Engine higher in the car, thus raising the center of mass of the car and the subsequent longitudinal weight transfer during acceleration. This increase in longitudinal weight transfer would theoretically provide more tractive ability for the rear tires during an acceleration run. The second objective of the chain layout was to reduce the wheelbase as much as possible.

We found that there was not too much benefit gained by the increased height of the center of gravity during acceleration. In addition, the negative effect this higher center of gravity had on lateral handling and rollover **completely** outweighs any positive acceleration effects gained by a raised engine. Ideally, the bottom of the engine should be placed as low as possible for the best handling car. The shorter wheelbase accomplished with the unique packing of the driveline did help with navigating tight obstacles in maneuverability, as demonstrated by our 22nd finish in maneuverability at the 2014 competition in Illinois (See results compilation in appendix for full placement details).

This car was also used to verify the effectiveness of the gear reduction ratio calculated from theory that will presented later in this report. From this testing it was found that only a slight modification to the originally calculated gear ratio was necessary to reach a well-performing vehicle.

Figure 2: ZB14 Chain Drive Transmission w/ Comet 780 Series CVT and Polaris RZR Axles

Chain Drive versus Spur Gear Drive

Before the design of the powertrain for the 2015 Akron Zips Baja Racecar (ZB15) could begin, an important decision needed to be made. This decision was whether to use an uncovered chain reduction similar to ZB14 or to use an enclosed, continuously-lubricated spur gearbox. Both selections have strengths and weaknesses and each aspect of what makes a sound Baja powertrain must be examined to determine which type is better. Each of the categories that will serve as the performance metrics by which to compare the transmission types will be measured on a scale from 1 to 5. If a transmission type receives a 5 in any particular category, it will mean that that particular transmission type fulfills that performance metric better than any other transmission type known to this application.

Weight

As weight is of overwhelming pertinence in a racing application, it must be the first metric under consideration. The double chain reduction has been shown to be a lightweight setup, but requires many heavy mounting tubes on the frame in its original configuration. A preliminary survey of teams with well-tested single-speed gearboxes shows that an optimized weight for gears and gear case is around 12-15 pounds. Combining this with 30 pounds for the CVT/belt and 50 pounds for the engine yields an approximate transmission weight (ignoring CV axles) of roughly 92-95 pounds. This is in comparison to the 109.46 pounds that the transmission on the Akron Baja 2014 car weighed in at. However, some consideration must be made that this gearbox would be a first iteration for our team, so we must add at least 10 to 15 pounds in order to be conservative until we learn how to make gears well. Even with the extra conservatism, the gear drive will receive a rating of **4** for weight while the chain drive receives a rating of **3** for weight.

Cost

Chain drives are very common for this type of application in the industry. For example, most lawn equipment and ATVs will use a chain or belt drive as a primary means of power. For this reason, commercially available parts make constructing a chain-drive transmission cheap and quick. For this reason, the double chain reduction of the ZB14 racecar receives a rating of **5** for cost. Lightweight, racing spur gears are typically “one off” jobs that are highly labor intensive. This is because the face width of the gear is not necessary throughout the web of the gear, so plenty of machining must be done to optimize weight. Gears must also be heat treated to provide acceptable tooth performance, which adds in addition cost and time. For these reasons, a spur gear reduction receives a rating of **2** in cost for a Baja racing application.

Design Simplicity

For chain drives, much of what comes commercially available needs no modifications to be ready for service on a Baja car. This makes for a transmission that is more “plug-and-play” than a spur gear drive. Chain drives also have the advantage of many teeth on a sprocket sharing load, such that teeth may be very small and sprockets may be made out of low quality

steel. For a gear drive, only one tooth on each pinion/gear is carrying the load much of the time, so great care must be exercised in designing the teeth properly. In addition to additional complexity of a spur gearbox in loading, appropriate center-to-center distances between shafts are more complicated and need to be much more precise on a spur gear drive. On a chain drive, any errors in center-to-center distances may be easily accommodated by creating a longer or shorter chain. However, on a spur gear drive, even an error of .030 inches could be deadly to the proper meshing of spur gears. For this reason, the chain reduction receives a rating of **5** when considering design simplicity, and the gear drive receives a **3**.

Familiarity

Obviously, since a chain drive has always been used on Akron Baja teams, and a spur gear reduction has never (or has rarely) been used, the chain drive receives a rating of **5** while the spur gear reduction receives a rating of **3**.

Ease of Manufacture

The metric ease of manufacture comes in two parts. The first part is the ease by which the rotating components of the transmission may be manufactured. The second is the ease by which the components may be attached to the chassis of the vehicle. For the chain reduction, component machining was simple. It only took some simple lathe and mill work to manufacture the required shafts and hubs. However, the chain reduction of the ZB14 car required a very intricate setup on the chassis to accommodate each of the rotating shafts. The spur gearbox required plenty of CNC (Computer numerical control) machining not only to lighten the gears, but also to manufacture the housing which would surround the gears and contain bearings. On the other hand, the gearbox mounts into the car with simply two bolts. In summary, chain drives require more intricate mounting on the chassis itself while requiring less machining work to manufacture. Furthermore, gear drives require much more complex machining while their mounting schemes into a chassis may be much simpler. Thus, in terms of ease of manufacture, we rate ZB14’s chain reduction **4** and the spur gear reduction **5**.

Lubrication

Properly lubricating metal surfaces which are in sliding or rolling contact is of great interest in engines and transmissions. This aspect of transmission design is something that design judges at competition regard as sacred. The chain reduction setup of ZB14 was not protected from dirt and mud, and was therefore acceptable to increased wear from the elements. This was seen as a major downfall of the design of the system by judges at competition. A spur gear reduction, when properly designed, has a gear case which has an oil bath which continuously lubricates the gears during operation. This extends the life of metal surfaces and can, with the right oil, allow even smaller face widths than with a lower quality oil. Because of the great weight placed on this aspect of the transmission at competition, the spur gear reduction earns a rating of **5** while the double chain reduction earns a **1**.

Powertrain Efficiency

For a chain reduction, studies on bicycle roller chain have shown average efficiencies of roughly 92-95% [1]. For an external spur gear reduction, the efficiency associated with each gear set meshing is normally in the range of 97-99% [2]. Assuming that the efficiency of the rubber belt CVT and final drive Rzeppa joint axles are 94% each and using the following equation we may find the minimum possible efficiency for both drives:

$$\eta_{chain\ drive\ overall} = \eta_{belt} * (\eta_{chain})^2 * \eta_{cv\ axle} = 0.94 * (0.92^2) * 0.94$$
$$= 0.75\ (75\%\ Eff\ min)$$

$$\eta_{spur\ gear\ drive\ overall} = \eta_{belt} * \left(\eta_{gear}\right)^2 * \eta_{cv\ axle} = 0.94 * (0.97^2) * 0.94$$
$$= 0.83\ (83\%\ Eff\ min)$$

It is clear from this equation that a spur gear reduction is far more efficient than a chain reduction. For this reason, the chain reduction of the ZB14 is rated **2** for powertrain efficiency and the spur gear drive is rated **4**. Note that the spur gear drive, once optimized for fluid bath level, heat treatment (distorting teeth), and shaft misalignments, would be rated 5. However, with our first year design we must account for errors in mesh losses and fluid churning losses associated with a new design.

Packaging

In the ZB14 chain reduction, center-to-center distances had to be large to avoid excessively large chain angle with respect to the line connect the centers of the shafts. If this angle were to become very large, the number of teeth that were carrying the load on the smaller sprocket would become less and potentially cause sprocket damage. With a spur gear reduction, center-to-center distances are small and the gears can be oriented in a way which optimizes space usage, without concern of the necessity to tension chains. For this reason, the chain reduction receives a rating of **3** for packaging while the spur gear reduction receives a rating of **5**.

Figure 3: Packaging and Serviceability of ZB14 Drivetrain

Serviceability

The term serviceability in this category refers to the ease by which rotating parts may be removed from the driveline, replaced, and then put back onto the car. The chain reduction involves the de-tensioning of chains and bolts which hold bearing housings to the chassis, making for roughly 2 hours of work to swap everything on the system. The spur gear reduction

will obviously involve draining of transmission fluid, removal of the gearbox from the car, separation of the gearbox, replacing components, and then re-sealing the gearbox, a process that could take up to 12 hours. However, with some clever design work, components which need to be removed often, such as axles and CVT pulleys, may be designed in such a ways as to allow their removal without removing the gearbox itself. The internal components of the gearbox, however, will always be more difficult to replace than the components of a chain drive. For this reason, the chain drive receives a score of **4** in serviceability while the spur gear reduction receives a score of **3**.

Safety

Operator safety is always a huge concern, whether you are at a Baja competition or in the industry. The safest transmission would not allow any accidents to happen during operation. Clearly, with a stationary gear case surrounding rotating gears of the transmission, a bystander is not likely to get his/her hands caught amongst rotating components. However, with the chain reduction of ZB14, even with the chain guards in place, it is still possible to injure oneself with some degree of ease on the rotating components (or the sharp-edged chain guards themselves). Moreover, if a piece of some rotating component were to become detached from the drivetrain during operation, a gear case is much more likely to contain the wreckage than chain guards, making it safer. For these reasons, the double chain reduction receives a score of **2** for safety while the spur gear reduction earns a **5**.

Final Results

The spur gear reduction has emerged victorious! Observing table 2 below, we see by the final tally of the design decision matrix that the spur gear reduction is a more optimal design

Transmission Design Decisions	Weight	Cost	Design Simplicity	Familiarty	Ease of Manufacture	Lubrication	Powertrain Efficiency	Packaging	Serviceability	Safety	**TOTALS**
ZB 2014 Double Chain Reduction	**3**	**5**	**5**	**5**	**4**	**1**	**2**	**3**	**4**	**2**	**34**
ZB 2015 Double Spur Gear Reduction	**4**	**2**	**3**	**3**	**3**	**5**	**4**	**5**	**3**	**5**	**37**

Table 2: Transmission Decision Matrix Comparing ZB14 to ZB15

than the chain reduction of ZB14.

Testing: Engine Dynamometer W.O.T. Performance

After the preliminary hardware selection stage, a powertrain designer's next point of interest is in designing the appropriate gear ratios that the vehicle will need to perform. These gear ratios will inevitably, after some hardware tuning during setup, allow maximum performance usage of the internal combustion engine. It then follows that a comprehensive understanding of how the engine performs over its rotational speed spectrum is absolutely critical to the aspiring powertrain designer.

To test the single cylinder engine for the Baja car, we took the engine to one of our sponsors, namely Cuyahoga Valley Career Center's automotive development labs. At the lab, we were able to setup and use a small engine water-brake dynamometer to get performance curves for the engine. The water-brake dynamometer (or dyno) was equipped with data acquisition and load control for the user, but did not have any throttle control attached at the time. With the help of some other members of the team, the curves seen below in figure 4 were obtained in a few simple steps:

1. The engine was fired and allowed to warm up to optimum operating temperatures.
2. After warm-up, the throttle was opened until the engine was running at maximum speed with no load (approximately 3800 rpm).
3. With the data acquisition system started, the engine was slowly loaded using water jets in the dyno, a process which was controlled by the user who sat in front of the main heads-up-display.
4. As the engine was loaded down (still at Wide open throttle (W.O.T.)), the rotational speed of the engine dropped until it reached roughly 3000-3100 rpm. At this point, which we may now see as the maximum torque point of the engine, the engine began to stall out.

5. Rather than stalling the engine, once this maximum torque position was reached, the load was removed from the engine and subsequently it sped back up to 3800 rpm. Steps 3 and 4 of the process were then repeated 10 times to obtain a valid data set.
6. To complete the performance curves of the engine, W.O.T. data was still needed for the idle to maximum torque speeds of the engine (roughly 1800 rpm to 3000 rpm). To obtain this information, the engine throttle and load were brought up together to maintain a constant rotational speed of approximately 2400 rpm. Once W.O.T. conditions were achieved at this constant rotational speed, the water jets were used to load and unload the engine, which subsequently slowed the engine down and sped it up again. This process was used to obtain the W.O.T. performance information between idle speed and 3000 rpm.
7. Step 6 was repeated 10 times to obtain an acceptably diversified data set.
8. The data was removed from the dyno using a USB drive and inputted into excel. In excel, the raw data were sorted, organized, and then plotted to obtain figure 4 below

From figure 4 on the next page, we may make some important observations about the gross torque (averaged over two revolutions of the crankshaft). It appears that the higher we set the idle speed of the engine, the more torque will be available during clutch slipping at the start of motion. Another interesting observation is that the torque performance of the engine over the useful rpm spectrum (1800-3500 rpm) of the engine is, for all practical purposes, constant. The next important observation is that the maximum torque of the engine does actually meet the specification of the engine (14.5 ft-lbs).

Turning our attention now to the horsepower graph (the lower line represented as squares in figure 4), a few more observations can be made. First and foremost, we note that the maximum horsepower of the engine does not come anywhere close to the advertised 10 horsepower. In reality, the maximum horsepower of the engine is right around 8.9 horsepower. Finally, from the graph we see that the horsepower of the engine drops off significantly after 3600 rpm. This tells us that at maximum vehicle speed, the engine is not

likely to spin faster than 3600 rpm, which will help us when designing our high gear ratio in the next section of the report.

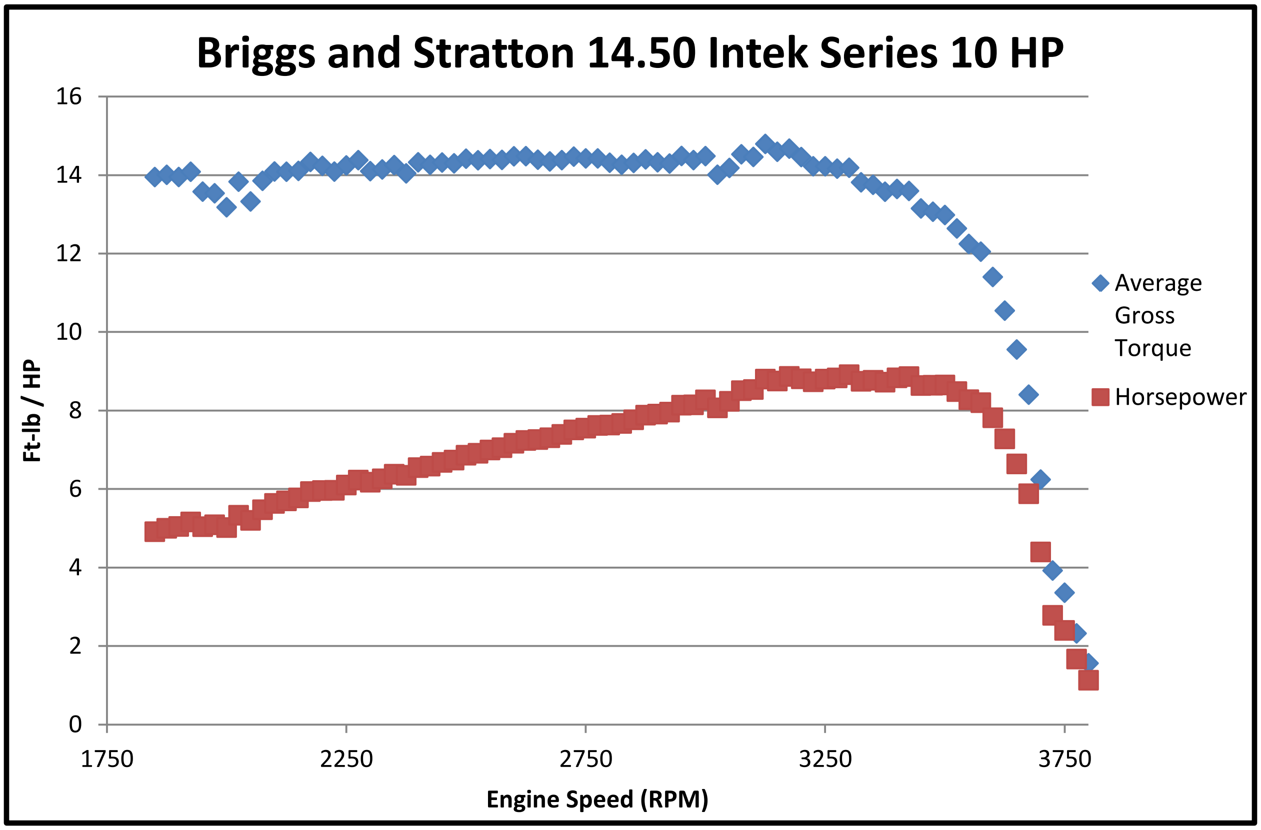

Figure 4: Water-Brake Dynamometer Wide-Open-Throttle (W.O.T.) Performance of the Baja Engine

Design: Gear Ratios

The transmission has the responsibility of delivering ample low-end torque to the wheels, which can be determined by a combination of tire gripping capabilities and the maximum load the vehicle is expected to taxi. Furthermore, the transmission must ensure that the internal combustion engine is allowed to operate in a spectrum where it is most effective.

Low Gear: Tractor Pull and Tire Tractive Ability

The accumulation of the information that will allow a proper low gear ratio calculation may be attributed to two team members from the 2013 Zips Baja race season. The model for a sled pull and calculations associated to maximum tire tractive ability can be found in the report of Peter Fetzer [3]. The tire data from which these maximum tire tractive ability calculations come from is contained in the report of Mack Monegan [4]. For our purposes here, we will present and explain the model of the sled pull contained in Fetzer's analysis as well as his final conclusion for the maximum required rear axle torque to pull the sled with 22 inch diameter tires. Further, the maximum tire tractive ability during sled pull will be presented without proof in this report, and the author refers interested readers to Fetzer's work for a full explanation.

Fetzer created a model for the necessary rear axle torque to tow the sled based off of approximate information collected from an Akron Baja SAE team member from the 2011 Kansas competition hosted by Pitt State. Below is a schematic which accurately represents the mechanics of the tractor pull, with the 2013 Baja car shown.

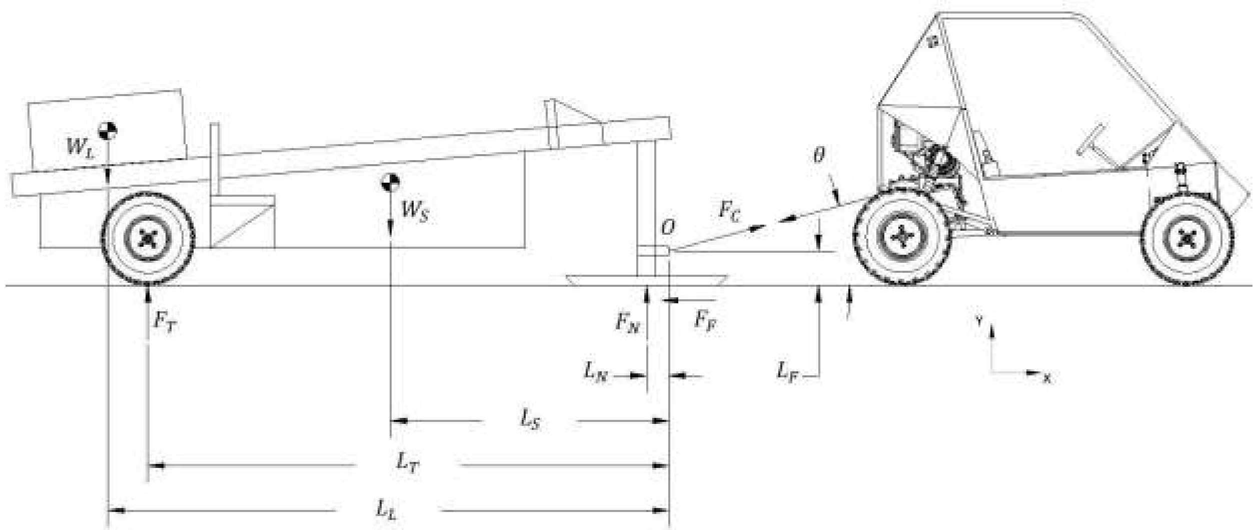

Figure 5: Schematic model of a SAE Baja car in a Sled-Pull Event (Courtesy of Fetzer [3])

The overall objective of the sled pull model is to determine the amount of force that the rear tires of the Baja car must deliver to the ground in order to taxi the load of the sled. A free body diagram on the Baja car yields the relationship between the tension in the cable which connects the tractor to the Baja car and the required rear axle torque of the Baja car with 22 inch tires. This is shown in equation form below:

$$T_{Low\ GR} = \frac{11}{12} * F_C * cos\theta \quad \text{(ft-lbs)} \tag{1}$$

Executing a free body diagram on the sled and using the linear and rotational forms of Newton's second law, Fetzer [3] derived a symbolic relation for the tension in the cable:

$$F_C = \frac{-W_L*(L_L-L_T)+W_S*(L_T-L_S)}{L_T\frac{cos\theta}{\mu}+L_T sin\theta - L_N\frac{cos\theta}{\mu}-L_F cos\theta} \tag{2}$$

Below is a table which shows the constants Fetzer used in his analysis, as well as a description of each for clarity:

PARAMETER	VALUE	DESCRIPTION
$\boldsymbol{\theta}$	15^o	Approximate Angle of Cable at 2012 Wisconsin Tractor Pull
$\boldsymbol{W_L}$	1,125 lb	Weight of the payload on the sled
$\boldsymbol{L_L}$	4 ft	Height of the hitch point on the sled to the center of mass of the movable sled weight
$\boldsymbol{L_T}$	11 ft	The length from the hitch point on the sled to the centerline of the sled axle
$\boldsymbol{W_S}$	1000 lb	Approximate Weight of Sled
$\boldsymbol{L_S}$	6 ft	The length from the hitch point on the sled to the center of mass on the sled
$\boldsymbol{L_N}$	2 ft	The length from the hitch point on the sled to the centerline of the support plate
$\boldsymbol{\mu}$	0.45	The coefficient of friction between the sled and the driving surface
$\boldsymbol{L_F}$	0.5 ft	The length from the hitch point on the sled to the center of mass of the movable sled weight

Table 3: Constants used by Fetzer [3] in evaluating equation 2

Fetzer then evaluated equation 2 using the constants in table 3 above to find:

$$F_C = 437\ lbs$$

This result may be substituted into equation 1 to find the minimum required rear axle torque to tow a sled as:

$$T_{Low\ GR} = 387\ ft - lbs$$

My objective, however, in designing the powertrain was to give far more torque to the rear axle than what could actually be delivered, thus giving the driver the ability to lose traction with his rear tires. This technique is highly advantageous not only for maneuverability, but also for hill climb, endurance, and suspension/traction events. From very extensive testing, it has been determined by our team that having extra torque beyond what the tires can actually deliver is critical to climbing rocky slopes and exhibiting over-steer characteristics in cornering (a desirable trait). According to Chris Mileti, who spent a great deal of time designing race tires for asphalt as well as off-road tires at Goodyear, the tread lugs on off-road tires do not behave like a formula car tire, and extra torque is advantageous to allow the tread lug to act as a lever to propel the vehicle over obstacles.

Using real tire testing data from the report of Mack Monegan [4], the normal and lateral forces under the tire may be obtained for different slip ratios. Using this data as well as correction factors for asphalt (since the test was done on sandpaper), Fetzer obtained **very** approximate values for the coefficient of friction between the off-road tires and an asphalt surface.

$$\mu_{min} = 0.788$$

$$\mu_{min} = 1.183$$

Using our approximated weight under a single rear tire as 200 lbs (roughly one tire of the ZB15 total weight goal plus a one-fourth of a drive), we find the minimum and maximum driving forces until slip (for one rear tire) as

$$F_{D_{min}} = F_N * \mu_{min} = (200\ lbs) * 0.788 = 158\ lbs$$

$$F_{D_{max}} = F_N * \mu_{max} = (200\ lbs) * 1.183 = 237\ lbs$$

Using this information, we can calculate the maximum theoretical torque that can be put through the rear axle before tire slip is certain to occur as the following:

$$T_{Traction\ Limited} = 2\ tires * F_{D_{max}}\ per\ tire * \frac{11}{12} ft = 2 * 237 * \frac{11}{12}\ ft * lbs$$

$$T_{Traction\ Limited} = 434.5\ ft * lbs$$

Thus, we need a low gear ratio which will provide a rear axle torque which is much greater than 435 ft*lbs if we want to guarantee slip capability on dirt. In the next section, I will explain the requirements of the high gear ratio and then excel will be used in conjunction with engine data to select the best possible CVT and post-CVT gear reduction for the two criteria. With an engine that outputs 14.5 ft*lbs of torque, this means our minimum gear ratio in low gear must be:

$$GR_{min} = \left(\frac{435}{14.5}\right) = 30$$

In summation:

PARAMETER	REQUIRED VALUE
Low Gear Ratio	30:1 or greater
High Gear Ratio	To be determined

Table 4: Current knowledge of required gear ratios

High Gear: Maximum Speed and Road-Load Power

The objective of the high gear ratio of the ZB15 racecar is to allow the maximum amount of available engine power propel the car forward. Top gear (high gear) must not inhibit the vehicle from attaining its maximum velocity at the rotational speed of the engine which provides the most horsepower. The CVT of the vehicle was chosen such that the required high gear ratio and low gear ratio could be obtained with a single speed reducer following the CVT. In order to determine what high gear ratio we wanted, I first looked at the maximum possible speed from a road-load power equation. To derive this equation, we look at a free body diagram of the vehicle as shown below:

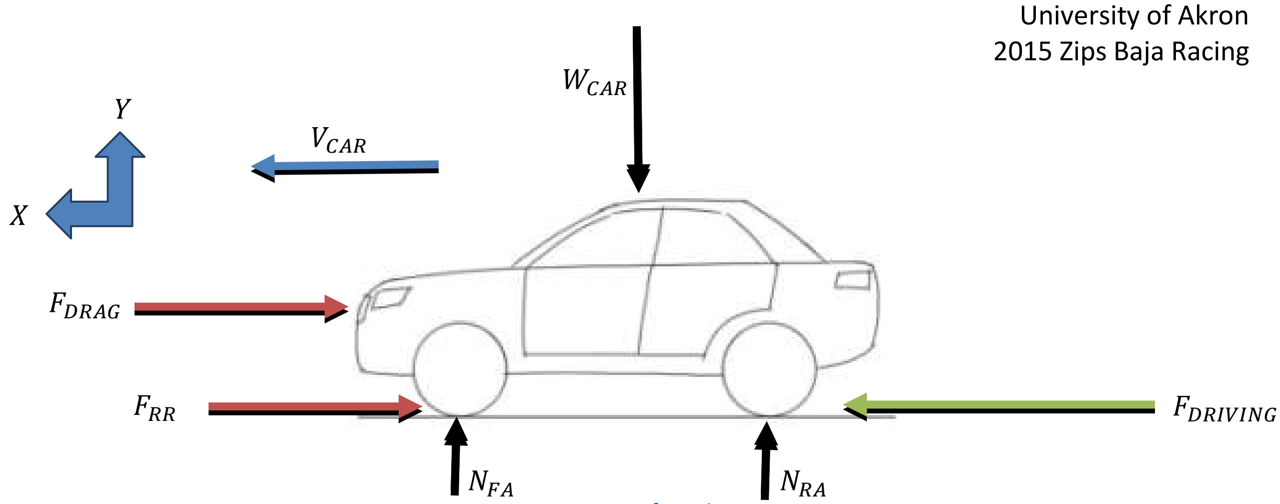

Figure 6: Free Body Diagram of a Vehicle in Motion

Using Newton's Second law in the x direction, with positive to the left, we find the following for a vehicle moving at a constant maximum speed:

$$\sum F_x = m * a_x \rightarrow F_{DRIVING\ MAX} = F_{DRAG} + F_{RR}$$

Expanding this equation to show the components of aerodynamic drag and rolling resistance, we have:

$$F_{DRIVING\ MAX} = \frac{1}{2} * \rho_{air} v_{max} A_{Frontal} C_D + C_{RR} W_{CAR}$$

To obtain our final expression which involves power, we multiply the entire equation by v_{max}, the maximum constant velocity of our vehicle.

$$Power_{MAX} = \frac{1}{2} * \rho_{air} {v_{max}}^2 A_{Frontal} C_D + C_{RR} W_{CAR} v_{max} \qquad (3)$$

Equation 3 is more commonly known as the road-load-power equation for a vehicle moving at constant maximum speed. Given all of the constants in the equation, the maximum possible vehicle velocity can be found through iteration. In 2013, Austin Beery conducted coast down testing on a University of Akron Baja Race vehicle to experimentally determine the drag coefficient and rolling resistance values. The values he obtained will be used in this equation for a decent approximation. The tabular values of the necessary constants in equation 3 are shown below in table

PARAMETER	VALUE	DESCRIPTION
$\boldsymbol{\rho_{air}}$	$2.38 * (10^{-3})\dfrac{slugs}{ft^3}$	Density of Air at Standard Atmosphere
$\boldsymbol{A_{Frontal}}$	$14\ ft^2$	Frontal Surface Area Including Firewall, Control Arms, Chassis, and Tires
$\boldsymbol{Power_{MAX}}$	$4112\dfrac{ft * lb}{s}\ or\ (7.48\ hp)$	Maximum Power Available at Rear Wheels (Obtained from Max horsepower of 8.9 on figure 4 and approximate efficiency of 84% as described on page 12)
$\boldsymbol{C_D}$	1.08	Aerodynamic Drag Coefficient determined by Beery [5]
$\boldsymbol{C_{RR}}$	0.068	Coefficient of Rolling Resistance determined by Beery [5]

Table 5: Parameters of the Road-Load Power Equation for ZB15

Using MATLAB, the maximum possible vehicle speed for the ZB15 racer was evaluated to be (See Appendix for MATLAB code):

$$v_{max} = 34.3\ mph$$

To evaluate what our high gear ratio needs to be for the engine to be able to push the car at this speed or higher, we carry out the following calculation

$$GR_{Top} = \frac{N_{EMP}*2*\pi*R_{Tire}*3600}{v_{max}*60*5280} \qquad (4)$$

Using the values in the table below, the required high gear ratio of the car may be calculated.

PARAMETER	VALUE	DESCRIPTION
$\boldsymbol{N_{EMP}}$	$3600\ rpm$	Engine rotational speed at maximum power (Found from performance curve on figure 4)
$\boldsymbol{R_{Tire}}$	$\frac{11}{12} feet$	Radius of the rear tires
$\boldsymbol{v_{max}}$	$34.3\ mph$	Maximum Possible speed from the road-load power equation

Table 6: Necessary Constants to Evaluate Equation 4

$$GR_{Top} = 6.87$$

Thus our high gear ratio must be no greater than 6.87 if we hope to achieve our maximum speed. Alternatively, the gear ratio limited top speed is described by the following equation (see appendix for nomenclature):

$$V_{car_{GR\ Limited}} = (N_{EMP}/GR_{Top}) * ((2 * \pi)/60) \frac{rad * min}{rev * sec} * (R_{Tire}) * (3600/5280) * \frac{sec * miles}{hour * feet} \doteq miles\ per\ hour\ (mph)$$

Below is a table which summarizes what we know about our desired gear ratio for ZB15.

PARAMETER	VALUE
Low Gear Ratio	30:1 or greater
High Gear Ratio	6.87:1 or less

Table 7: Information about the required Gear Ratios for ZB15

CVT Selection from Gear Ratio Requirements

Using excel, 4 different types of rubber belt CVTs from 3 different manufacturers were compared to determine which CVT would allow performance to be maximized with a single speed gear reduction following the CVT. The following is a list of the rubber belt CVTs that were included in the analysis:

1.) Comet 790 Series (Used on Akron Baja Cars in late 1990s and early 2000s)
2.) Comet 780 Series
3.) CVTech
4.) Gaged

The conclusion that I arrived at was that the **Comet 780** Series Provided the best balance of optimizing engine performance while in high gear while also allowing a prescribed amount of tire slip in low gear.

Figure 7 below shows a small snippet of the excel document which was used to compare the CVTs. This figure shows a comparison between the Comet 780 Series CVT and the Comet 790 Series CVT (The 790 Series has been used for Akron Baja for as far back as I could gather). In the leftmost column we see various possible values for a gear ratio following the CVT, meaning the gear reduction of a chain drive or gearbox after the rubber belt CVT. At the top left of the figure we see the low and high ratios of each CVT model, which I have adjusted to be more practical from the theoretical values given by the manufacturer. The values in the 2nd through 5th columns (some of which are highlighted orange and blue) are representative of the driving force at the interface between the rear tires and the ground for each CVT model depending on the following gear reduction (leftmost column). Remember from our earlier discussion that the maximum force the two tires can, combined, deliver to the ground is in the neighborhood of **475 lbs**. Thus, a value that is likely to cause slippage is highlighted orange while a value that will not cause slippage is highlighted blue. Recall from our earlier discussion that we want to be right in the middle of the orange range.

The values in the last two columns on the right of figure 7 are the engine speeds that the high gear ratio of the CVT would produce if the car was traveling at its maximum speed. These engine rotational speeds were cross referenced with figure 4 (engine performance curves) to determine if, at that rotational speed, the engine actually produces its maximum horsepower. The values that are highlighted green fall within, or very close to, the maximum horsepower rpm range of the engine.

[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	480.3657143	[redacted]	437.4857143	[redacted]	[redacted]	[redacted]
[redacted]	495.3771429	[redacted]	451.1571429	[redacted]	[redacted]	[redacted]
[redacted]	510.3885714	[redacted]	464.8285714	[redacted]	3034.43115	[redacted]
[redacted]	525.4	[redacted]	478.5	[redacted]	3123.679125	[redacted]
9	540.4114286	[redacted]	492.1714286	[redacted]	3212.9271	[redacted]
9.25	555.4228571	[redacted]	505.8428571	[redacted]	3302.175075	[redacted]
9.5	570.4342857	[redacted]	519.5142857	[redacted]	3391.42305	[redacted]
9.75	585.4457143	[redacted]	533.1857143	[redacted]	3480.671025	[redacted]
10	[redacted]	[redacted]	546.8571429	[redacted]	3569.919	[redacted]
[redacted]	[redacted]	[redacted]	560.5285714	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	2904.8382
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	2970.85725
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	3036.8763
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	3102.89535
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	3168.9144
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	3234.93345
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	3300.9525
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	3366.97155
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	3432.9906

Figure 7: Portion of CVT comparison Excel Document which compares the 780 Series Comet CVT to the 790 Series Comet CVT

From this information, the following hardware and post-CVT drive gear ratios were selected:

- **CVT Selection**
  - Comet 780 Series Rubber Belt CVT
    - Nominal High Ratio: 0.7
      - Estimated Actual High Ratio: 0.73-0.74
      - Tested Actual High Ratio: 0.74-0.78
    - Nominal Low Ratio: 3.7
- **Post CVT Gear Ratio Selection**
  - Speed Reducing Ratio: 9.1875:1

These selections yield the following final drive ratios:

$GR_{Final\ High}$ =6.79 (Meets requirement of less than 6.87)

$GR_{Final\ Low} = 34$ (Meets requirement of greater than 30)

Design: Rotating Transmission Components

Shaft Design

Keeping in mind that the transmission shafts used for this application are not likely to see more than a few thousand maximum load cycles, the yield strength of the material will be used as an approximation to the fatigue strength. Thus a mere static yielding stress analysis will be performed on the input shaft to the transmission and the intermediate shaft inside the transmission. This elementary analysis yields a shaft of competitive size in this racing application.

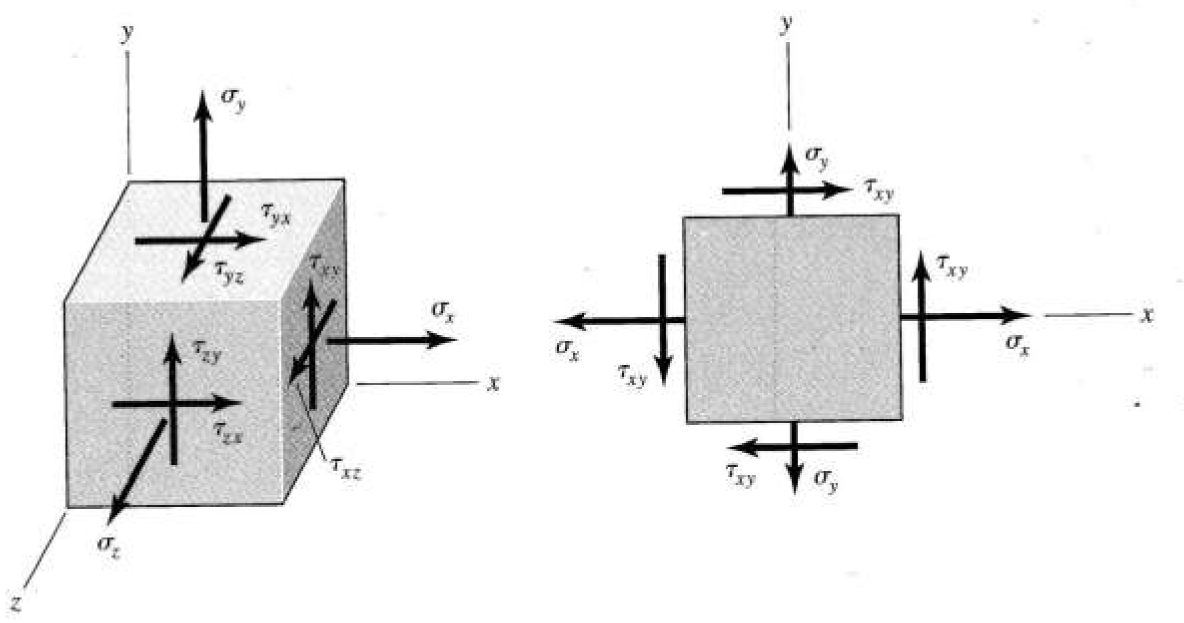

Figure 8: A material element with a complex 3-dimensional stress state shown

For hand calculations the 3-dimensional stress state shown in figure 11 was combined using the Distortion Energy Theory, which is shown by the following equation.

$$\sigma_{von\ mises} = \frac{1}{\sqrt{2}}\sqrt{\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + (\sigma_z - \sigma_x)^2 + 6 * ({\tau_{xy}}^2 + {\tau_{yz}}^2 + {\tau_{zx}}^2\right]}$$

As previously stated, the yield strength of the material (1045 Cold Finish Steel) shall be used as the distortion energy stress limit. Finally, in the interest of weight, factors of safety between 1 and 1.2 were used, even though in a production vehicle this would cause eventual failure of components. This low factor of safety is justified because of the limited application time of the races and the push for very low weights. The shaft sizes obtain using these safety factors have proven themselves time and time again in testing to be satisfactory. Table 8 below shows the final design information of the transmission shafts:

SHAFT	DIAMETER	MATERIAL	ADDITIONAL HEAT TREATMENT
Input Shaft	0.75 inches	1045 Cold Finish Steel	No
Intermediate Shaft	0.875 inches	1045 Cold Finish Steel	No

Table 8: Description of the final design of the transmission shafts

Hub Design

To connect the final drive gear to the Rzeppa joint CV axles, a hub is needed. The material that was selected for this application was 1045 Cold Finish to provide the best combination of cost, lack of need for additional heat treatment, and strength. This hub has a bolt circle which fastens it to the gear and a spline which accepts the inboard joint of the CV axle (continuous velocity axle). These hubs were analyzed using Solidworks FEA simulation with a maximum torque of (14.5 ft*lbs)*34 = 493 ft*lbs. Material was removed from the hubs until the factor of safety fell below 1.5. This was deemed acceptable for two reasons:

1.) By the time power reaches the final drive gear, the torque pulses coming from the engine are fairly smoothed out and the final drive hubs are likely to see something which approximates a smooth 493 ft*lbs. This is what I observed during my manual transmission testing at the Lubrizol Corporation.
2.) This sizing was used with zero failures on the ZB14 racecar.

CV Axle Shaft Design

Similar to ZB14, stock Polaris RZR 900 axles will be used again. However, the model that I have selected is a few years old and is much smaller and lighter than the axles used on ZB14. In total, these older style axles will be 12 pounds lighter than the axles on ZB14 (6 pounds per axle). In order to accommodate this older style axle, I need to design the shaft that connects the two ends of the CV axle (or the two "joints" if you like). The diameter of this axle is dictated by the geometry of the ends to be around 0.78 inches in diameter. However, Polaris Industries has established and tested a very thorough heat treatment procedure that involves some blend of through hardening and induction hardening. Therefore, I needed to be very cautious about my materials selection choice as well as any heat treatments I applied.

Figure 9: Outboard End of the CV Axle, showing the axle shaft (bottom left), bearing balls, inner cage, and outer housing

Considering how a CV axle works and observing figure 9 above, I noticed that the splines on each end of the axle actually put it in a purely torsional stress state (ideally). This lack of bending not only simplifies calculations, but also allows for a smaller axle than with bending. Assuming roughly 500 ft-lbs of torque at the rear axle, similar to the hubs, it is safe to say that

under maximum load, the axle shafts are likely to see roughly 250 ft-lbs each. However, to be conservative with the understanding that these axles are likely to be put in some interesting transient loading scenarios, 500 ft-lbs will be used for each axle individually. From an elementary torsional stress standpoint we have the following equation for the axles:

$$\tau_{maximum} = \frac{T_{RA} * r_{axle\ shaft}}{J} = \frac{T_{RA} * \frac{d_{axleshaft}}{2}}{\frac{\pi}{32} * \left(d_{axleshaft}\right)^4}$$

Substituting our values of $T_{RA} = 500\ ft * lbs$ and $d_{axleshaft} = 0.78\ inches$, we find that the maximum torsional stress in the shaft, in our conservative model, is:

$\tau_{maximum} = 64392.8\ psi$ (Safety factor of 2)

$\tau_{maximum} = 130{,}000\ psi$ (Safety factor of 4)

Referring to Shigley's Mechanical Engineering Design Book [6], we see the correlation between the hardness of a material and its ultimate tensile strength as the following

$$S_U = 0.5 * H_B \tag{5}$$

Using our value for stress with a safety factor of 4 in equation 5, we come out with a required Brinell hardness of approximately 260. This is equivalent to a Rockwell C hardness of 28-29 Rc.

Therefore, for the application of the axle shafts, two different materials will be used and tested. The information pertaining to the selected materials is given in table 9 below:

MATERIAL	INITIAL HARDNESS	FURTHER HEAT TREATEMENT	FINAL HARDNESS
4140 Hot Rolled and Annealed Alloy Steel	17-20 Rc	Yes, through hardening (quench and tempering)	38-42 Rc
4340 Hot Rolled Quenched and Tempered Alloy Steel	30-35 Rc	NO	N/A

Table 9: Materials Selection Choices for CV Axles

Spur Gear Design

The spur gears which will provide the post-CVT reduction require a number of steps to design. Since the gear ratio has already been determined, one of the steps has been eliminated. There will be four gears in the gearbox, two pinions and two gears. The word pinion describes the smaller gear in a meshing gear set and the word gear describes the larger gear in a meshing gear set. Figure 10 below shows a gear and labels the critical portions of a gear that will be referred to without explanation in the rest of the paper.

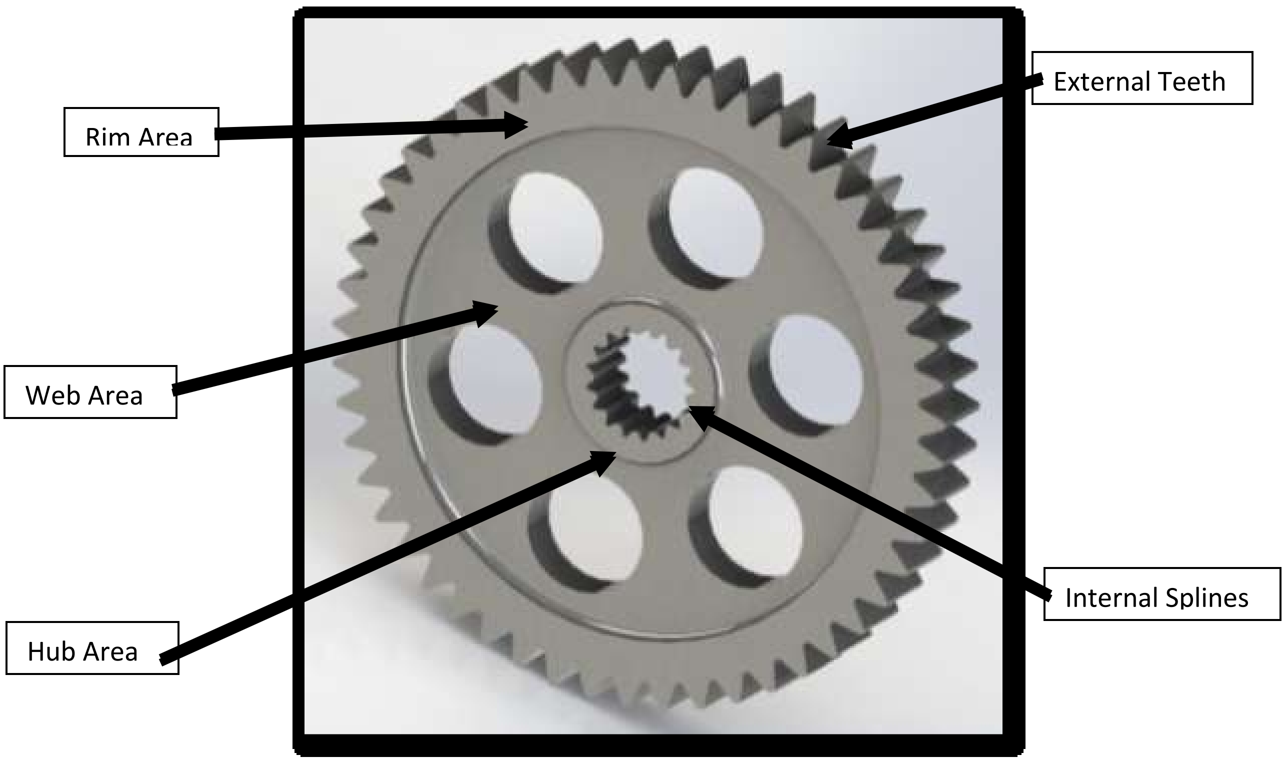

Figure 10: Labeled Picture of a Gear

Two reductions will be used to avoid undesirably large gears and accompanying issues with accomplish low static CV-axle angles. The remaining steps of the gear design can be listed as follows:

1.) Selection of Gear Type
2.) Selection of Pressure Angle
3.) Selection of Tooth Numbers

4.) Selection of Diametral Pitches
5.) Selection of Face Width
6.) Design of Rim, web, and hub of gear
7.) Selection of Gear material and Heat Treatment

Selection of Gear Type

For this particular application, the main choices of gear type would be helical or spur gears. In order to avoid unwanted axial loads on the shafts, helical gears will not be selected for this application. Spur gears will be selected for simplicity in calculations and to keep axial loads negligible, thereby avoiding the need for bearings and shaft designs which would handle said axial loads.

Selection of Pressure Angle

Some typical choices of pressure angles for gears include 14.5°, 20°, 22°, and 25°. Through my research I found that higher pressure angles would be more advantageous for the design of a Baja transmission. Therefore, the pressure angle of 25° was chosen for the ZB15 transmission. The reasons why are as follows, and are all recommendations from Dudley's handbook of Practical Gear Design and manufacture [7]:

- The 25° pressure angle form makes the teeth thicker at the base, thus more resistance to transient bending stresses (very characteristic of this heavily shock loaded application). Furthermore, even though contact stresses are higher in larger pressure angle gears, vehicles can handle a bit of surface damage such as micro-pitting and still function. However, a broken tooth, caused by exceeding the bending stress, puts a vehicle gear out of action immediately
- The 25° pressure angle form nearly eliminates the issue of undercutting during machining. This will help to ensure that the root fillets on the teeth of the gears are strong and no unwanted stress concentrators arise during machining.
- The 25° pressure angle form allows a standard addendum and dedendum size to be used with as few as 15 teeth on the pinion (smaller gear in a mating set of gears). This is

less teeth than the smaller pressure angles, hence the 25° pressure angle form can accommodate smaller center-to-center distances. Smaller center-to-center distances means better packaging, which is a major design goal in a racing application such as this one.

<u>Selection of Tooth Number</u>

For a 25° pressure angle spur gear, the minimum number of teeth without undercutting is 12, but the minimum amount of teeth allowed with a standard size addendum and dedendum is 15. In accordance with keeping the design and machining of this gear set simple and practical, the number of teeth on the first and second pinion of the gearbox are kept above 15. When observing industry recommendations for the minimum number of teeth that should be on a pinion to balance bending and contact stresses, a number of teeth that was smaller than the recommendation, but larger than 15, was selected [7]. The number of teeth on the pinion in the first reduction (Pinion 1) was chosen to be 17 and the number of teeth on the second reduction was chosen to be 16. This means that, in accordance with the desired gear ratios from the transmission, the number of teeth on gear 1 is 51 and the number of teeth on gear 2 is 49.

<u>Selection of Diametral Pitches and Face Widths</u>

Now that I have chosen the type of gear, pressure angles, and the number of teeth that will be present in the gear box, the final step in forming each gear is deciding what diametral pitch and face width each gear should have. Diametral pitch is a quantity which relates how many teeth are on the gear to its pitch diameter.

$$P_{diametral} = \frac{Number\ of\ Teeth}{Pitch\ Diameter\ of\ Gear\ or\ Pinion}$$

In accordance with the above relation, a smaller value for diametral pitch is associated with larger teeth, and a larger value of diametral pitch means smaller teeth for a given pitch diameter. In our case however, since the number of teeth are specified, the diametral pitch selection primarily determines the center-to-center distance between the gears, a topic of

great importance in gear design. Face width is a very self-explanatory term, and simply means the width along the tooth perpendicular to the radial direction of the gear.

Spur Gear Design: AGMA Bending Stress

In order to properly select these final two parameters, we need a bit more background on gear design. We must introduce the AGMA bending and contact stress equations, as well as the concept of contact ratio, in a tabular fashion to optimize the selection of diametral pitches and face widths. The AGMA equation for bending stress on a gear tooth is the following [6]:

$$\sigma_{bend} = \frac{W_t * P}{F} * K_a * K_m * K_s * K_v * K_t \quad (6)$$

Table 10 on the next page presents a brief explanation of constant term in this expression, as well as what value was used with a brief explanation as to why. Some of the terms which will vary depending on the selection of diametral pitch in the excel sheet. These terms are listed in bullet point fashion below:

- P= Diametral Pitch of Gear (1/inches)
- F= Face width of Gear (inches)
- W_t= Transmitting Load at tooth interface (calculated at pitch diameter) (lb)
- K_v= Dynamic Load Factor, accounts for the adverse effects of overloads caused by the engine or road conditions (will change as pitch diameter changes)

PARAMETER/DESCRIPTION	VALUE	EXPLANATION
K_a: Application Factor, takes into account the smoothness of loading, both at the driving and driven ends of the gear set.	2.2	Both Shigley's and Dudley's books recommend a value between 2.2 and 2.5 for heavy shock loading scenarios such as ours [6,7]
K_s: Size Factor, takes into account excessive size effects of gears that distort physical enhancements such as heat treating and affect machining accuracy.	1	Shigley's recommends a value of 1 be used for gears with a diametral pitch greater than 5 (which is what we will look at) [6].
K_m: Load Distribution Factor, takes into account the nonlinear pressure distribution across the face width of the gear.	1.2	A complex empirical relation exists, but for simplicity Dudley's suggests using a number between 1 and 1.4 [7].
K_t: Geometry Factor: Handles unknowns associated to the shape of the tooth, load sharing between teeth, and the stress concentration at the root area	Roughly 3.4	Read from a table 5.34 in Dudley's handbook of practical gear design and manufacture. [7]

Table 10: Constants of the AGMA Bending Stress Equation

Spur Gear Design: AGMA Contact Stress

Now we take a brief look at the AGMA equation for contact stress [6]

$$\sigma_{contact} = C_p * \sqrt{\frac{W_t}{F*d} * K_m * K_s * K_v * K_o * \frac{C_f}{I}} \tag{7}$$

The constants in equation 7 are given and explained in table 11 as they were for the AGMA bending stress equation in table 10. Some of the terms in this expression vary with the diametral pitch of the gear of the gear, similar to terms in the bending stress equation. These variable terms are:

- F= Face width of Gear (inches)
- d= pitch diameter of gear (inches)
- W_t= Transmitting Load at tooth interface (calculated at pitch diameter) (lbs)
- K_v= Dynamic Load Factor, accounts for the adverse effects of overloads caused by the engine or road conditions (will change as pitch diameter changes)

PARAMETER/DESCRIPTION	VALUE	EXPLANATION
$\boldsymbol{K_o}$: Application Factor, takes into account the smoothness of loading, both at the driving and driven ends of the gear set.	2.2	Both Shigley's and Dudley's books recommend a value between 2.2 and 2.5 for heavy shock loading scenarios such as ours [6,7]
$\boldsymbol{K_s}$: Size Factor, takes into account excessive size effects of gears that distort physical enhancements such as heat treating and affect machining accuracy.	1	Shigley's recommends a value of 1 be used for gears with a diametral pitch greater than 5 (which is what we will look at) [6].
$\boldsymbol{K_m}$: Load Distribution Factor, takes into account the nonlinear pressure distribution across the face width of the gear.	1.2	A complex empirical relation exists, but for simplicity Dudley's suggests using a number between 1 and 1.4 [7].
$\boldsymbol{C_p}$: The elastic coefficient	$2300\sqrt{psi}$	This value may be used for steel spur or helical gears [6,7]
$\boldsymbol{I}$: Surface Strength Geometry Factor	Approximately 0.1436	Empirical Relation involving the pressure angle and speed ratio of each gear mesh
$\boldsymbol{C_f}$: Surface Condition Factor: Associated to the shape of the tooth, load sharing between teeth, and the stress concentrations	1	Dudley's states "unless a detrimental surface condition exists, use 1" [7]

Table 11: Constants of the AGMA Contact Stress Equation

Spur Gear Design: Contact Ratio

The final factor which will weigh into our diametral pitch and face width decisions is the contact ratio. The contact ratio is, in a way, a description of the load sharing between teeth during gear meshing. Shigley's recommends that the contact ratio (m_p), which is described by equation 8 below, should never fall below 1.2 in a properly engineered gear mesh [6].

$$m_p = \frac{\sqrt{\left({d_o}'/_2\right)^2 - \left({d_b}/_2\right)^2} + \sqrt{\left({D_o}'/_2\right)^2 - \left({D_b}/_2\right)^2}}{\varrho_b} \qquad (8)$$

The terms in the above expression are elaborate upon below:

- ${d_o}'/{D_o}'$: The effective outer diameter of the pinion and gear, respectively. This is the diameter to the outer tip of the external teeth. (in inches)
- d_b/D_b: The base diameter of the pinion and gear, respectively. This is the diameter to the base circle of the gear, which can be found by drawing a circle tangent to the line that passes through the pitch point (point of tangency of the two pitch radii of the mating gears) of the gear at the pressure angle of the gear. (in inches)
- ϱ_b: the base pitch of the gear mesh (equal to $\frac{\pi * \cos(pressure\ angle)}{diametral\ pitch}$) (in inches)

Spur Gear Design: Final Selection of Geometry

Since so many factors in the bending stress, contact stress, and contact ratio change when the diametral pitch changes, Microsoft excel will be used to optimize the reduction. An optimized gear reduction will provide an acceptable contact ratio, exceptional packaging (small center-to-center distances), and acceptable bending and contact stresses for a life of 10,000 – 100,000 cycles according to AGMA 170.01 standard for vehicle spur and helical gears [7]. This cycle count is a conservative estimate based off of percentage of 3 full endurance races of max load condition (12 hours total). The factor of safety in fatigue on the final gear geometry shown in table 13 is roughly 1.5-1.76 for 10,000-100,000 cycle, a fairly conservative gearset

Diametral Pitch (Teeth/inch)	Addendum (in)	Dedendum (in)	Pitch Diameter Pinion 1 (in)	Pitch Diameter Gear 1 (in)
4	0.25	0.31	4.25	12.75
6	0.17	0.21	2.83	8.50
7	0.14	0.18	2.43	7.29
8	0.13	0.16	2.13	6.38

Pitch Diameter Pinion 2 (in)	Pitch Diameter Gear 2 (in)	Base Diameter Pinion 1	Base Diameter Gear 1	Base Diameter Pinion 2	Base Diameter Gear 2
4.00	12.25	3.85	11.56	3.63	11.10
2.67	8.17	2.57	7.70	2.42	7.40
2.29	7.00	2.20	6.60	2.07	6.34
2.00	6.13	1.93	5.78	1.81	5.55

Circular Pitch Reduction 1(in)	Circular Pitch Reduction 2 (in)	Base Pitch (in)	Base Pitch (in)	CD 1 (in)	CD 2 (in)	Contact Ratio Reduction 1	Contact Ratio Reduction 2
0.79	0.79	0.71	0.71	8.50	8.13	1.38	1.37
0.52	0.52	0.47	0.47	5.67	5.42	1.34	1.33
0.45	0.45	0.41	0.41	4.86	4.64	1.32	1.31
0.39	0.39	0.36	0.36	4.25	4.06	1.30	1.29

Transferred Load A (lb)	Pitch Line Velocity (ft/min)	Dynamic Load Factor	.5 inch Face width Bending Stress A (psi)	.75 inch face width Bending Stress A (psi)	1 inch face width Bending Stress A (psi)	.5 inch face width Contact Stress A (psi)	.75 inch face width Contact Stress A (psi)
303	1053	1.344	29241	19494	14621	136501	111452
454	702	1.284	62832	41888	31416	200090	163373
530	601	1.263	84175	56117	42088	231595	189096
606	526	1.247	108519	72346	54259	262960	214706

Table 12: Small segment of Excel Document Used to analyze and optimize various geometrical and mechanical properties of the reduction

Table 12 above is a small snippet of the analysis of four different diametral pitches and 3 different face widths, two of the diametral pitches shown were chosen for the final design (6 and 8). Excel was used to analyze nine different diametral pitches and three face widths for each reduction individually. A few things to note from table 12 is that all distances (pitches, diameters, center distances, etc.) are in inches, all stresses are in psi, and the transferred load is in pounds force. The stresses displayed at the bottom of the table are for the first reduction in the gearbox.

Using excel, an optimized double reduction gear set was created with the parameters seen below in table 13. Information relating to the material selection and heat treatment of the gears is also included. These specifications were largely taken as recommendations from Kenmore Gear and Machine Company in Akron, OH. Note that it was recommended in Dudley's that the face width of the pinion be slightly larger than the gear and that the pinion be roughly 2-3 Rockwell C points harder than the gear to help with load distribution and break in [7].

PARAMETER (unit)	**VALUE FOR REDUCTION 1**	**VALUE FOR REDUCTION 2**
Tooth Profile	Involute	Involute
Pressure Angle (degrees)	25	25
Teeth on Pinion	17	16
Teeth on Gear	51	49
Diametral Pitch (1/inches)	8	6
Contact Ratio	1.30	1.33
Pinion Face Width (inches)	1	1.75
Gear Face Width	.813	1.63
Pinion AGMA Bending Stress (psi)	Approx. 55,000	Approx. 53,000
Gear AGMA Bending Stress (psi)	Approx. 63,000	Approx. 55,000
Pinion AGMA Contact Stress (psi)	Approx. 186,000	Approx. 187,000
Pinion AGMA Bending Stress (psi)	Approx. 200,000	Approx. 250,000
Gear Material	4140 HRA	4140 HRA
Hardness of Pinion (Rc)	42-44	42-43
Hardness of Gear (Rc)	39-41	39-41

Table 13: Final Selections for gear geometry, material, and heat treatment. Further information relating to gear stress

Design: Gear Webbing

Once the rims of the gears were designed, finite element analysis was used on each gear to reduce the amount of material in the web of the gear by facing and drilling (see figure 10). FEA was used to ensure that the stress in the web remained less than half the stress at the root fillet on the tooth during heavy shock loading scenarios. This method was used to reduce the weight of the gears by over 20 pounds. Figure 11 below shows an example of gear Solidworks FEA. The loading scenario was 14.5 ft*lbs of torque multiplied by 4 and the appropriate intermediate gear ratio and then translated into a force along the line of the pressure angle using the relationship

$$Torque = force * radius.$$

The internal splines of the gear were fixed in this analysis, simulating a situation in which the tire is jammed between rocks or in a crevice and cannot spin.

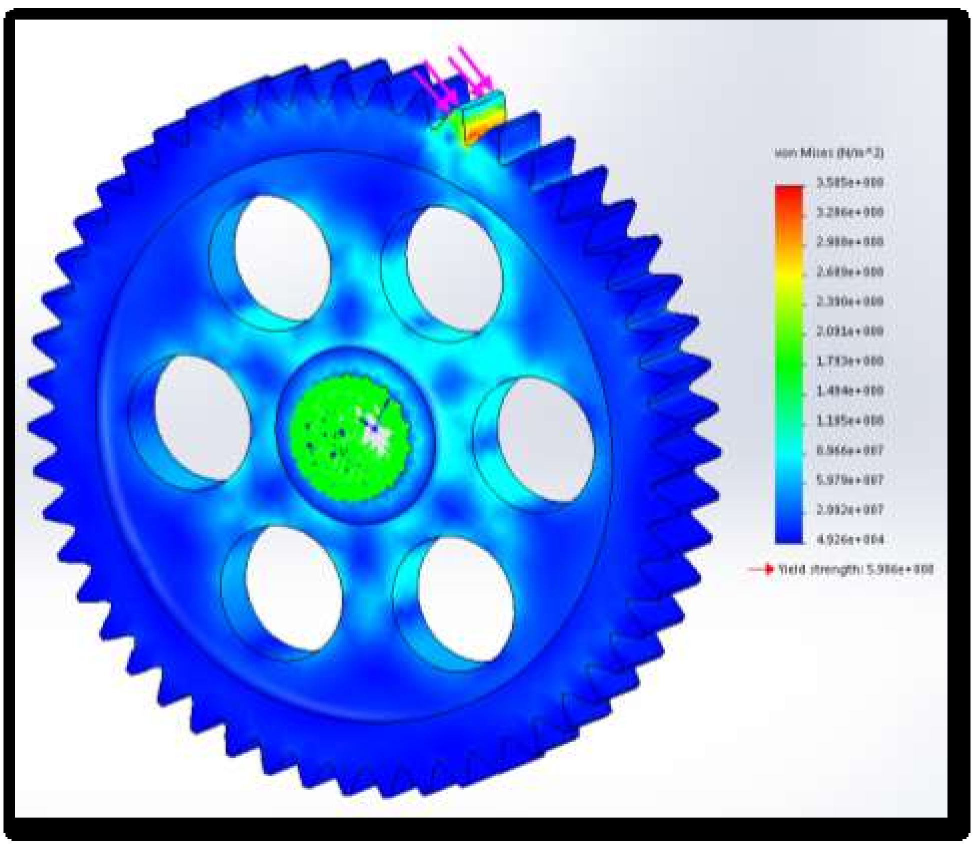

Figure 11: Finite Element Analysis to reduce weight in the web of the gears

Design: Gear Casing

Elementary static bending analysis with Newton's second law was used to calculate bearing forces generated by the action of each shaft. These bearing forces, and their respective 3-dimensional orientations, were applied to the Solidworks model of the gear casing using Solidworks finite element analysis (FEA). A minimum factor of safety of 3 was used for reducing weight from less-stressed areas of the gearbox. A total of 4 pounds was reduced through this method, making the gear casing just over 10 pounds. Figure 12 below shows a picture of the finite element analysis on the gear. Each of the mounting bolts is a fixed support and the bearing bores have the respective forces applied in the exact planes in which they will act.

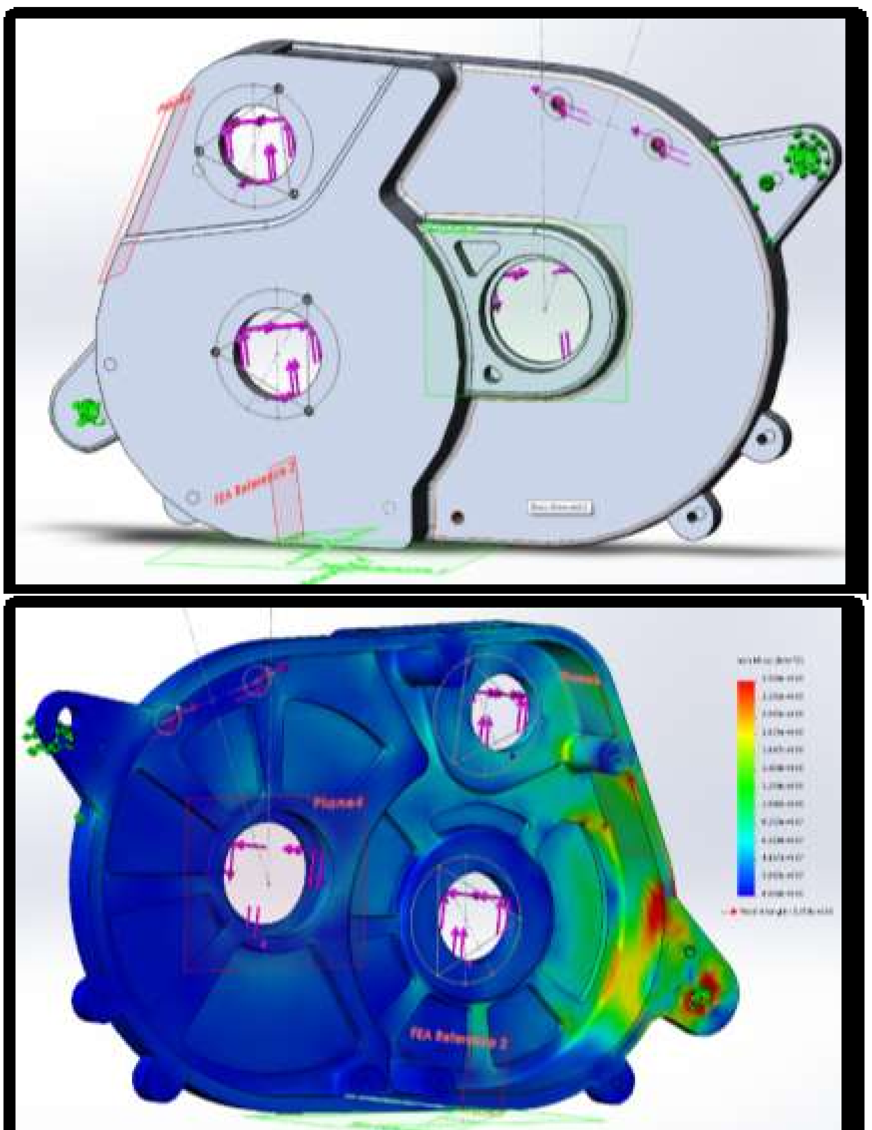

Figure 12: (Top) Loading and Fixture Scenario on right half of gearbox housing. (Bottom) Results of finite element analysis on right half of gearbox housing.

Testing

The car performs rather well with the double spur gear reduction. Our max speed has been increased to 35 mph from the previous top speed of 31 mph attained by ZB14 last year. We are continuing to test with the least stiff torsion spring in the driven pulley of the CVT (black, 8 coils), as it yields the highest top speed for the car.

- Positive Performance of ZB15
  - 7th in suspension/traction event at Alabama
  - 13th in hill climb event at Alabama
- Negative Performance of ZB15
  - 54th in acceleration at Alabama

Acceleration continues to be a problem for ZB15 as it was for ZB14. Most of this is attributed by the author to the very large rotating mass of the driveline. Recommendations for reducing this rotating mass will be given in the next section of the report. Tables 14 and 15 show testing data in an attempt to improve acceleration time. From these tables we may discover two important things:

1.) Putting a stiffer torsion spring in the driven pulley of the CVT only yields acceleration gains of less than a tenth of a second, hardly statistically significant given the size of our data set.
2.) Power-braking the engine at 3000 rpms has been shown to increase acceleration run times by up to a half a second. The author cautions continuous use of this method during testing, as it tends to damage clutch components after a short amount of time.

Powertrain Testing				
Session 1				
	Date	8/6/2014		
	Drive Springs	Yellow		
	Drive Weights	91 grams, Blue with Blue Side Cones		
	Torsion Spring	Black, 8 Coils		
	Acceleration Data	(12 parking spaces, approx 100 ft)		
		Without Power Brake		time (s)
			Trial 1	5.35
			Trial 2	5.2
			Trial 3	5.33
			Trial 4	5.31
			Trial 5	5.25
			AVERAGE	5.288
		3000 rpm Power Brake		time (s)
			Trial 1	4.88
			Trial 2	5.1
			Trial 3	4.95
			Trial 4	4.78
			AVERAGE	4.9275

Table 14: Acceleration Testing with a less stiff torsion spring in the Driven Pulley of the Comet 780 Series Clutch

Powertrain Testing					
Session 2					
	Date	8/20/2014			
	Drive Springs	Yellow			
	Drive Weights	91 Grams, Blue with Blue Side Cones			
	Torsion Spring	Red, 6 coils			
	Acceleration Data	approx 100 ft			
		Without Power Brake		time (s)	time (s)
			trial 1	5.29	5.16
			trial 2	5.08	5.31
			trial 3	5.18	5.33
			trial 4	5.2	5.38
			trial 5	5.33	5.18
			AVERAGE	5.216	5.272
		3000 rpm Power Brake		time (s)	
				4.78	
				4.61	
				4.83	
				4.91	
				4.7825	

Table 15: Acceleration Testing with a more stiff torsion spring in the Driven Pulley of the Comet 780 Series Clutch

Final Recommendations

The author first and foremost advises any future powertrain leaders to **thoroughly** examine the gear ratios, clutches, and component sizing that previous cars use. This will allow the young powertrain designer to understand engine performance and how to translate that performance using appropriate gear ratios.

<u>Some recommendations for improvement on the ZB15 powertrain may be seen below:</u>

- If the Comet 780 Series CVT sees continued use in the next car, reduce the post-CVT reduction from 9.1875 to either 8.7 or 8.8.
- Reduce weight in the driveline by reducing each gear's face width by ½ of an inch.
- Experiment with different heat treatment techniques, namely, case hardening techniques (Hardening the surface of the tooth without hardening the web and the mesh.
- Try experimenting with slightly smaller CV Axles, perhaps .5 inches in diameter to .75 inches in diameter, with the same material and heat treatment that is currently used (4140 HRA through hardened to 38-43 Rockwell C)
- The gears on ZB15 were designed with a fatigue factor of safety of 1.5-1.7 for a cycle count of 10^4-10^5 cycles. Reduce this fatigue factor of safety to 1 or below in the future.
- It may be advantageous to experiment with CVTech or Gaged CVTs. Even though these CVTs do not provide the optimal ratios as we have been attracted to in this report, they do have some positive qualities:
    - They are lighter than the Comet CVTs
    - The Driven Pulley of CVTech CVTs and Gaged CVTs is not as large as the pulley on the Comet CVTs, thus it will be allowed to fully expand without first running into the heat shield on the side of the cylinder head.
- Some Comments on heat treatment
    - Per recommendation from Kenmore Gear and Machine Company as well as the faculty of the University of Akron graduate machine shop, do not through harden 4140 steel above 50 on a Rockwell C scale. When 4140 is through hardened past this value, it becomes very brittle.
    - Discuss case hardening options with local machine shops and see what depth they normally case harden to. Start with this value and experiment to see if failures occur as you reduce the face widths of gears. Case hardening, if done to the right depth, would be much more advantageous for our gears by avoiding

much of the post-heat treatment machining that I had to do to eliminate distortion effects.

- Bearing Press Fits
    - Per recommendation from Bill Wenzel in the University of Akron machine shop, the interference value that has been used for pressing a steel bearing into an aluminum housing for the past two years has been 0.0005-0.001 inch of press fit for each inch of bearing **OD.**
        - This has worked fairly well

<u>Some general comments about what I learned as a two-year running powertrain designer:</u>

- **Heat Treatments will distort parts.**
    - Not terribly enough to cause problems with axle shafts (we only had to file the tip of the splines to get them to fit in the ends)
    - More noticeable with the gears
        - Inner splines and register surfaces for the hubs were distorted between .010 inches and .030 inches, requiring time consuming re-machining of the internal splines on the first gear and the register surfaces of the hubs.
- **PASS ON KNOWLEDGE**
    - When we began designing the car for the 2014 race season, we had absolutely nothing to go off of in terms of hardware or proper designs.
    - I had some useful testing data given to me, but had to learn the hard way about many aspects of proper transmission design and vehicle dynamics.
    - The ZB15 car is a **work in progress**, keep the progress going. Move forward, not backward, by reducing component weight and exploring reverse mechanisms as well as different CVTs. **No complete redesign of concept should be needed on this car**.
- Know who to talk to and when to do your own research
    - There will be people on the team who claim to know a lot about something or other. It is a very dangerous trap to fall into to listen to people who have more interest in hearing themselves talk than in proper machining and design of components.
    - If you do your own research, make your own conclusions, design your own parts, machine your own parts, and specify your own heat treatments, then the uneducated words of others will not bother you and you can continue getting things done.
    - On the other hand, **talk to other powertrain designers at competition**. There are some fantastic designs out there, and teams get away with having ridiculously

light gears (total gearbox weight of 10 to 15 pounds with a reverse gear!). Learn from them, it will be fun!

- Use a rubber belt CVT
    - Even if you want to break away from the pack because you think there is some oasis of hope with using a centrifugal or diaphragm clutch connected to a multi-speed gearbox, **DON'T DO IT.**
    - The rubber belt CVT is the most lightweight, efficient, reliable, and tested clutch for this application. This is why it is used in such a widespread fashion in the industry
- Good Surface Finishes on heavily loaded transmission parts matter.
    - Steel is fairly sensitive to surface flaws, and this effect is only amplified when it is hardened.
    - If you ever have a question, talk to Bill or Mike in the machine shop. These two are very knowledgeable and can give you great recommendations on machine component design, machining, and heat treatment.
    - To get a good surface finish when turning the OD or ID of a part.
        - Make sure you are running at a good lathe rpm to get close to the proper surface feet per minute specification for the tool and part.
        - On your final pass, take a cut which is larger than the radius of the cutting tool tip (.040 inch to 0.050 inch cuts for the turning tool in 2015). This will avoid chipping of the material and provide the best surface finish. **Do not take very small cuts of 1 or 2 thousandths of an inch to get a good surface finish.**
- <u>**I wish the best of luck to future Baja teams. Have fun, learn something, and make a fast car! Go Zips!**</u>

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Appendix

MATLAB Programs: Road-Load Power Max Velocity Calculation Script

```
% This Program will allow for a maximum vehicle velocity to be calculated
% from a number of given inputs

% Maximum Power at the rear wheels (assuming 85% efficient driveline)
Pmax = 550*.84*(8.9); % (ft*lb/s)

% Frontal Car Area
A = 2016/144; % sq ft

% Drag Coefficient
Cd = 1.08;

% Density of Air at standard conditions
rho = 2.38 * (10^(-3)); %slugs/(cubic foot)

% Vehicle weight
W = 530; %lbf

% Coefficient of Rolling Resistance
Crr = .068;


% Maximum Vehicle Velocity (Just an initialization)
Vmax = 0; % ft/s
roots = 0;
while (1)
    RHS = (1/2)*(Cd)*(A)*(rho)*(Vmax^3) + (Crr*W)*Vmax;

    eapprox = abs((Pmax - RHS)/Pmax)*100;

    if eapprox < .1;

        fprintf('The maximum attainable vehicle velocity is approximately\n')
        fprintf(' %.4f  mph \n',Vmax*(3600/5280))
        break



    else
        Vmax = Vmax + .01;
    end
end
```

Results Compilation

ZB 2014 Racer

2014 Kansas Competition

Overall: 61st
Endurance: 74th
Suspension: 72nd
Sled Pull: 34th
Maneuverability: 50th
Acceleration: 76th
Design:23rd
Cost: 38th

2014 Illinois Competition

Overall: 22nd
Endurance: 33rd
Hill Climb: 10th
Rock Crawl: 69th
Maneuverability: 22nd
Acceleration: 51st
Design: 23rd
Cost: 37th

ZB 2015 Racer

2015 Auburn Competition

Overall: 34th
Endurance: 38th
Hill Climb: 13th
Maneuverability: DNF
Suspension/Traction: 7th
Acceleration: 54th
Design: 30th
Cost: 57th

Pictures of Completed Design

Figure 13: Rendering of Completed Gearbox Design in Solidworks (Half of the Gearbox Removed for clarity)

Figure 14: Completed Drivetrain on the Car at Auburn Alabama Competition