2012

Design Charts for Circular Fins of Arbitrary Profile Subject to Radiation and Convection with Wall Resistances

G. Zhang
University of Akron Main Campus

Benjamin T.F. Chung
University of Akron Main Campus, bchung@uakron.edu

Open Access

Design Charts for Circular Fins of Arbitrary Profile Subject to Radiation and Convection with Wall Resistances

G. Zhang[1] and B. T. F. Chung[2,*]

[1]*Royal & Sun Alliance Insurance Co. of Canada, 10 Wellington St E, Toronto, ON, Canada*

[2]*Department of Mechanical Engineering, The University of Akron, Akron, Ohio, 44325-3903, USA*

Abstract: In this work, the optimization for a radiative-convective annular fin of arbitrary profile with base wall thermal resistances is considered. A fourth order Runge-Kutta method is used to solve the associated non-linear governing equations. Further differentiations yield the optimum heat transfer and the optimum fin dimensions. To facilitate the thermal design, design charts for optimum dimensions are proposed. Furthermore, the fin effectiveness for the optimal annular radiative-convective fins is presented to check the practicality of the design.

Keywords: Optimum fin dimensions, annular fin, arbitrary profile, optimum fin effectiveness, convection and radiation, wall thermal resistances.

1. INTRODUCTION

Circular fins are used extensively in heat exchange devices to enhance the heat transfer rate. For a given weight or volume, the fin can dissipate different amounts of heat because of the different shape and geometry. The goal of fin optimization is to find the shape of the fin which would minimize the fin volume for a given amount of heat dissipation or to maximize the heat dissipation for a given fin volume.

During the past decades, numerous studies have been presented on the performance of the annular fin [1-5]. For the optimization of a circular fin, only a few papers have appeared in the literature. Razelos and Imre [6] studied convective circular fins of three profiles: rectangular, triangular, and trapezoidal; they considered the effect of curvature and the thermal properties of the fin. Ullmann and Kalman [7] employed a numerical method to investigate the convective radial fin of rectangular, triangular, hyperbolic, and parabolic profiles. They presented the fin efficiencies and the optimum dimensions for these four different annular fin shapes. Zubair *et al.* [8] investigated the optimum circular fin dimensions with variable profile and temperature-dependent thermal conductivity. Chung and Ma [9] included the wall thermal resistance effect but for convective fins only. A minor typographical error is found in their expression of overall wall resistance, but the numerical results are not affected.

The aforementioned optimization results were restricted to the case of a linear boundary condition or a uniform base wall temperature. As pointed out in Aziz's review paper [10], the optimization of radiation-convection fins is practically non-existent and calls for more research endeavors. The current literature does not cover the combined effect of convection and radiation from the fin surface and wall thermal resistances at the fin base, using an arbitrary fin profile. Furthermore, very few previous studies have included fin effectiveness calculations for their optimal fin designs. Recently, Chung *et al*. [11] proposed the ranges of optimum design under different thermal and physical conditions. The previous work was restricted to annular fins of trapezoidal profile only and also did not include the fin effectiveness calculations. The purpose of current study is to determine the optimal dimensions of a radiating-convecting annular fin using an arbitrary profile and more specifically to present convenient design charts for the thermal designers. Those charts are not available in the open literature.

2. MATHMATICAL ANALYSIS

2.1. Physical Model

The present analysis is based on the following assumptions:

1. Heat conduction in the fin is steady and one-dimensional.
2. The fin material is homogeneous and isotropic.
3. The fin material has constant properties, and fin surface is diffused.
4. The heat transfer coefficient over fin surface is uniform.
5. The heat transfer at fin tip is negligibly small.
6. The temperature of the fluid inside the pipe is constant; the ambient temperature and environment temperature around the fin are also uniform.
7. The radiative interaction between the base wall and fin is neglected.
8. The curvature effect of the fin is negligible.
9. There is no heat generation inside the fin.

*Address correspondence to this author at the Department of Mechanical Engineering, The University of Akron, Akron, Ohio, USA;
Tel: 3309727739; Fax: 3309722011; E-mail: bchung@uakron.edu

1874-396X/12 2012 Bentham Open

2.2. Governing Equations

Considering an annular fin shown in Fig. (**1**), a general fin profile function is expressed by

$$f(r)=\frac{\delta}{2}\left(\frac{r_o}{r}\right)^n \tag{1}$$

Here n refers to the fin profile number. The energy balance on a control volume shown in Fig. (**1**) results in

$$\frac{d}{dr}\left[r\left(\frac{r_o}{r}\right)^n\frac{dT}{dr}\right]=\frac{2hr}{k\delta}(T-T_\infty)+\frac{2\sigma r}{k\delta}\left(\varepsilon T^4-\alpha T_e^4\right) \tag{2}$$

For fluid with constant temperature T_f inside the pipe, the convective heat transfer coefficient h_f between the fluid and inner pipe wall is given. Considering the energy transfer from hot fluid inside the pipe to the interface between the outer pipe radius and fin and the heat loss from the fin base (see Fig. **1**), we obtain the following boundary condition:

$$-k\frac{dT}{dr}=\frac{T_f-T}{\dfrac{r_o}{r_ih_f}+\dfrac{r_o}{k_w}\ln\left(\dfrac{r_o}{r_i}\right)+R_{tc}} \quad @\ r=r_o \tag{3}$$

In the denominator of the right hand side of Eq. (3), the first term represents the thermal resistance between the hot fluid and the inner surface of the pipe; the second term designates the thermal resistance inside the pipe; and the third term, R_{tc} is the total contact resistance between the primary and the extended surfaces at the base. If all resistances approach zero, Eq. (3) is reduced to the prescribed wall temperature boundary condition as presented in all current heat transfer text books.

From the physical model 5 mentioned above, the fin tip is insulated. This assumption is reasonable if either (i) the fin cross sectional area is small, which is generally true in practice; (ii) the fin length is long enough that the tip temperature approaches to the environment temperature and (iii) the Harper-Brown approximation [12] is applied. The approximation states that a convective fin tip can be replaced by an insulated fin tip when the length of the fin is extended by one half of the fin thickness of the fin. Consequently, numerous investigators [2, 13-20, just to mention a few] have assumed the zero temperature gradient at the fin tip which is also adopted in the present analysis, i.e.

$$\frac{dT}{dr}=0 \quad @\ \mathrm{r}=r_t \tag{4}$$

In engineering practice, most materials can be considered gray bodies, and the environment temperature is assumed to be the same as ambient temperature, i.e.

$$\varepsilon=\alpha,\ T_e=T_\infty.$$

Substituting the following non-dimensional parameters

$$r'=\frac{r}{r_0},\ \rho=\frac{r_t}{r_0},\ \theta=\frac{T}{T_f},\ \theta_b=\frac{T_b}{T_f},\ \theta_e=\frac{T_e}{T_f},$$

into Eqs. (2-4) yields the dimensionless energy in the form of:

$$\frac{d}{dr'}\left[r'^{(1-n)}\frac{d\theta}{dr'}\right]=\frac{2hr_o^2}{k\delta}r'(\theta-\theta_e)+\frac{2\sigma r_o^2T_f^3\varepsilon}{k\delta}r'\left(\theta^4-\theta_e^4\right): \tag{5}$$

The corresponding boundary conditions are

$$\frac{d\theta}{dr'}=\frac{\theta-1}{R_w} \quad @\ r'=1 \tag{6a}$$

$$\frac{d\theta}{dr'}=0 \quad @\ r'=\rho \tag{6b}$$

where

$$R_w=\frac{k}{r_ih_f}+\frac{k}{k_w}\ln\left(\frac{r_0}{r_i}\right)+\frac{k}{r_0}R_{tc} \tag{7}$$

To determine the optimal dimension of the annular fin, we maximize the heat transfer for a given fin volume which is given by

$$V=\int_{r_o}^{r_t}4\pi r\cdot f(r)dr$$

Using Eq. (1), the above expression gives the relationship between δ and ρ for a specified fin profile number n

$$\delta=\begin{cases}\dfrac{V(2-n)}{2\pi r_o^2\left(\rho^{2-n}-1\right)} & n\neq 2\\[2ex] \dfrac{V}{2\pi r_o^2\ln\rho} & n=2\end{cases} \tag{8}$$

The dimensionless fin base width can be defined as a function of radius ratio, ρ and profile number, n from Eq. (8)

$$\frac{\delta r_o^2}{V}=\begin{cases}\dfrac{2-n}{2\pi\left(\rho^{2-n}-1\right)} & n\neq 2\\[2ex] \dfrac{1}{2\pi\ln\rho} & n=2\end{cases} \tag{9}$$

Substituting Eq. (8) into the Eq. (5), yields the following expressions

$$\frac{d}{dr'}\left[r'^{(1-n)}\frac{d\theta}{dr'}\right]=m_c\frac{\rho^{(2-n)}-1}{2-n}r'(\theta-\theta_e)+m_r\frac{\rho^{(2-n)}-1}{2-n}r'\left(\theta^4-\theta_e^4\right)\quad \text{if}\quad n\neq 2 \tag{10}$$

$$\frac{d}{dr'}\left[r'^{(1-n)}\frac{d\theta}{dr'}\right]=m_c\ln\rho r'(\theta-\theta_e)+m_r\ln\rho r'\left(\theta^4-\theta_e^4\right)\quad \text{if } n=2 \tag{11}$$

where

$$m_c=\frac{4\pi hr_0^4}{kV} \tag{12}$$

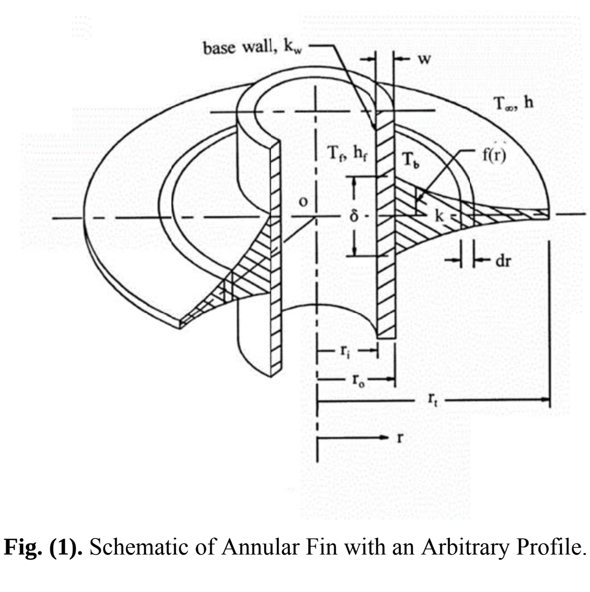

Fig. (1). Schematic of Annular Fin with an Arbitrary Profile.

$$m_r = \frac{4\pi\varepsilon\sigma r_0^4 T_f^3}{kV}. \tag{13}$$

The three non-dimensional parameters, namely, the convection characteristic number, m_c, the radiation characteristic number, m_r, and the overall thermal resistance at the fin base, R_w, play important roles in heat dissipation and optimum fin design.

2.3. Heat Dissipation

At steady state the energy dissipated from the fin surface is equal to the heat transfer at the fin base. The dimensionless heat transfer Q can be expressed as

$$Q = \frac{q}{VkT_f / r_0^2} = \begin{cases} -\dfrac{2-n}{\rho^{(2-n)}-1} \left.\dfrac{d\theta}{dr'}\right|_{r'=1} & n \neq 2 \\ -\dfrac{1}{\ln\rho} \left.\dfrac{d\theta}{dr'}\right|_{r'=1} & n = 2 \end{cases} \tag{14}$$

From the expressions of the energy and heat dissipation equations, the parameters affecting heat transfer are as follows: heat transfer coefficient along the fin surface, h, and that inside the pipe, h_f; base wall thermal conductivity, k_w ,dimensions r_o and r_i; contact thermal resistance between the fin and primary pipe; R_{tc}, fin shape profile number n; fin base width δ; and radius ratio ρ; fin material properties, such as thermal conductivity k, emissivity ε, environment temperature T_e and fluid temperature, T_f.

Due to the non-linear characteristic of Eqs. (10 and 11), the present authors employed a fourth order Runge-Kutta method to solve these equations and used the bisection method to accelerate the convergence speed of the computed temperature and temperature gradient at the fin base.

2.4. Fin Optimization

Once the heat dissipation is obtained, the optimal dimensionless fin tip radius, ρ^* can be obtained by solving

$$\frac{\partial Q}{\partial \rho} = 0 \tag{15}$$

In the present analysis, the numerical Golden Section Search method [21] is employed to determine the optimum radius ratio ρ* and profile number n* for each specified condition. The Golden Section Search Method is an algorithm that can be used to find the maximum (or minimum) of a function, say $f(x)$. First it is assumed that we have found a region in which $f(x)$ has one and only one maximum. Let x_1 and x_4 ($x_1 < x_4$) be points that bracket the peak value region. Interior points $x_2 = 0.618x_1 + 0.382x_4$ and $x_3 = 0.382x_1 + 0.618x_4$ are next examined. If $f(x_2)$ is less than $f(x_3)$, then point x_1 is discarded, and. x_2, x_4 are now known to bracket a peak value region. Let new $x_1 = x_2$, and the new interior points x_2 and x_3, which are calculated the same as above, will be examined. On the other hand, if $f(x_3)$ is less than $f(x_2)$, then point x_4 is discarded, the new $x_4 = x_3$, and new interior points of x_2 and x_3 are examined. If the difference between x_2 and x_3 is less than 10^{-6}, then $f[(x_2+x_3)/2]$ is our maximum value and $(x_2+x_3)/2$ is the corresponding number resulting in the maximum value.

Once ρ^* is calculated, the optimum fin width at the base, δ* is computed from Equation (8).

2.5. Fin Effectiveness

The fin effectiveness is another important variable in the fin design. In the present work, it is defined as the ratio of the actual heat dissipated from the fin to that dissipated from a bare pipe with zero wall resistance. The actual heat dissipated from the fin can be obtained from Eq. (14). If the thermal resistance inside the pipe and the conduction resistance through the pipe wall are neglected, the heat dissipation for the bare pipe can be expressed in the form of

$$q_p = 2\pi r_o \delta_o h(T_f - T_e) + 2\pi r_o \delta_o \sigma\varepsilon(T_f^4 - T_e^4) \tag{16}$$

For convenience, the following mathematical expression for fin effectiveness will be adopted

$$\xi = \frac{q}{q_p} \tag{17}$$

Substituting Eq. (12) into Eq. (16) gives

$$\begin{aligned} q_p &= \frac{VhT_f}{r_o}\frac{2-n}{\rho^{2-n}-1}(1-\theta_e) + \frac{VhT_f^4}{r_o}\frac{2-n}{\rho^{2-n}-1}(1-\theta_e^4) \quad n \neq 2 \\ q_p &= \frac{VhT_f}{r_o}\frac{1}{\ln\rho}(1-\theta_e) + \frac{VhT_f^4}{r_o}\frac{1}{\ln\rho}(1-\theta_e^4) \quad n = 2 \end{aligned} \tag{18}$$

After some manipulations, the fin effectiveness can be expressed as

$$\xi = -C\frac{1}{m_c(1-\theta_e) + m_r(1-\theta_e^4)} \left.\frac{d\theta}{dr'}\right|_{r'=1} \tag{19}$$

where

$$C = \frac{4\pi r_o^3}{V} \tag{20}$$

This parameter, C relates to the fin geometry and volume, and is called the fin geometry characteristic number.

3. RESULTS AND DISCUSSION

3.1. Optimum Fin Profile Number

Our numerical computations indicate that the increase of fin base wall thermal resistance and environment temperature will reduce the total heat transfer, as expected. Figs. (**2** and **3**) show the relationship between the optimum fin profile number n* and radius ratio ρ for pure convection and pure radiation, respectively. These figures reveal that the optimum fin profile number could approach the limiting value of 2 when R_w and θ_e are zero, the convection and radiation characteristic numbers are very small, and the fin height is very large. However, in practice, R_w and θ_e cannot be exactly zero (except in outer space); very small m_c and m_r do not have any practical meaning; and a very large fin height is also not realistic. Therefore the optimum fin profile number n* must be greater than 2. However, when the fin profile number is greater than 2, the fin shape will be very sharp; a sharp fin is not easy to fabricate and is also easy to break at the tip. In practice, the most frequently used shapes are rectangular, trapezoidal, triangular, and hyperbolic, for which the values of n are less than 2.

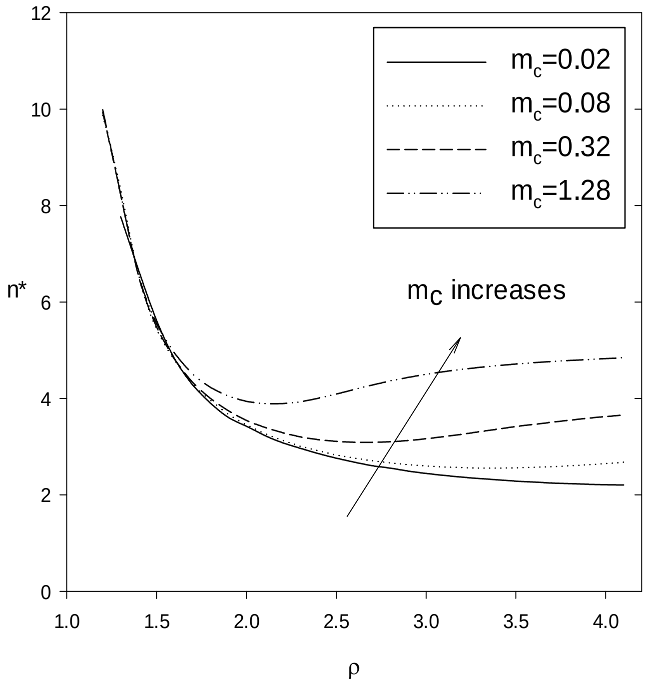

Fig. (2). n^* vs. ρ with m_c Varying, m_r=0.0, R_w=0.0, θ_e=0.0.

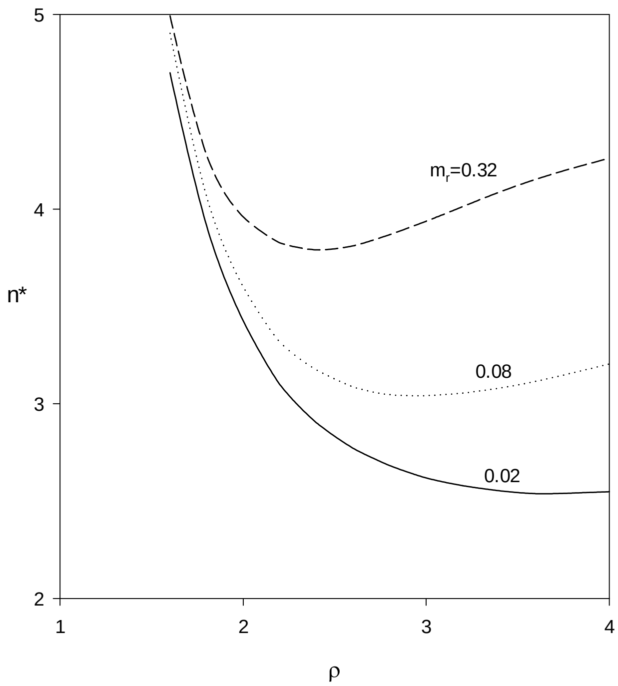

Fig. (3). n^* vs. ρ with m_r Varying, m_c=0.0, R_w=0.0, θ_e=0.0.

3.2. Optimum Radius Ratio

The following discussion focuses on the optimized fin radius ratio and the associated heat transfer. Typical values for θ_e = 0.5 and R_w = 0 & 1 are used. Figs. (**4-6**) describe the variation of the optimum heat dissipation, Q^* with m_r for the condition of R_w = 0 at n=0, 1, and 2 respectively. Figs. (**7-9**) describe the condition of R_w = 1 corresponding to the above profile numbers. From these figures, the effect of optimum

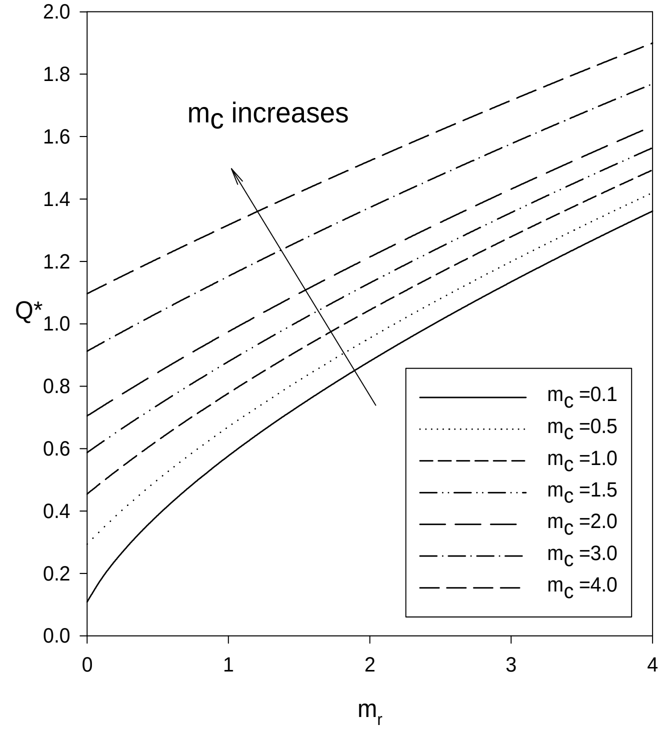

Fig. (4). Q^* vs. m_r with Various m_c, n=0.0, R_w=0.0, θ_e=0.5.

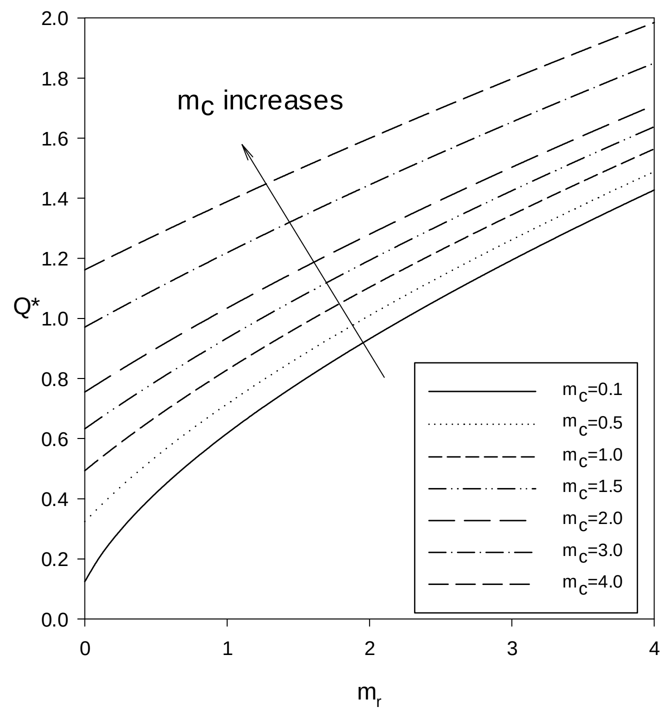

Fig. (5). Q^* vs. m_r with Various m_c, n=1.0, R_w=0.0, θ_e=0.5.

heat transfer is observed. Generally, the heat dissipation increases with the increase of m_r. When m_c is small, the effect of m_r is stronger than that when m_c is large. Comparing Figs. (**4** to **6**) with one another, we find that with the increase of n, not only does the heat dissipation increase, but the slopes of the curves with same m_c are somewhat different as well. This implies the effect of m_r is different for different profile numbers. Similar results can be observed from Figs. (**7** to **9**)

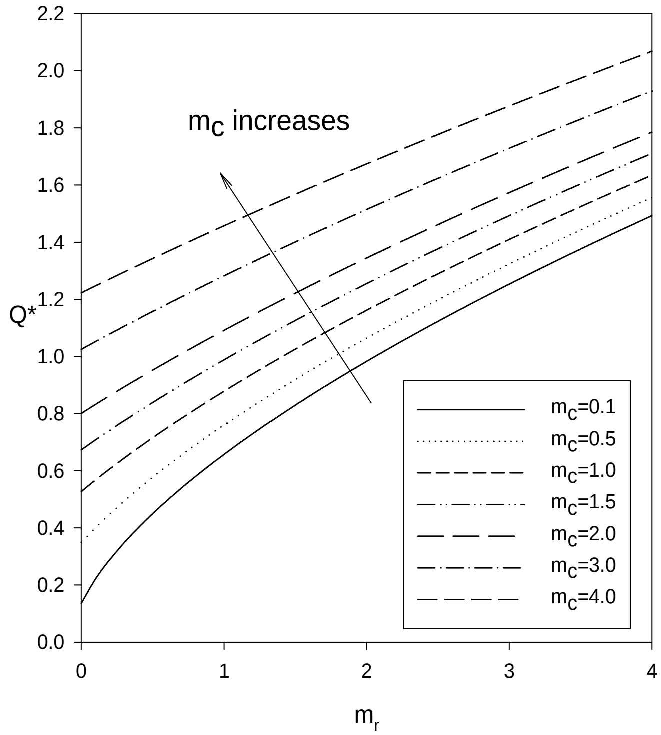

Fig. (6). Q^* vs. m_r with Various m_c, n=2.0, R_w=0.0, θ_e=0.5.

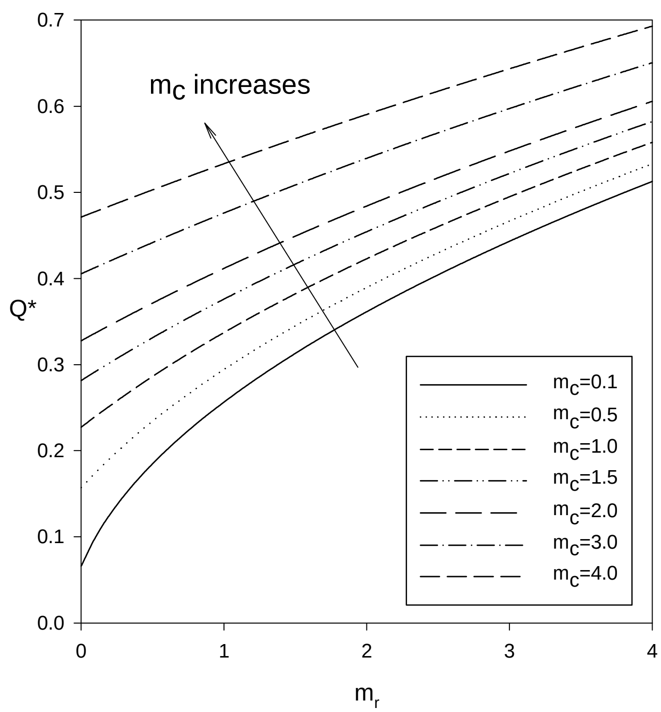

Fig. (7). Q^* vs. m_r with Various m_c, n=0.0, R_w=1.0, θ_e=0.5.

where R_w is unity. From the heat transfer point of view, the higher order hyperbolic profile is better than the hyperbolic profile, which is in turn better than the rectangular profile. Comparing figures with same profile number but with different base wall resistances, it is found that when R_w increases, not only does the heat transfer decrease, but also the effect of m_r decreases. These figures imply that for optimum design, the radiation part should not be neglected, even when the

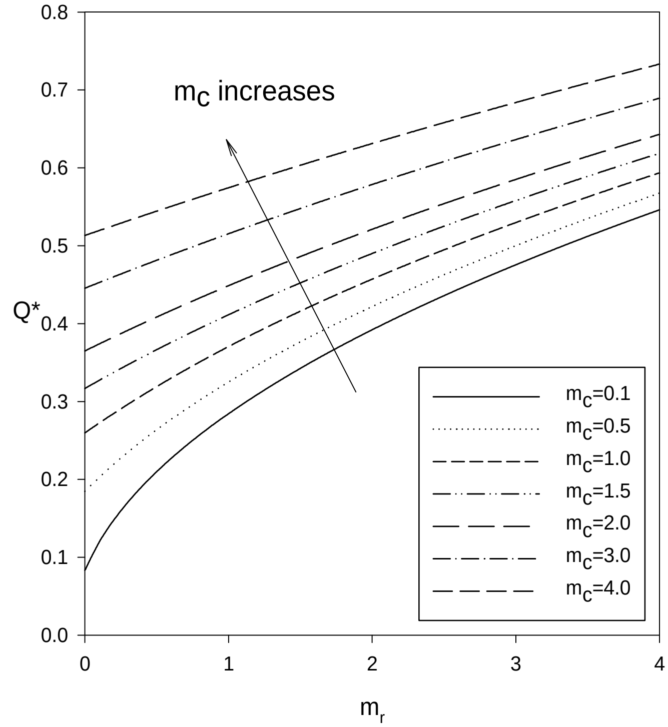

Fig. (8). Q^* vs. m_r with Various m_c, n=1.0, R_w=1.0, θ_e=0.5.

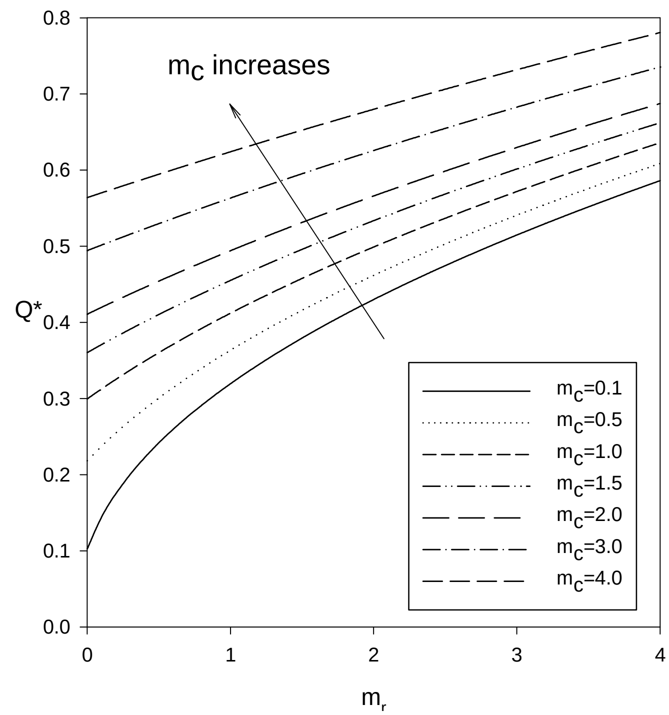

Fig. (9). Q^* vs. m_r with Various m_c, n=2.0, R_w=1.0, θ_e=0.5.

convection is dominant. Neglecting the radiating effect in the previous analyses could lead to a gross error in predicting the maximum heat transfer; especially for the case of free convection (i.e. m_c is small).

The effect of radiation on optimum radius ratio ρ^* is shown in Figs. (**10** to **15**) for different combinations of wall resistance, and profile number, with the convection characteristic number as a parameter. As found in the case of heat

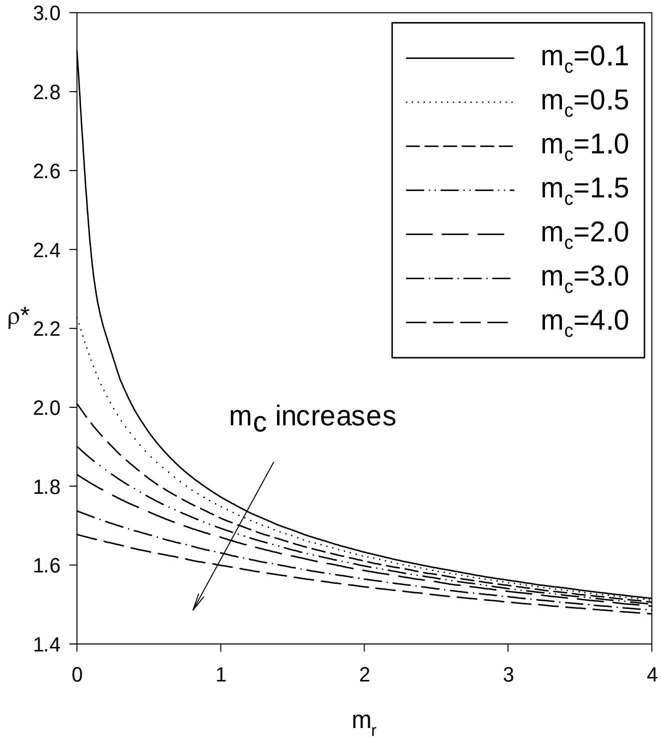

Fig. (10). ρ^* vs. m_r with Various m_c, n=0.0, R_w=0.0, θ_e=0.5.

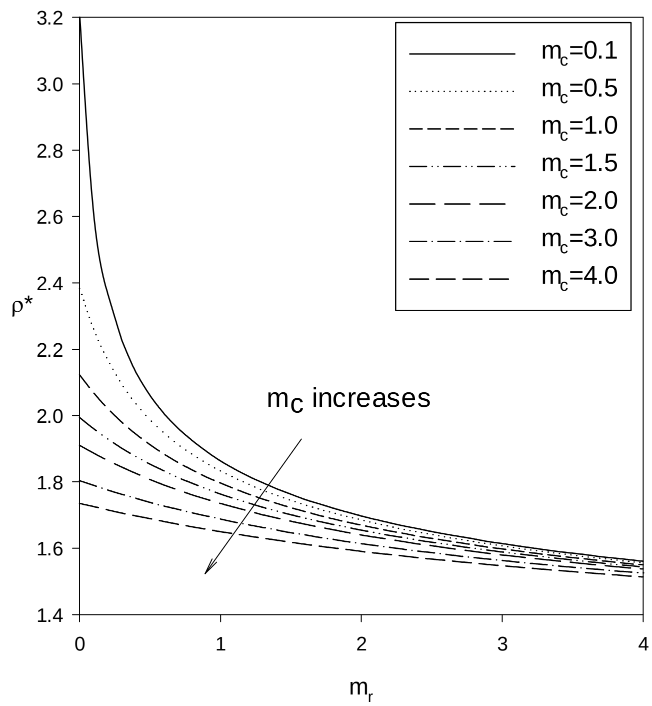

Fig. (11). ρ^* vs. m_r with Various m_c, n=1.0, R_w=0.0, θ_e=0.5.

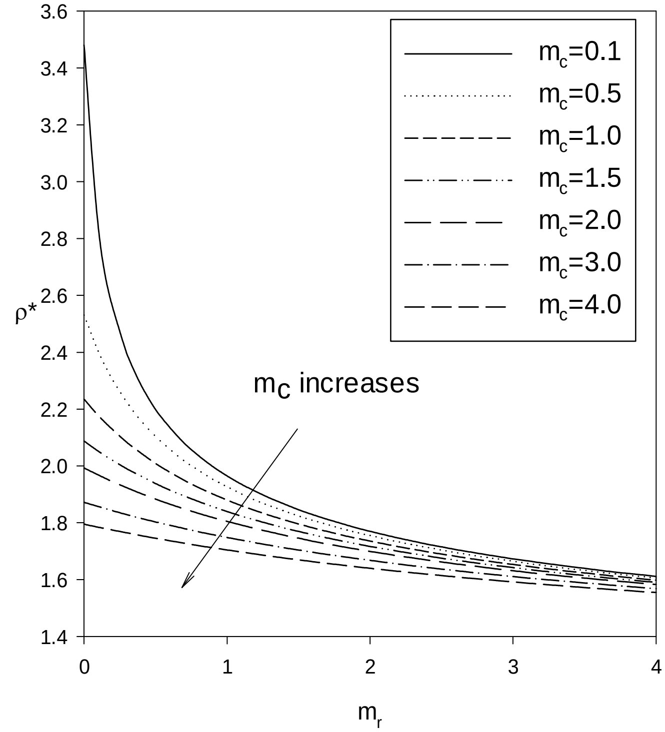

Fig. (12). ρ^* vs. m_r with Various m_c, n=2.0, R_w=0.0, θ_e=0.5.

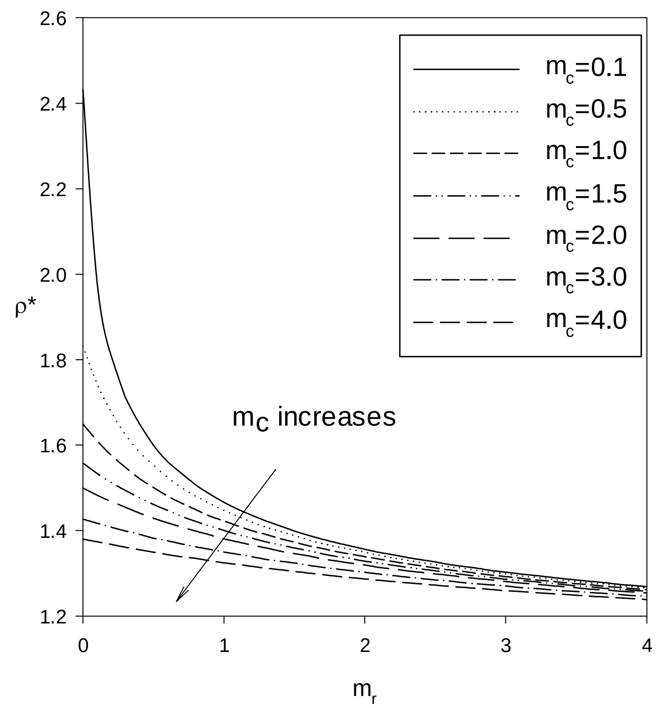

Fig. (13). ρ^* vs. m_r with Various m_c, n=0.0, R_w=1.0, θ_e=0.5.

transfer, when m_c is small, m_r has a strong effect on ρ*; the increase of m_r causes ρ* to decrease drastically. When the convection parameter increases, the percentage that radiation contributes lessens, and the radiation effect decreases. Comparing the above figures at different profile numbers, we find that ρ* also increases slightly, when n increases.

Figs. (**4-15**) represent a set of convenient design charts for the thermal designer. The methodology of applying the charts will be briefly described below: For fixed fin shape and fin volume V, one can obtain the maximum heat dissipation from Figs. (**4-9**) and the optimum fin tip radius from Figs. (**10-15**), given that the base wall thermal resistance, R_w, convection and radiation characteristic numbers, m_c and m_r,

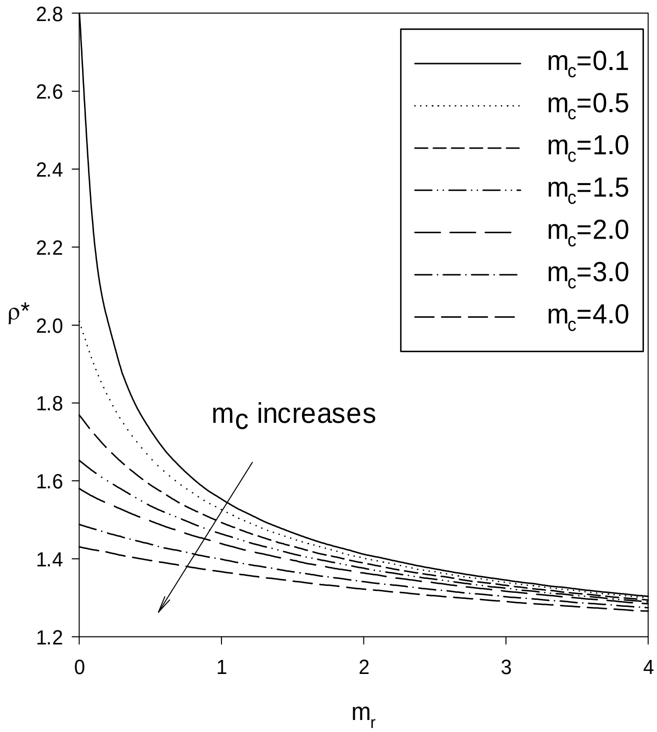

Fig. (14). ρ^* vs. m_r with Various m_c, n=1.0, R_w=1.0, θ_e=0.5.

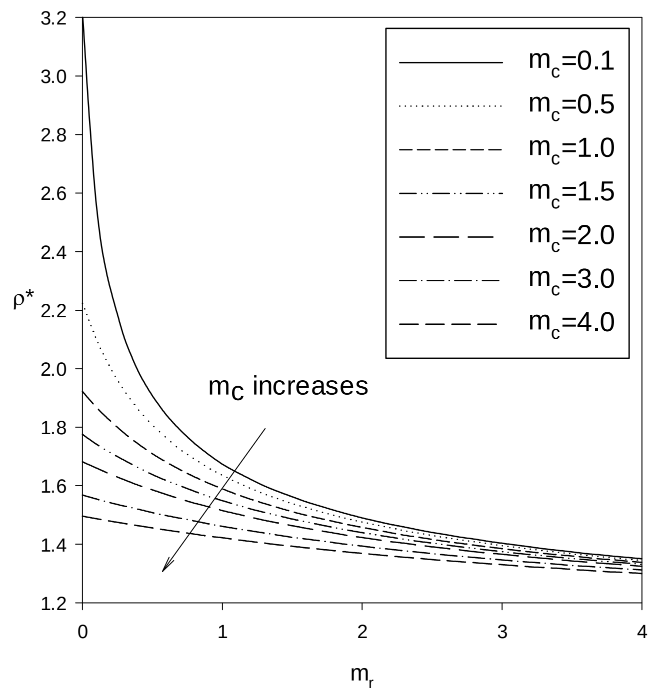

Fig. (15). ρ^* vs. m_r with Various m_c, n=2.0, R_w=1.0, θ_e=0.5.

are known. Consequently, the optimum fin base thickness is calculated from Eq. (9). In the case that the heat transfer instead of fin volume is specified, an iteration procedure is needed to determine the optimum fin volume and optimum fin dimensions. This will be outlined below: We first assume a fin volume, V and then calculate the dimensionless Q from the first part of Eq. (14). After m_c, and m_r, are obtained from Eqs. (12 and 13), respectively, an optimum Q^* can be found from Figs. (**4-9**). If Q^* does not approach the specified heat transfer, the newly calculated Q will be used to compute the new volume, V using the first part of Eq. (14). The above procedures are repeated until the solution for V or Q converges. Once Q^* is found, the corrected parameters, are automatically determined. Then the optimal dimensions can be obtained directly from one of the optimum fin radius design charts, Figs. (**10** to **15**) (or by interpolation between two adjacent figures).

3.3. Fin Effectiveness

The fin effectiveness is computed from Eqs. (19 and 20) once the temperature distribution is obtained. Equation (19) shows an important linear relationship between C and fin effectiveness. Physically, C represents the inverse of dimensionless fin volume. In the present work, a typical value of C =10 is adopted. If the actual value of C is not equal to 10, the fin effectiveness obtained from Figs. (**16** to **19**) can be easily modified by multiplying a factor of C/10.

Figs. (**16** and **17**) show the fin effectiveness ξ^* for the rectangular and hyperbolic profile when R_w=0. Figs. (**18** and **19**) are the counterparts when R_w=1. When m_c or m_r, increases the fin effectiveness decreases, even though heat dissipation increases. This implies that when the heat transfer coefficient is large enough, fin may not be needed. Comparing Fig. (**16**) to Fig. (**17**) or Fig. (**18**) to Fig. (**19**), we found that the smaller fin profile number has higher fin effectiveness. Numerical comparisons show that the rectangular profile fin has the largest fin effectiveness; the effect of n is quite small as compared to the corresponding case of zero wall resistance. We also detect that the overall wall thermal resistance has a strong effect on fin effectiveness. Our numerical computations indicate that when R_w increases, ξ^* decreases sharply. Comparing Figs. (**16** to **18**) or Figs. (**17** to **19**) shows the same. Therefore, in order to improve the fin effectiveness, one should reduce the base wall thermal resistance or to choose the rectangular fin profile (n=0).

It should be noted that under the optimal condition, Q^* increases with the increase of fin profile number, but ξ^* decreases. Therefore, if the total heat transfer is the dominant factor for the design, the second order hyperbolic fin profile is recommended. On the other hand, if the fin effectiveness is the desired factor, the rectangular fin is a better choice. It is found in those figures, the optimal fin effectiveness can be very small at certain conditions. This means that some optimal fin designs may not be realistic.

4. CONCLUSIONS

In the present work, the combined effect of radiation and convection on circular fin optimization with a general profile is investigated. For both convection and radiation, the numerical computations show that the optimum fin profile number n^* is greater than 2. The previous approaches which neglect the effect of radiation may lead to a significant error in predicting the heat dissipation and optimal dimensions for the free convection case. Three important dimensionless parameters control the optimum fin dimensions, namely the base wall thermal resistance, the convection characteristics number, and the radiation characteristics number.

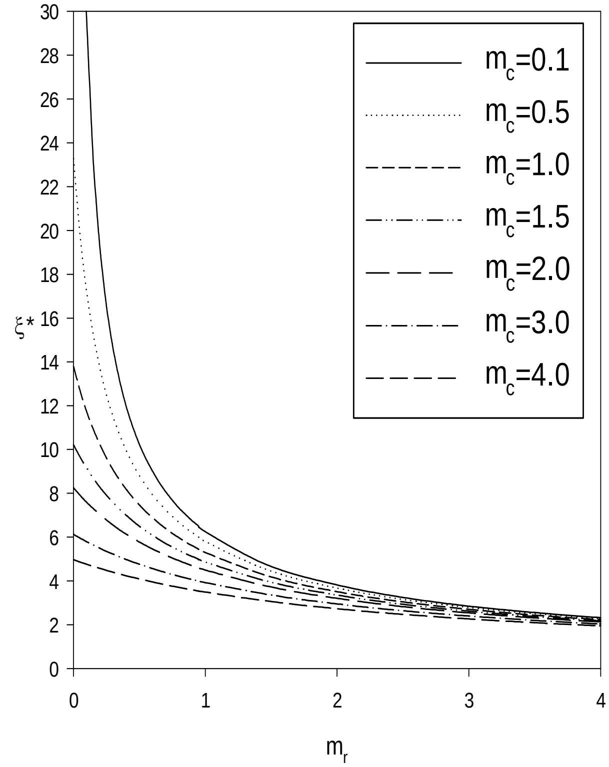

Fig. (16). Optimum Fin Effectiveness ξ^* vs. m_r with m_c Varying n=0.0, R_w=0.0, θ_e=0.5, C=10.

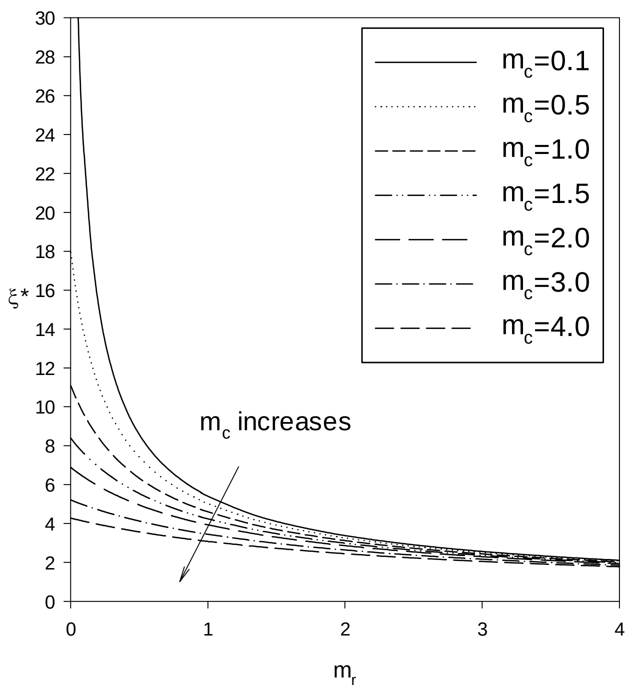

Fig. (17). Optimum Fin Effectiveness ξ^* vs. m_r with m_c Varying n=1.0, R_w=0.0, θ_e=0.5, C=10.

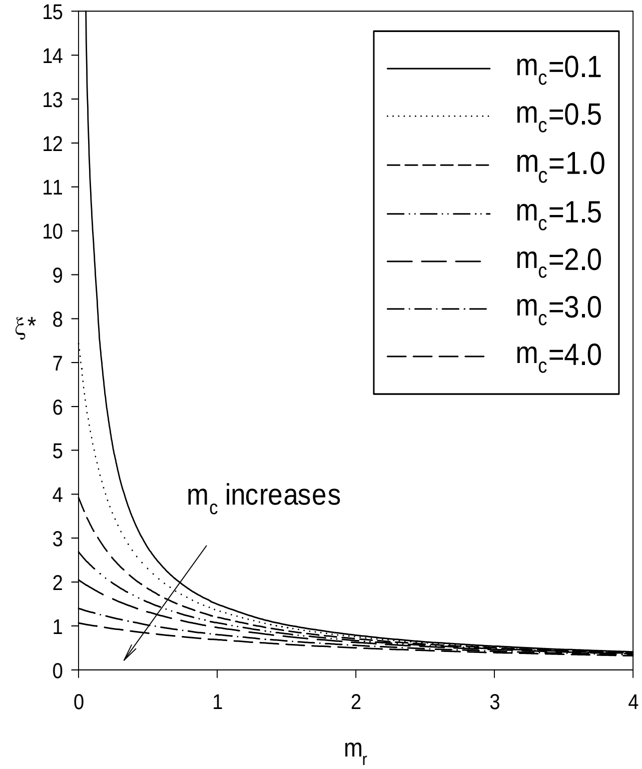

Fig. (18). Fin Optimum Effectiveness ξ^* vs. m_r with m_c Varying n=0.0, R_w=1.0, θ_e=0.5, C=10.

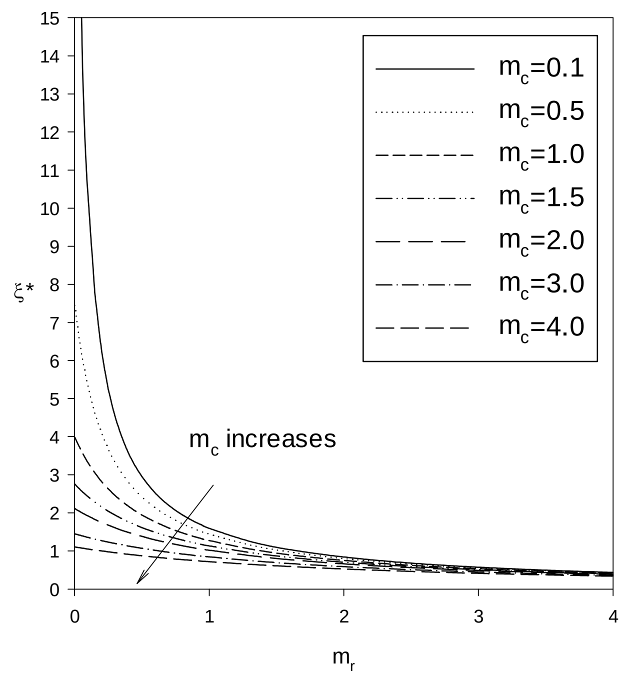

Fig. (19). Fin Optimum Effectiveness ξ^* vs. m_r with m_c Varying n=1.0, R_w=1.0, θ_e=0.5, C=10.

Design charts are presented for the optimal heat dissipation and optimal dimensions of the rectangular and hyperbolic annular fins, subject to simultaneous convection and radiation. For a given fin volume and profile number, the optimal dimensions and heat transfer can be obtained directly from the present charts. For a specified heat transfer, an iteration scheme coupled with the use of design charts is needed to obtain the optimal dimensions.

Fin effectiveness is presented for some typical optimum designs. The wall thermal resistance tends to decrease the fin effectiveness. Even for the limiting case of R_w=0, the fin effectiveness may not be large enough for certain optimal designs.

From a heat transfer point of view, the higher-order hyperbolic profile is always better than the lower-order hyperbolic fin, which in turn is better than the rectangular fin. However, from the effectiveness point of view, a rectangular profile appears to be the best shape in the annular fin family. The present numerical results show that the fin effectiveness is less than one for certain optimum designs. Therefore, the authors strongly recommend to always checking the value of fin effectiveness during fin design.

NOMENCLATURE

C = geometry constant

dr = the increment of fin radius

h = heat transfer coefficient along the fin surface, w/m^2-K

h_f = heat transfer coefficient inside the primary pipe, w/m^2-K

k = conductivity of fin material, w/m-K

k_w = conductivity of primary pipe, w/m-K

m_c = convection characteristic number, $\frac{4\pi h r_o^4}{kV}$, dimensionless

m_r = radiation characteristic number, $\frac{4\pi\sigma\varepsilon r_o^4 T_f^3}{kV}$, dimensionless

n = fin profile number, dimensionless.

q = heat transfer, w

Q = dimensionless heat transfer, $\frac{q}{VkT/r_o^2}$

r = radius of annular fin, m

r_o = outside radius of primary pipe, m

r_i = inside radius of primary pipe, m

r_t = radius of fin tip, m

r' = dimensionless fin radius, r/r_o

R_{tc} = contact thermal resistance, m^2-K/w

R_w = dimensionless base wall thermal resistance, $\frac{k}{r_i h_f} + \frac{k}{k_w}\ln\left(\frac{r_o}{r_i}\right) + \frac{k}{r_o} R_{tc}$

T = absolute temperature, K

V = fin volume, m^3

GREEK SYMBOLS

ρ = radius ratio, r_t/r_o

σ = Stefan-Boltzmann constant, 5.6696x10^{-8} w/m^2-K^4

θ = dimensionless temperature, T/T_f

δ = fin base width, m

ε, = emissivity dimensionless

α = absorptivity, dimensionless

ξ = fin effectiveness, q/q_p

SUBSCRIPTS

b = fin base

e = environment

f = fluid inside pipe

i = inside of pipe

o = outside of pipe

t = fin tip

p = bare pipe

∞ = surrounding adjacent to fin

SUPERSCRIPT

* = optimal condition

ACKNOWLEDGEMENTS

None declared.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

REFERENCES

[1] R. L. Chambers, and E.V. Somers, “Radiation fin efficiency for one-dimensional heat flow in a circular fin,” *J. Heat Transf.*, vol. 81, pp. 327-329, 1959.

[2] H. H. Keller, and E. S. Holdredge, “Radiation heat transfer for annular fins of trapezoidal profile,” *J. Heat Transf.*, vol. 92, pp. 113-116, 1970.

[3] H. V Truong, and R. J. Mancuso, “Performance predictions of radiating annular fins of various profile shapes,” *ASME Paper*, 80-HT-31, 1980.

[4] P. J. Smith and J. Sucec, “Efficiency of circular fins of triangular profile,” *J. Heat Transf.*, vol. 91, pp. 181-182, 1969.

[5] S. Sikka, and M. Iqbal, “Temperature distribution and effectiveness of a two-dimensional radiating and convecting circular fin”, *AIAA J.*, vol. 8, pp.101-106, 1970.

[6] P. Razelos, and K. Imre, “The optimum dimensions of circular fin with variable thermal parameters,” *J. Heat Transf.*, vol. 102, pp. 420-425, 1980.

[7] A. Ullmann, and H. Kalman, “Efficiency and optimized dimensions of annular fin of different cross-section shapes,” *Int. J. Heat Mass Transf.*, vol. 32, pp. 1105-1110, 1989.

[8] S. M. Zubair, A. Z. Al-Garni, and J. S. Nizami, “The optimal dimensions of circular fins with variable profile and temperature-dependent thermal conductivity,” *Int. J. Heat Mass Transf.*, vol. 39, pp. 3431-3439, 1996.

[9] B. T. F. Chung, and Z. Ma, “Optimization of the annular fin with base wall thermal resistances”, *Proceedings of Symposium on Energy Engineering in The 21st Century*, Hong Kong, vol. 2, 2000, pp. 581-587.

[10] A. Aziz, “Optimum dimensions of extended surfaces operating in a convective environment,” *Appl. Mech. Rev.*, vol. 45, pp. 155-173, 1992.

[11] B. T. F. Chung, Y. Zhou, Y. Wang, and L.T. Yeh, “A new look at optimum design for convecting-radiatiting annular fins of trapezoidal profile,” *Thermodyn. J.*, vol. 5, pp. 93-103, 2011.

[12] B. Harper, and D. R. Brown, "Mathematical equations for heat conduction in the fins of air-cooled engines," *Tech. Rep. NACA Report* 158, National Committee on Aeronautics: Washington D.C, 1922.

[13] G. Ahmadi, and A. Razani, " Some optimization problems related to cooling fins," *Int. J. Heat Mass Transf.*, vol. 16, pp. 2369-2375, 1973.

[14] A. Aziz, "Optimization of rectangular triangular fins with convective boundary condition", *Int. Commun. Heat Mass Transf.*, vol. 12, pp. 479-482, 1985.

[15] H.S. Chu, C.I. Weng, and C. K. Chen, "Transient response of a composite straight fin", *J. Heat Transf.*, vol. 105, pp. 307-311, 1983.

[16] B. T. F. Chung, and B. X. Zhang, "Optimization of radiating fin array including mutual irradiations between radiator element", *J. Heat Transf.*, vol. 113, pp. 814-822, 1991.

[17] C. K. Krishnaprakas, "Optimum design of radiating longitudinal fin array extending from a cylindrical surface", *J. Heat Transf.*, vol.19, pp. 857-860 , 1997.

[18] P. Razelos, "The optimization of longitudinal convective fins with internal heat generation", *Nucl. Eng. Des.*, vol. 54, pp. 289-299, 1979.

[19] B. T. F. Chung, and B. X. Zhang, "A novel design for a radiative fin array", *J. Franklin Inst*., vol. 330, pp. 465-478, 1993.

[20] S. S. Wu, C. L. Shiu, and W. J. Wu, "Analysis on transient heat transfer in annular fins of various shapes with their bases subjected to heat flux varying as a sinudoidal time function", *Comput. Struct*., vol. 61, pp. 725-734, 1996.

[21] R. W. Daniel, "*An Introduction to Numerical Methods and Optimization Techniques*," Elsevier: North-Holland, New York, 1978.

Received: March 06, 2012 Revised: May 26, 2012 Accepted: May 30, 2012