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A Formal Anaylysis Of Hohfeldian Relations

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A FORMAL ANALYSIS OF HOHFELDIAN RELATIONS

by

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It has long been recognized that opacity in legal phraseology may place a major stumbling block in the road to clarity in legal analysis.

> As our law develops it becomes more and more important to give definiteness to its phraseology; discriminations multiply, new situations and complications of fact arise, and the old outfit of ideas, discriminations, and phrases has to be carefully revised. Law is not so unlike other subjects of human contemplation that clearness of thought will not help us powerfully in grasping it. If terms of common legal use are used exactly, it is well to know it; if they are used inexactly, it is well to know that, and to remark just how they are used.[1]

Quoting the above language, Professor Wesley N. Hohfeld set about attempting to clarify the important legal terms "right," "duty," "privilege," "no-right," "power," "immunity," "disability," and "liability.[2]

In his analysis, Hohfeld developed what has come to be viewed as a system of deontic logic.[3] That system predated the development of the field by von Wright[4] and even the early work in deontic logic of the 1920s and 1930s.[5] Hohfeld's work lacks the formality of later work in deontic logic, but understandably so, since it also predates the development of the systems of modal logic on which the formal deontic systems are based.[6] The relatively recent recognition of Hohfeld's work as a con-

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1 J. THAYER, PRELIMINARY TREATISE ON EVIDENCE 190 (1898).

2 Hohfeld, *Some Fundamental Legal Concepts as Applied in Judicial Reasoning*, 23 YALE L.J. 16 (1913) (quoting J. THAYER, at 29 n.25) [hereinafter Hohfeld (1913)]. "No-right" does not, of course, appear to be a legal term in need of clarification. It is, in fact, a term devised by Hohfeld to fill a need. A concept existed for which there was no common term, *see infra* notes 20-25 and accompanying text. Hohfeld had to employ the concept to round out his scheme and supplied the term so as to be able to label the concept.

3 Cullison, *Logical Analysis of Legal Doctrine: The Normative Structure of Positive Law*, 53 IOWA L. REV. 1209, 1211 (1968); Mullock, *Holmes on Contractual Duty*, 33 U. PITT. L. REV. 471, 474 (1972). Deontic logic is the logic of obligation. *See generally* Prior, *Deontic Logic*, 4 ENCYCLOPEDIA OF PHILOSOPHY 509 (P. Edwards ed. 1967); G. VON WRIGHT, AN ESSAY IN MODAL LOGIC ch. 5 (1951).

4 *See* G. VON WRIGHT, supra note 3.

5 *See, e.g.*, Grelling, *Zur Logik der Sollsaetze*, UNITY OF SCIENCE FORUM 44 (Jan., 1939); Reach, *Some Comments on Grelling's Paper "Zur Logik der Sollsaetze"*, UNITY OF SCIENCE FORUM 72 (April, 1939).

6 While modal logic, the logic of necessity and possibility, was discussed even as early as Aristotle, *see* ARISTOTLE, DE INTERPRETATIONS chs. 11-12; ARISTOTLE, PRIOR ANALYTICS, Book I, chs. 8-22, the formal development of modal logic stems from C. LEWIS & C. LANGFORD, SYMBOLIC LOGIC (1932).

tribution to deontic logic has led those interested in the relationship between logic and law to begin a formal analysis of his work.[7]

The formalizations of Hohfeld's analysis do more than point out his contribution. That contribution is clear from the many non-formal examinations of his work.[8] The real impact of formalization is the additional clarification it provides to the definitions of, and relations between, the terms involved. That additional clarity, while it has always been important, is becoming even more important.

> With the advent of the digital computer and the power of electronic information retrieval systems, the precise usage and definition of words rises from the level of merely aiding the efficiency of a transaction between legal entities to that of being virtually essential, where computers are involved, if the transaction is to take place at all. Man learns by example and possesses the creativity to resolve ambiguities; . . . machines are considerably less sophisticated than men in taking into account the relevant features of the total context in dealing with problems. In general, a computer requires a clearer and more precise specification of the question to be resolved.[9]

While various authors have added some formalism to Hohfeld's terminology,[10] the analysis has generally not taken place within a formal system. Without the inference rules of such a system, there can not be a real Hohfeldian logic. An exception to the lack of formality is the work of Professor Layman Allen. Professor Allen has provided a formal system in which Hohfeld's concepts of right, duty, no-right and privilege may be expressed,[11] and he has shown that the relations Hohfeld finds among his concepts are provable in his system.[12] The system Professor Allen presents, however, does not encompass Hohfeld's concepts of power, immunity, liability and disability.[13] Those concepts are more complex in that they concern the

[7] *See, e.g.*, Allen, *Formalizing Hohfeldian Analysis to Clarify the Multiple Senses of "Legal Right:" A Powerful Lens for the Electronic Age*, 48 S. CAL. L. REV. 428 (1974); Anderson, *The Logic of Hohfeldian Propositions*, 33 U. PITT. L. REV. 29 (1971); Andrews, *Hohfeld's Cube*, 16 AKRON L. REV. 471 (1983); Cullison, *supra* note 3, Finan, *Presumptions and Modal Logic: A Hohfeldian Approach*, 13 AKRON L. REV. 19 (1979); Mullock, *supra* note 3.

[8] *See, e.g.*, Corbin, *Legal Analysis and Terminology*, 29 YALE L.J. 163 (1919); Finnis, *Some Professorial Fallacies About Rights*, 4 ADEL. L. REV. 377 (1972); Goble, *A Redefinition of Basic Legal Terms*, 35 COLUM. L. REV. 535 (1935); Halpin, *Hohfeld's Conceptions: From Eight to Two*, 44 CAMBRIDGE L.J. 435 (1985); Husik, *Hohfeld's Jurisprudence*, 72 U. PA. L. REV. 263 (1924); Kocourek, *The Hohfeld System of Fundamental Legal Concepts*, 15 ILL. L. REV. 24 (1920); Kocourek, *Various Definitions of Jural Relation*, 20 COLUM. L. REV. 394 (1920); Radin, *A Restatement of Hohfeld*, 51 HARV. L. REV. 1141 (1938); Randall, *Hohfeld on Jurisprudence*, 41 L.Q. REV. 86 (1925); Stone, *An Analysis of Hohfeld*, 48 MINN. L. REV. 313 (1963); Williams, *The Concept of Legal Liberty*, in ESSAYS IN LEGAL PHILOSOPHY 121 (R. Summers ed. 1968).

[9] Allen, *supra* note 7, at 428-29.

[10] *See supra* note 7.

[11] *See* Allen, *supra* note 7.

[12] *Id.* at 483 n. 44.

[13] Professor Allen has, with Professor Saxon, expanded his system to include the remaining Hohfeldian relations. The work is unpublished but was delivered under the title "Analysis of the Logical Structure of Legal Rules by a Modernized and Formalized Version of Hohfeld's Fundamental Legal Conceptions" at

creation and extinction of legal relations[14] rather than the simple existence of such relations as expressed in "right," "duty," "no-right," and "privilege."[15]

It is the goal of this article to provide a formal system in which all the Hohfeldian terms are formalized and in which all the relations Hohfeld requires among his concepts may be proved to hold. The effort will commence with a brief look at Professor Hohfeld's work.[16] The development of the formal system will then begin with a reformulization of Professor Allen's work.[17] It will be a reformulation in that there will be a change in notation and his inference rules will be changed to an equivalent set of rules.[18] The system must then also be extended to provide the logical machinery necessary to allow for changes in legal relations.[19]

THE HOHFELDIAN RELATIONS

Hohfeld's system includes eight legal relations: right, duty, no-right, privilege, power, liability, immunity and disability. Hohfeld explained the nature of these basic legal relations by noting the relations existing between them. He presented tables of jural opposites and jural correlatives. The jural opposites are paired as follows:

right -- no-right
privilege -- duty
power -- disability
immunity -- liability,

while jural correlatives are paired as:

right -- duty
privilege -- no-right
power -- liability
immunity -- disability.[20]

Consiglio Nazionale delle Ricerche, IInd International Congress, LOGICA INFORMATIC DIRITTO: Automated Analysis of Legal Texts, Florence, Italy, September, 1985.

Professor Allen's expanded system requires the addition of a time notation to express the non-existence of a legal relation at the time of the act exercising a power and the subsequent existence of that legal relation. The system presented here will not employ a time notation but will instead rely on a logic representing counterfactual conditionals. *See infra* notes 107-49 and accompanying text.

[14] *See infra* notes 26-31 and accompanying text.
[15] *See infra* notes 21-25 and accompanying text.
[16] *See infra* notes 20-46 and accompanying text.
[17] *See infra* notes 47-79 and accompanying text.
[18] Professor Allen's system will also be extended slightly. A different treatment of the concept of natural necessity indicates that a slightly stronger axiom regarding the possibility of not committing a violation might be employed. *See infra* note 73 and accompanying text. One of the relevance requirements employed by Professor Allen will also be weakened slightly. *See infra* note 61.
[19] *See infra* notes 80-149 and accompanying text.
[20] Hohfeld (1913), *supra* note 2, at 30; Hohfeld, *Fundamental Legal Conceptions as Applied in Judicial*

He then explained the correlations by noting that "if X has a right against Y that he shall stay off the former's land, the correlative (and equivalent) is that Y is under a duty toward X to stay off the place."[21] Thus, a right is enjoyed by an individual as against another individual that the second shall do or refrain from doing something for the first. A duty is simply a right viewed from the vantage point of the individual who must do or refrain from doing the act in question. The sense in which they are correlative is then clear. X has a right against Y with regard to act A, if and only if Y has a duty to X with regard to act A.

Hohfeld could have taken either "right" or "duty" as a primitive term and defined the other in terms of the primitive term. Instead, he appears to have taken both as primitive and noted the relation between the two. Alternatively, one could, as Professor Allen[22] and others[23] have done, define both "right" and "duty" in terms of obligation. "Obligation to do A" may then be defined by noting that the failure to do A implies some sort of violation -- if legal rights and duties are at issue, a legal violation.

Having presented the concepts of rights and duties, Hohfeld then used those concepts to explain "no-right" and "privilege."[24] They are, respectively, the opposites of "right" and "duty." That is, an individual has a no-right against another individual with regard to a particular act if and only if that individual does not have a right against the second individual with regard to that act. Similarly, an individual has a privilege against a second individual with regard to a particular act if and only if the individual does not have a duty toward the second individual with regard to that act.

The terms "privilege" and "no-right" are also correlatives. X has a privilege against Y with regard to act A if and only if Y has a no-right against X with regard to A. This status as correlatives follows from "privilege" and "no-right" being opposites of terms that are themselves correlatives. If rights and duties must always be paired, then no-rights and privileges must also always be paired.[25]

Hohfeld provides more detail as to what he means by "power" than he did for "right" or "duty,"[26] but similarly relies on the correlative and opposite relations to define "liability," "disability" and "immunity" in terms of "power." In

Reasoning, 26 YALE. L.J. 710, 710 (1917) [hereinafter Hohfeld (1917)].

[21] Hohfeld (1913), *supra* note 2, at 32.

[22] *See* Allen, *supra* note 7, at 482; *see also infra* notes 47-79 and accompanying text.

[23] *See, e.g.*, Mullock, *supra* note 7, at 475-76.

[24] *See* Hohfeld (1913), *supra* note 2, at 32-33.

[25] To see that this is the case assume that a privilege exists between two individuals with regard to a particular act. If X has a privilege against Y, X does not have a duty toward Y. Hence by the correlative relation between rights and duties, Y does not have a right against X, which would give Y a no-right against X.

[26] Hohfeld still provides only "an approximate explanation sufficient for all practical purposes," saying that "[T]oo close an analysis might seem metaphysical rather than useful." Hohfeld (1913), *supra* note 2, at 44. "Power" too seems to be a primitive term but with some explanation of how the term is being used.

commencing his discussion of powers and liabilities Hohfeld noted that one way a change in legal relations could result would be "from some superadded fact or group of facts which are under the volitional control of one or more human beings.... [T]he person (or persons) whose volitional control is paramount may be said to have the (legal) power to effect the particular change of legal relations."[27] The changes with which Hohfeld was concerned were the creation or termination of any of his eight legal relations.[28] Thus, a person with a power is capable of performing some act which has the effect of creating or terminating a legal relation.

A liability is the correlative of a power,[29] that is, it is a power viewed from the point of view of the person whose legal relations may be changed. A person X is under a liability, if there is an act another person can perform that will affect the legal relations of X.

Disabilities and immunities are the opposites of powers and liabilities.[30] If X does not have a power with regard to individual Y, then X is under a disability with regard to Y. Similarly, if Y is not under a liability with regard to X, then Y has an immunity with regard to X.

Disabilities and immunities are also correlatives of each other.[31] If X has a disability with regard to Y, that is, X has no power to affect Y's legal relations, then Y is immune from having his or her legal relations affected by X. Similarly, if Y is immune with regard to X, then X is under a disability with regard to Y.

It should be emphasized that Hohfeld's legal relations exist at the individual or specific, rather than the general, level. A legal relation holds or fails to hold between an individual X and an individual Y. Hohfeld does admit of what he calls "multital" rights which seem to be held against an aggregate of persons. However, he notes: "A multital right... is always *one* of a large *class* of *fundamentally similar* yet separate rights . . . residing in a *single* person (or single group of persons) but availing *respectively* against persons constituting a very large and indefinite class of people."[32] More generally, Professor Corbin, in his discussion of Hohfeld, states:

> The term "legal relation" should always be used with reference to two persons, neither more nor less. One does not have a legal relation

[27] Hohfeld (1913), *supra* note 2, at 44.
[28] *See, e.g., id* at 45 ("Thus, X the owner of ordinary personal property . . . has the power to extinguish his own legal interests (rights, powers, immunities, etc.) . . . and . . . to create in the other persons privileges and powers."); *id.* ("X has the power to transfer his interest to Y,--that is, to extinguish his own interest and comcomitantly create in Y a new and corresponding interest."); *id* at 53 ("this enactment imposed . . . a liability to have duty created").
[29] *Id.* at 44.
[30] *Id.* at 44, 55.
[31] *Id.* at 55.
[32] Hohfeld (1917), *supra* note 20, at 718. It would seem more accurate to say that the multital right is a class of rights against separate persons rather than is one of a class.

> to himself. Nor does one have a legal relation with two others; he has separate legal relations with each. A so-called legal relation to the State or a corporation may always be reduced to many legal relations with the individuals composing the State or the corporation, even though for convenient discussion they may be grouped.[33]

Even if it would be desirable to discuss group legal relations, it would appear that Hohfeldian relations between individuals are more basic. Any logical treatment of group relations could proceed by replacing the individual with individual variables and employing the proper quantification.[34]

There has been disagreement over Hohfeld's choice of terms for his legal relations. Professor Radin criticized Hohfeld for including as privileges certain legal relations commonly referred to as rights, citing as an example the Hohfeldian privileges found in the Bill of Rights.[35] He, nonetheless, recognized the importance of the distinction between Hohfeldian rights and Hohfeldian privileges and was concerned only over whether Hohfeld's "terminological reform" would overcome entrenched usage.[36] Glanville Williams found similar difficulty in the use of the term "privilege" and suggested the use of "liberty" or "liberty not."[37] H.J. Randall disagreed with the use of "liability," because it was to be used in Hohfeld's scheme even when beneficial, and he thought "beneficial liability" to be a contradiction in terms.[38]

While arguments may be offered that an alternative term might be superior to one chosen by Hohfeld, that does not lessen Hohfeld's contribution. Hohfeld recognized that for any two individuals and an act, there either is or is not an obligation that the act be done, and the obligation or lack thereof may be looked at from the point of view of the person who would perform the act or the of person for whom the act is to be performed. Similarly, for any two individuals one can do an act that will affect the other's legal relations or he or she cannot so affect the other's legal relations. This too may be looked at from the point of view of the person who would perform the act or of the person whose legal relations would or would not be affected. The importance is the recognition that eight situations exist. How those situations are labelled is of lesser importance, and in fact, any use of English terms as labels is bound to lead to the sort of problems found by Radin, Williams and Randall as ambiguous natural language terms are attached to unambiguous, or at

[33] Corbin, *supra* note 8, at 165.
[34] *See infra* notes 148-49 and accompanying text.
[35] Radin, *supra* note 8, at 1148-49.
[36] *Id.* at 1149.
[37] Williams, *supra* note 8, at 128. Interestingly, Williams also noted that, since "liberty" means "no duty not," it contains two negatives "logically independent of each other." *Id.* at 130. He thought this to be "perhaps a unique phenomenon in our language." *Id.* Actually such a phenomenon merely points to the modal logic nature of the relations. Just as "liberty" means "no duty not," "necessary" means "not possible not" and "obligatory" means "not permissible not." *See infra* notes 66-71 and accompanying text.
[38] Randall, *supra* note 8, at 92-93.

least less ambiguous, logical constructs.[39]

Another criticism aimed at Hohfeld is that his set of legal relations may be reduced to fewer than eight and that "[t]o have more terms than necessary is just as harmful to a clear understanding as to have fewer terms than necessary."[40] Reductions to various combinations of terms have been suggested. Halpin claimed that the other six relations are all reducible to rights and duties,[41] while Goble claimed that all the other relations derive from powers.[42] Corbin, on the other hand, argued that the remaining legal relations may all be derived from duties and powers.[43]

Based on the logic to be presented, Corbin appears to have been correct. In fact one may select, as it were, one from column A -- right, duty, no-right, privilege -- and one from column B -- power, liability, disability, immunity -- and express the remaining relations in terms of the two selected. Each of those remaining from the first choice may be expressed as the negation, correlative or negation of the correlative of the term chosen. The same is true of the second choice. Each term in each group of four will be equivalent to its correlative, with a change in point of view that is not of great logical relevance, and will hold whenever its negation does not. Since they are logically related, they may be expressed in terms of each other. Attempts to reduce all eight to one term or to two terms from the same column are questionable. If the logic presented here is an adequate representation, the terms in the two groups are sufficiently different that the members of each should not be expressible in terms of the other.

Even recognizing the validity of Corbin's reduction, the greater than necessary number of terms in Hohfeld's scheme does not constitute a serious criticism. Logic tells us, for example, that we can get along without the use of the word "and", since it may always be replaced by a combination of "not" and "or."[44] That does not mean, however, that a logical system should not employ "and" as a logical connective. Its regular occurrence justifies its inclusion,[45] and indeed the system would be cumbersome without it.[46] Similarly, while duties and powers, or rights and

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[39] Even the translation of the connective "or" from English to symbolic logic is not without difficulty, since in English "or" may be either inclusive or exclusive. The logician simply chooses one meaning to identify with the logical symbol and then expresses the other meaning through a different symbol or a combination of symbols.

[40] Husik, *supra* note 8, at 267.

[41] Halpin, *supra* note 8.

[42] Goble, *A Redefinition of Basic Legal Terms*, 35 COLUM. L. REV. 535 (1935).

[43] Corbin, *Jural Relations and Their Classification*, 30 YALE L.J. 226 (1921).

[44] *p and q* is equivalent *to not-(not-p or not-q)*.

[45] Sometimes a symbol for "and" will not be included as a primitive symbol but will be defined in terms of the symbols for "or" and "not". Other times a symbol for "and" may be so included and the relation between "and" and "or" and "not" stated as a theorem.

[46] A more striking example is presented by the Sheffer stroke. All the logical connectives may be represented through various combinations of the Sheffer stroke. The Sheffer stroke, however, does not correspond to any term in English. Thus, while use of only the stroke presents a minimalist logic, it is not clear that it is superior to a logic employing "or", "and", "not", "implies" or any combination of those connectives.

powers, may be sufficient to express the other relations, greater clarity and correspondence with everyday language justify the inclusion of the additional concepts, while keeping in mind the relations between the terms.

A LOGIC OF RIGHTS, DUTIES, NO-RIGHTS AND PRIVILEGES

Professor Allen has provided a logic capable of expressing and working with the concepts necessary to analyze rights, duties, no-rights and privileges.[47] The system that follows is simply a reformulation and slight extension of his work. The Polish notation in which he works has been changed. His inference schemes have also been changed, but with one exception,[48] the changes are without logical significance. They are intended only to match the change in notation and perhaps to provide some slight simplification. The minor extension comes about as a result of a differing treatment of the laws of nature or natural necessity.[49] The treatment employed here indicates that a slightly more powerful axiom than one employed by Allen can be included in the system.[50]

As with any logic, the development of the system must begin with a vocabulary.

VOCABULARY

Propositional variables: p, q, r, . . .
Act variables: a, b, c, . . .[51]
Person variables: x, y, z, . . .
Constants: V1, V2, V3, . . .[52]
Connectives: &, V, ->,[53] ~, L, L', M, M'[54]
Predicates: D_, a one place predicate, and B__ and F__,
two place predicates.

The development of the system continues with the presentation of a grammar for the construction of meaningful sentences using the vocabulary.

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[47] *See* Allen, *supra* note 7.
[48] The exception is a change in one of Professor Allen's relevance requirements. *See infra* note 61.
[49] *See infra* note 68 and accompanying text.
[50] *See infra* note 73 and accompanying text.
[51] The act variables represent changes in states of affairs.
[52] The constants V1, V2, etc. represent specific violations of legal norms within the legal system being analyzed.
[53] While &, v and ~ represent the *and, or* and *not* of propositional logic, the connective -> is not the material implication of standard propositional logic. It is what Professor Allen calls genuine implication and it requires relevance between the antecedent and the consequent. The restricted formulation of the deduction theorem, presented as transformation rule (9), provides the necessary relevance.
[54] The connectives L, L', M, and M' are modal operators, with L and M representing logical necessity and possibility and L' and M' representing a form of natural necessity and possibility. *See infra* notes 67-68 and accompanying text.

FORMATION RULES

1. Any propositional variable or constant standing alone is a well-formed formula (wff).[55]

2. If u and v are wffs, so are u&v, u∨v, u->v, ~u, Lu, L'u, Mu, and M'u.

3. If a is an act and x is a person, Da, Bax and Fax are wffs.[56]

4. If a string of symbols is not a wff by a combination of (1), (2), and (3), it is not a wff.[57]

A defined connective, *p <-> q*, may also be added to the system. *p <-> q* is defined as *(p -> q) & (q -> p)*. Given the definition, it is clear that if *u* and *v* are wffs, then by several applications of rule (2), *u <-> v* is also a wff.

With the grammar now in place, transformation rules may be presented. The transformation rules allow the derivation one formula from another or from a combination of other formulae. They are also central to the construction of a proof.

A proof is a series of wffs each of which is either an assumption, an axiom,[58] or a previously proven thesis,[59] or is derived from earlier steps in the proof by means of the transformation rules. The wffs that make up a proof are accompanied by a second column providing the justification for each step in the proof. That justification column indicates that the step in question is either an assumption, axiom or thesis or indicates the transformation rule employed and the previous steps relied on.

Professor Allen also includes an assumption dependence column in the construction of his proofs. That column, a third column in the presentation, requires the assignment of a number to each assumption as the assumption is introduced into the proof. If an axiom or thesis is introduced, no additional number is placed beside the axiom or thesis in the third column. As the transformation rules are applied, the assumption dependence column must also be manipulated. In general, if a step in the proof results from the application of a transformation rule to a single previous step, the numbers in the assumption dependence column of that previous step are

[55] The well-formed formulae are the grammatically correct sentences of the logic.
[56] *Da* is read as "a is done", that is, the change in affairs represented by a has been brought about. *Bax* is read as "a has been done by x" and *Fax* is read as "a has been done for x." *Bax* then means simply that the change represented by a has been brought about and that x is the person who caused the change. *Fax* is a little less straightforward. It also means that the change represented by a has been brought about but also indicates that the change was for the benefit of x.
[57] The later extension of the logic will require that this rule be amended to include other formation rules. *See infra* notes 128, 138 and 149.
[58] Axioms are the basic assumptions of the logic. They are statements taken to be true and on which the truth of all the derived statements rest.
[59] A thesis is a wff that may be proven in the system without being dependent on any assumption.

reproduced in the third column of the derived step. If a step results from the application of a transformation rule to two previous stops, each number written in either step's third column is included in the assumption dependence column for the derived step. The only exceptions to this accumulation of numbers in the third column are the deduction theorem and indirect inference, rules 9 and 10 infra. The handling of the assumption dependence column for those rules will be discussed as the rules are presented. Rules 1, 9 and 10 also have requirements regarding the numbers in the third columns of the steps to which the rules are applied.[60]

TRANSFORMATION RULES

1. From u&v derive u or v, and from u and v derive u&v.[61]

2. (Modus ponens) From u->v and u derive v.

3. (Modus tollens) From u->v and ~v derive ~u.

4. From u⋁v derive ~(~u& ~v) and vice versa.

5. From u&(v⋁w) derive (u&v)⋁w.

6. From ~(~u) derive u.[62]

7. From u derive u⋁v or v⋁u.

8. From u&v derive ~(~(u&v)).

9. (Deduction Theorem) If u is taken as an assumption and v is proved from that assumption, then u->v may be derived.

In the deduction theorem proof of *u->v*, *v* must be dependent on *u*. That dependence is shown in the assumption dependence column. Since *u* was taken as an assumption it was assigned a new number. The number assigned *u* must appear in the third column of the step in which *v* is derived. Once *u->v* is derived, that formula is no longer dependent on *u* and the number assigned *u* is dropped from the assumption dependence column for *u->v*, while the remaining numbers in the third

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[60] *See infra* notes 61, 63 and 64 and accompanying text.

[61] Professor Allen's system requires that, to inter *u&v* from *u* and *v*, the third columns of the steps consisting of *u* and *v* must share at least one number in common. That rule is altered in the system presented here. The rule is weakened slightly to allow the inference of *u&v* from *u* and *v* where *u* and *v* are not dependent on any assumptions and thus have no numbers in their third columns. Thus, two theses, be they axioms, tautologies or previously proven theorems, may be conjoined.

[62] Professor Allen does not allow the derivation of ~(~u) from u, as would be allowed in standard propositional logic. Double negations may be derived only in certain specific instances provided for in the transformation rules, as in Rule 8 *infra*.

column for v[63] are retained in the third column for *u->v*.

A note of caution is required in using the deduction theorem. If the deduction theorem is used within a longer proof, steps which were part of the proof of *v* from *u* and are dependent on u may not be used outside the context of proving *u->v*. If *u* is an assumption made for use of the deduction theorem, the number assigned *u* should not appear in the assumption dependence column of the concluding line of any proof.

10. (Indirect Inference) If u is taken as an assumption and v and ~v are both derived from u, then derive ~u.

Here too the proofs of *v* and ~*v* must both be dependent on *u*. That dependence is again shown in the assumption dependence column. The assumption *u* was assigned a new number, and that number must appear in the third column of the steps in which *v* and ~*v* are derived. Once the contradiction is derived, ~*u* may be derived. ~*u* is not dependent on u and the number assigned *u* is not included in the assumption dependence column for ~*u*, while the remaining numbers in the third columns for *v* and ~*v*[64] are retained in the third column for ~*u*.

The caution required in using the deduction theorem is also required for indirect inference. If indirect inference is used within a longer proof, steps which were part of the proof of *v* and ~*v* from *u* and are dependent on *u* may not be used outside the context of proving *v* and ~*v*. If *u* is an assumption made in employing indirect inference, the number assigned *u* should not appear in the assumption dependence column of the concluding line of any proof.

The following proof provides an example of the deduction theorem, indirect inference, and the general construction of the three columns involved. In the justification column "Assumption" is abbreviated as "Ass.", "Transformation Rule" is abbreviated as "TR", "Deduction Theorem" as "DT", and "Indirect Inference" as "II". Modus Ponens and Modus Tollens are transformation rules and can be justified simply by using the number of the rule. They are, however, used so frequently that, rather than referring to the rule number, the steps involved will be justified simply as "MP" or "MT". Whenever the justification appeals to a transformation rule, the step or steps to which the rule has been applied is or are indicated.

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63 The remaining numbers in the assumption dependence column for *v* result from the fact that previous steps in the proof may be combined with *u* in the derivation of *v*.

64 As in note 63, the remaining numbers in the assumption dependence columns for *v* and ~*v* result from the fact that previous steps in the proof may be combined with *u* in the derivations of *v* and ~*v*.

PROOF: s-> ~t, p->(r&q), s->((p->q)->t) |- ~s[65]

STEP	JUSTIFICATION	ASSUMPTIONS
1. s	Ass. for II	1
2. p	Ass. for DT	2
3. p->(r&q)	Ass.	3
4. r&q	MP 3, 2	2 3
5. q	TR 1, 4	2 3
6. p->q	DT 2-5	3
7. s->((p->q)->t)	Ass.	4
8. (p->q)->t	MP 7, 1	1 4
9. t	MP 8, 6	1 3 4
10. s-> ~t	Ass.	5
11. ~t	MP 10, 1	1 5
12. ~s	II 1-11	3 4 5

Additional transformation rules are necessary for the manipulation of several other concepts required in the analysis of Hohfeld's relations.

11. From Lu derive u.
12. If u is a thesis, derive Lu.
13. From Lu derive L(~(~u)).
14. From Mu derive ~L(~u) and vice versa.
15. From u derive Mu.
16. From L'u derive u.
17. If u is a 'thesis,[66] derive L'u.
18. From L'u derive L'(~(~u)).
19. From M'u derive ~L'(~u) and vice versa.
20. From u derive M'u.

Transformation rules (11) through (20) are used to manipulate a combination of two forms of modalities. *L* and *M* represent the alethic modalities of logical necessity and logical possibility. A wff is logically necessary if and only if its truth follows from the rules of logic. A wff is logically possible if and only if it does not represent a logical contradiction. The concept is sometimes presented as a consideration of possible worlds.[67] If a wff would have to be true in every possible world, it is logically necessary. If a wff could be true in some possible world, even if not true in the actual world, it is logically possible.

[65] The symbol |- in the statement of a proof indicates that all the preceding formulae may be taken as assumptions in proving the formula following the symbol. An assumption free proof, that is the proof of a thesis, would have no formulae preceeding the |-.

[66] A 'thesis is not really a thesis in the logical sense; that is, it might not be provable only from the rules of logic. Instead, it is a formula provable from the theses of logic and the laws of nature.

[67] *See, e.g.*, Kripke, *Semantical Analysis of Modal Logic*, 9 ZEITSCHRIFT FÜR MATHEMATISCHE LOGIK UND GRUNDLAGEN DER MATHEMATIK 67 (1963).

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L' and *M'* represent another form of necessity and possibility, which Professor Allen labels natural necessity and natural possibility. For Professor Allen a proposition is naturally necessary if it logically follows from the laws of nature.[68] Similarly, a proposition is naturally possible if it does not contradict the laws of nature. Expressed in terms of possible worlds, a proposition is naturally necessary if it must be true in every world in which the laws of nature hold, and a proposition is naturally possible if it could be true in some world that adheres to the laws of nature.

Since in any world in which the laws of nature hold the laws of logic also hold, the set of naturally similar worlds is a subset of the set of logically similar worlds. Thus, if a proposition is logically necessary and is true in every (logically) possible world, it must be true in every world adhering to the laws of nature and hence be naturally necessary. Similarly, if a proposition is true in some world adhering to the laws of nature and is hence naturally possible, it is by that fact true in some possible world and is logically possible. This consideration leads to two additional transformation rules.

21. From Lu derive L'u.
22. From M'u derive Mu.

With a machinery for necessity and possibility now in hand, Professor Allen adds several defined terms and formulae to the vocabulary.

V is defined as $V_1 \vee V_2 \vee V_3 \vee$.....[69]
u->>v (read "u naturally implies v") is defined as
L'(u->v)
Ou is defined as ~u->>V[70]
Pu is defined as ~O(~u)[71]

Just as *p <-> q* was included in the logic as a short form for the formula *(p -> q) & (q -> p)*, it would be useful to include the formula *p <<->> q* as shorthand for *(p ->> q) & (q ->> p)*.

We may then return to the development of the transformation rules and add rules to manipulate the predicates involving action.

23. From Da and a->>b, derive Db.[72]

[68] The laws of nature are not specified. Clearly they include laws that are not mandated by logic alone but rest instead on basic truths of physics and the other sciences. Natural necessity is similar to the theoretical necessity of Professor Snyder. *See* D. SNYDER, MODAL LOGIC AND ITS APPLICATIONS 175-78 (1971).
[69] That is, *V* is the disjunction of all the individual violations. *V* might be read as "There is some violation."
[70] *Ou* is read as "u is obligatory" and is defined as "The failure of u naturally implies > violation" which is equivalent to "It is naturally necessary that the failure of u genuinely implies a violation."
[71] *Pu* is read as "u is permitted."
[72] *a->>b* means that the change in affairs represented by *a* naturally implies the change in affairs represented

24. From Bax and a->>b, derive Bbx.
25. From Fax and a->>b, derive Fbx.
26. From Bax derive Da.
27. From Fax derive Da.
28. From OBax and D(~a), derive B(~a)x.
29. From OFax and D(~a), derive F(~a)x.
30. From B(~a)x derive ~Bax.
31. From F(~a)x derive ~Fax.
32. From B(~(~a))x derive Bax and vice versa.
33. From F(~(~a))x derive Fax and vice versa.

Lastly, an axiom must be added and the Hohfeldian relations of right, duty, no-right and privilege represented within the logic.

AXIOM

M'(~V)[73]

DEFINITIONS[74]

Right-xay is defined as O(Bay&Fax).[75]
Duty-xay is defined as O(Bax&Fay).[76]
No-right-xay is defined as ~O(Bay&Fax).[77]
Privilege-xay is defined as ~O(F(~a)y&B(~a)x).[78]

With the system presented, Professor Allen was then able to prove that the correlation and opposition Hohfeld posited for his relations were proveable within the logic.[79]

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by *b*.

[73] The axiom simply states that it is possible that there is no violation; that is, it is possible to adhere to the law. This axiom is somewhat stronger than that adopted by Allen. Professor Allen assumes only that it is logically possible that there be no violation. Here the assumption is that it is naturally possible that there is no violation; that is, it is physically possible to adhere to the law. The laws of nature should not force one into a violation of the law.

[74] The definitions Professor Allen provided have been rewritten to read more naturally. Professor Allen wrote *Right-axy* which he read as "y has a right that a with respect to x." As presented here, *Right-xay* is read as "x has a right that a with respect to y."

[75] *Right-xay* or "x has a right that a with respect to y" is defined as "It is obligatory that a be done by y and a be done for x."

[76] *Duty-xay* or "x has a duty to a with respect to y" is defined as "It is obligatory that a be done by x and a be done for y."

[77] *No-right-xay* or "x has no right that a with respect to y" is defined as "It is not obligatory that a be done by y and be done for x."

[78] *Privilege-xay* or "x has a privilege that a with respect to y" is defined as "It is not obligatory that not-a be done for y and be done by x."

[79] *See* Allen, *supra* note 7, at 483 n.44.

AN INFORMAL LOOK AT THE POWER SET

Powers, immunities, liabilities and disabilities involve changes in legal relations. There are various ways in which a legal relations may change, but not all of them are the result of an exercise of a power, etc. A change in rights under a contract may, for example, result from the occurrence of a natural event.[80] Powers, immunities, liabilities and disabilities, however, exist in situations in which the potential change in legal relations is dependent on the volitional act of some person.

X is said to have a power over Y, if X, by doing some act, can change the legal relations of Y.[81] This change is either the creation or termination of any of Hohfeld's eight legal relations. Thus, a person with a power is capable of performing some act which has the effect of creating or terminating a legal relation.

Since a liability is the correlative of a power,[82] it too concerns a volitional act that changes a legal relation. However, with a liability the situation is viewed from the point of view of the person whose legal relations are changed, rather than from the point of view of the person with the capacity to effect the change through the performance of the act. Thus, a person Y is under a liability with regard to X, if there is an act X can perform that will affect the legal relations of Y.

Disabilities and immunities are the opposites of powers and liabilities,[83] so statements asserting the existence of disabilities and immunities also regard the possibility of creating or extinguishing legal relations. The statements, however, assert incapacity. If X is under a disability with regard to Y, X does not have a power with regard to individual Y, that is, there is no act X can perform that will affect Y's legal relations. Similarly, if Y has an immunity with regard to X, Y is not under a liability with regard to X. Once again, there is no act X can perform that will affect Y's legal relations. The difference is that, while disability stresses the incapacity of X, immunity stresses the protection of Y's legal relations against change, at least against change resulting from X's acts.

Given the interrelations between power, liability, disability and immunity, providing a logical analysis of any one term will go a long way toward the analysis of the remaining terms. The selection of either power or liability appears to be the best choice with which to begin the analysis, since either states a positive. Between power and liability, power appears to be the better starting point, since it is analyzed

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[80] A policy holder's right to payment under a flood insurance contract would, of course, require the antecedent occurrence of a flood.

[81] Hohfeld noted that a change in legal relations may result "from some superadded fact or group of facts which are under the volitional control of one or more human beings. . . . [T]he person (or persons) whose volitional control is paramount may be said to have the (legal) power to effect the particular change of legal relations." Hohfeld (1913), *supra* note 2, at 44.

[82] *Id.* at 44.

[83] *Id.* at 44, 55.

from the point of view of the actor. This analysis will proceed by analyzing powers.[84]

A person has a power over another person, if he or she can do some act that changes the legal relations of that second person.[85] Any change in legal relations can be viewed as a combination of the creation of some legal right or rights and the extinguishing of some other legal right or rights.[86] Thus, a power held by X over Y may be looked at as X's capacity to perform an act that will have the effect of either creating or extinguishing a legal relation to which Y is a party.[87]

It would appear that a power represents a sort of conditional. X's act is antecedent to the creation or termination of Y's legal relationship. Very roughly, to say that X has a power over Y is to say that there is some act X can perform that will create or terminate a legal relation between Y and some other person.[88] That the statement is a very rough approximation is shown by the length of the following analysis presented to explicate the concept.

Acts

The acts that can be the antecedents to the conditionals expressing powers must be acts with legal significance. If Y has made a contract offer to X, and that offer is still outstanding, X has the power of acceptance; that is, by accepting the contract offer, X creates legal relations to which Y is a party. Acceptance, except in cases of acceptance by silence, requires a physical (possibly verbal) act by X, yet to say that X may bind Y by signing his name or uttering some word or words may not sufficiently stress the need for legal context. In some situations X might accept by signing his or her name, but that is true only in case X's signing has the legal significance of constituting an acceptance.

If the view of Justice Holmes that "[a]n act is always a voluntary muscular contraction, and nothing else"[89] is accepted, then such an act might not be an adequate antecedent in the conditional expressing the power. The antecedent act

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[84] This choice is not required and should not affect the outcome. That is, a different choice might lead to a different expression of the Hohfeld relations, but the expression should be equivalent to that developed here. The choice made is simply one that it is believed will add to the clarity of the analysis presented.

[85] *See supra* notes 26-28 and accompanying text.

[86] For any legal relation that has changed, the change may be seen as the termination of the preexisting relation and the creation of the new legal relation.

[87] X need not be a party to the legal relationship created or terminated. For example, X, through the exercise of his power of acceptance to a contract offer from Y, may create a legal relation between Y and a third party beneficiary (perhaps in addition to rights Y would then enjoy against X or duties Y would owe X). Similarly, X may have a power as an agent to bind Y to contractual obligations between Y and X's principal, without X having and rights or duties with regard to Y under the contract.

[88] The other person is a person other than Y. The legal relation created or terminated may be between Y and X.

[89] O.W. HOLMES, THE COMMON LAW 91 (1881). *See also*, 1 J. AUSTIN, LECTURES ON JURISPRUDENCE 290 (R. Campbell ed. 1875) ("In truth, the only parts of the train [of incidents] which are my act or acts, are the muscular motions.").

must have legal significance. One way that legal significance can be expressed is to posit conditions under which the act, in the sense of a muscular contraction or series of such contractions, has the required legal significance. That is, to say that X has a power of acceptance is to say that "If X signs his name, then circumstances are such that X has accepted, and if X accepts, Y comes under a contractual obligation."

The approach suggested leads to needless complexity. There is nothing logically incorrect in going from the muscular contraction to the legal significance under the circumstances and then noting the creation of the legal relationship resulting from the now legally significant act. However, "act" need not be considered so narrow a term. As Sir John William Salmond has stated "We habitually include all material and relevant circumstances and consequences under the name of the act . . . not merely the muscular contractions by which this result is effected."[90]

We may sometimes speak of an act as a muscular contraction and all human acts may be argued to include a muscular contraction.[91] Yet, we may more often speak of acts as including Salmond's circumstances and consequences. It may be said that X murdered Y or that X accepted Y's offer of contract, despite the fact that each statement presents a legal conclusion that the muscular act constituted murder or acceptance under the relevant circumstances.

If there is no necessity to limit the ascription "act" to those basic acts constituting muscular contractions, then a step in the analysis may be omitted. When it is said that there is an act X may do that will create a legal relation to which Y is a party, X's act might be something as complex as acceptance. The logic is simplified by having to express only the relation between acceptance and the creation of the legal relation. On the other hand, the question of whether or not the antecedent has occurred becomes mired in the complexities of contract law. If the antecedent were instead a basic act, the contraction of muscles, it would be easier to determine whether the antecedent had occurred, but the same problem would arise in determining the legal significance of the act. Since it the relation between the legally significant act and the legal relation created that is of interest in examining powers, it would seem reasonable to drop questions of contract law from the logical analysis. Circumstances and consequences will be included in the acts of interest and legally significant acts will be among those that may replace the variable *a* in *Da*, *Bax* and *Fax*.

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[90] J. SALMOND, JURISPRUDENCE 370 (10th ed. 1947).

[91] This is not meant to contest the existence of mental acts. Rather, the position is that for a physical occurrence to be considered a human act, there must have been a muscular contraction in the causal chain leading to the event.

Powers as Conditionals

1. Material Implication

If powers are to be represented as some form of a conditional, it is clear that they may not be treated simply as material implications. Material implication is the traditional formal logic treatment of conditionals. Under that approach a conditional is true, unless its antecedent is true and its consequent false. Formal logic in general has found some difficulty with this approach. The paradoxes of material implication, for example the fact that a false proposition implies absolutely anything[92] and a true proposition is materially implied by any other formula,[93] have led to the development of modal logic with its strict implications requiring that the implication be necessary[94] and, where that logic fails,[95] relevance logic with its requirement that there be some connection of relevance between the antecedent and the consequent.[96]

The paradoxes of material implication come about because, when the antecedent is false, the conditional, even if true, tells one nothing about the truth of the consequent. Looked at from another point of view, if the antecedent is false, one need not even examine the truth of the consequent to know that the conditional is true. That fact raises a particular difficulty for the analysis of powers.

If a power is taken to express the material implication; "If X does act a, then Y has a legal relation created,"[97] the conditional contains no information with regard to Y's legal relations, if X has not done a. This is a particular problem for the expression of powers, because it would seem correct to say that X has a power over Y only when X has not yet done the act that will have the effect of changing Y's legal relations. If X has done the act, then Y's relations have already changed and X no longer has a power over Y, or at least not the same power as that which was exercised by performing the act in question. In fact, it would appear that a correct formalization of powers would have to include the fact that X has not yet done the act. A formalization using material implication would yield something of the form

~ Bax & (Bax -> Change-y).[98]

[92] If *p* is false, then *p* -> *q* is true, whatever *q* may represent, if -> is material implication. Since the antecedent *p* is false, the conditional does not have true antecedent and false consequent. Thus, the conditional is true.

[93] If *q* is true, then *p* -> *q* must be true. Since the consequent is true, the conditional does not have true antecedent and false consequent and, thus, is true.

[94] *See generally*, D. SNYDER, *supra* note 68; G. HUGHES & M. CRESSWELL, AN INTRODUCTION TO MODAL LOGIC (1968).

[95] *See infra* notes 99-100 and accompanying text.

[96] *See supra* notes 63-64 and accompanying text.

[97] The conditional could instead state that a legal relation of Y will be terminated. It should also be noted that the formulation eventually developed will be considerably more complex that the single statement is text. *See infra* notes 107-20 and accompanying text. The simple statement is presented only to show the difficulty presented by material implication, and that difficulty is one that would remain, even if a more complex, but still material, implication were developed.

[98] *Change-y* is temporarily used to mean Y's legal relations have changed. A better symbolization will be

Since the formalization includes the fact that the antecedent of the conditional expressing the relationship between act and change must be false, the conditional will be true whether Y's legal relations change or not. The formalization then tells us nothing with regard to the effect of the act on the legal relationships of Y.

2. Strict Implication

As a way of requiring more than simply the fact that X not have acted without Y's legal relations having changed, powers might be defined in terms of strict implication. Strict implication is related to material implication through necessitation. p strictly implies q if and only if it is necessary that p materially implies q. That is, in no accessible world may p be true, while q is false. Unfortunately, strict implication also raises problems.

First, strict implication also has its paradoxes. For example, a contradiction strictly implies absolutely anything,[99] and tautologies are strictly implied by any other formula.[100] If necessity is taken as natural necessity, and the accessible worlds as those in which the laws of nature prevail, then if a is an act the doing of which would contradict the laws of nature, $L(Bax \rightarrow Change\text{-}y)$ must be true. If that is how powers would be expressed, then X has powers over everyone to change their legal relations by levitating his body or doing any other naturally impossible act. Clearly, at least some requirement of natural possibility with regard to X's performance is needed.

A more important problem is the fact that powers do not generally represent logically or naturally necessary relations between acts and changes. Circumstances play an important role, and those circumstances are expressed by contingent, not necessary, propositions. To say that one has a power of appointment is not to say that in all possible worlds, or in all possible worlds adhering to the laws of nature, one enjoys such a power. Rather, the factual situation of the world in which the statement is made determines whether one indeed has the power. In some other accessible world, the testator granting the power of appointment may still be alive, and the power would not yet exist in that world. Alternatively, the law could be different in

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presented *infra* notes 141-45 and accompanying text. This is an also a further oversimplification. The expression of a power may not require the identification of the act to be done. That is, one may mean, in saying that X has a power over Y, that there is some act X may do that will affect Y's legal relations, without knowing or without expressing precisely what that act is. Some form of quantification will be needed to express the existence of such an unspecified act. *See infra* notes 137-45 and accompanying text.

[99] $L((p\& \sim p) \rightarrow q)$ is true, whatever q may represent. Since the antecedent of the material implication $p\& \sim p$ is a contradiction, it is false in every possible world. Thus, $(p\& \sim p) \rightarrow q$ is true in every accessible world, and $L((p\& \sim p) \rightarrow q)$ is true.

[100] $L(p \rightarrow (q \vee \sim q))$ is true, whatever p may represent. Since $q \vee \sim q$ is a tautology, that is a formula that is always true, it is true in every possible world. Thus, in no world may the material implication have true antecedent and false consequent. The material implication is then true in every accessible world and so $L(p \rightarrow (q \vee \sim q))$ is true.

some other accessible world, and no powers of appointment might exist. Thus, to say that X has the power to appoint Y can not be to say that, no matter how the contingent facts may vary, X may, by doing some act, change Y's legal relations.

3. Powers as Time-Indexed Conditionals

The difficulties with treating powers as necessary implications would appear to require a return to material implication or some relevance based, but not necessary, implication. The difficulty with the fact that the performance of the act must not yet have occurred remains as a problem for material implication,[101] and would likely remain for a relevance conditional.[102] One might attempt to resolve this problem by a form of time indexing of propositions.[103]

Time indexing would require that propositions may be asserted to be true only at specific times. The time at which any proposition is asserted to be true would have to be indicated as part of the symbolization of the proposition. In such a system, *p* would not be a well formed formula, since it asserts the truth of the proposition expressed by *p* without stating the time at which it is claimed the proposition is true. Some notation of that time would have to be included in the definition of well formed formula, and each proposition would have to carry a time identifier.[104]

The advantage to requiring time identifiers is that it eliminates the problem presented by the fact that, in expressing a power, one must include that the antecedent of the conditional embodying the power be false.[105] If X has a power relative to Y,

[101] *See supra* notes 97-98 and accompanying text.

[102] Even if relevance between antecedent and consequent is required, simply placing a requirement of relevance on top of material implication will be inadequate. If that is the choice, the conditional will be true if there is relevance and it is not the case that the antecedent is true and the consequent false. Thus, as long as there is relevance, if the formalization includes the falsity of the antecedent, the conditional will be true. That is, a power will exist to change any legal relation of Y to which the act of X has relevance.

While one might try to limit this problem by adopting a restrictive view of relevance, it is not clear how that definition should be phrased. For example, there is relevance between X signing a contract offered by Y and a change in Y's legal relations placing a new contract duty on Y. However, the contract may, for some reason, be void. Thus, while the consequent, that Y's legal relations change, is false, so would be the antecedent, where X has not yet signed, and the conditional, *If X signs the contract, Y's legal relations will change*, will be true. To go beyond mere relevance and avoid situations where some other factor, for example a law declaring such contracts void, controls the change in legal relations, some analysis of those other factors, in addition to the requirement of relevance between antecedent and consequent will be required. Such a logical system is presented *infra* notes 121-49 and accompanying text.

[103] This is the approach taken by Professors Allen and Saxon, *supra* note 13. Their system is much more sophisticated than that suggested in text here. However, the problems that lead to the rejection of time indexing are the result of the general approach, rather than the specifics of a particular time indexed system. Thus, there is no need to discuss the Allen and Saxon system in detail.

[104] It is possible that necessary propositions might be exempted from such a requirement. Necessary propositions are true under all circumstances, or in all possible worlds. A difference in time is a different circumstance or indicates a different world. In either case the necessary proposition would be true in that circumstance or world; that is, the truth of the necessary proposition does not depend on time.

[105] *See supra* notes 97-98 and accompanying text.

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X can not yet have done the act that exercises the power. The fact that X has not done the act may be represented by *~Bax* with a notation indicating that the truth of that proposition is tied to the time at which it is stated that the power exists. The remaining symbolization would have to capture the fact that, if at some time in the future X does the act, then at that time Y's legal relation will change. Thus, the *Bax* that is the antecedent in the conditional part of the symbolization is not the negation of the *~Bax* stating that the antecedent has not yet been done.

While time indexing may, thus, eliminate the problem presented by the approaches already presented, time indexing raises another problem that makes the approach inadequate. The problem is that circumstances may change between the time at which it is asserted that the power exists and the future time at which the act is performed.

If Y has made an offer of contract to X, X has a power of acceptance. There is an act X may do, that is, accept the offer, that will create rights and duties in Y. In a time indexed approach in asserting the current existence of the power one would be stating that at this point X has not accepted and if at some point in the future X does accept, Y's legal relations will change. But, suppose Y revokes his offer before X accepts. Can the logic reach back in time and make the power asserting statement false? If so, the original statement would seem to be of little import. If not, then the conjunct expressing the conditional that if X accepts, Y's legal relations will change must still be true, but it would appear to be false and so the original claim of a power would be false.

One way in which to argue that the conditional does not become false would be to hold that what X does at the later time does not constitute acceptance. Since there is no longer an offer to be accepted, the claim seems reasonable. But suppose that instead of the offer being revoked, the law changes so that no contract rights and duties arise under the sort of contract offered. Once again, at the time it was stated that the power exists, the claim may have been true. But at the time of the act of acceptance, no legal relations changed. It is more difficult to argue that no acceptance occurred here. There was offer and acceptance, but no legal relations were changed.

Even if it were argued that there could be no acceptance of an offer to enter a non-enforceable contract, other examples could be presented. The proposed contract could be, both at the time of offer and acceptance, perfectly legal, but frustration of purpose may make the contract unenforceable. Again the act of acceptance would not have the effect of changing legal relations. Once again, the definition of acceptance could be changed to make X's act fail to constitute acceptance when the purpose behind the offer accepted would be frustrated. This would, contrary to contract theory, treat frustration of purpose as a claim that no contract had ever been formed rather than as a defense to contract.[106]

It is clear that circumstance after circumstance could be postulated in which the power existed at the time it was claimed to exist yet by the time the act occurred, no change was wrought. Time and again the act of acceptance could be redefined so as to avoid the problem. But, what would remain at bottom would be that acts constituting acceptance are only those acts that change Y's legal relations. That turns the expression of a power almost into a tautology. *X has a power of acceptance over Y* becomes *There is some act X may do but has not yet done and if the act changes Y's legal relations Y's legal relations will change.* The later statement is almost tautologically true, resting only on the requirement that X have the capacity to act. There are many acts that I may do and have not yet done, and for each of those acts, if they change Y's legal relations, Y's legal relations will change, whoever Y may be. Yet, certainly if I may be said to have a power of acceptance over Y on that basis, the power of acceptance is without substance.

Time indexing fails to take into account the fact that circumstances may change. While the likelihood and scope of change might be limited by requiring that the time at which the power is said to exist and the time at which the act is done be in close temporal proximity, that would limit the use of the concept of power. Clearly, claims of power are made even when the act and change in legal relations will not occur for some time.

One last attempt to save the time indexing approach might be to freeze the circumstances. X would be said to have a power over Y if there is an act X has not yet done, and in the future, if the circumstances do not change and X does the act, Y's legal relations will change. The problem is that the circumstances do change. The change in time in itself is a change in circumstances. It may, of course, be argued that the change in time is not a circumstance affecting the ability of X's act to change Y's legal relations. It is only the relevant circumstances that must be frozen. However, capturing the relevance of changes in circumstances and expressing the requirement that those circumstances not change requires a logic that goes far beyond merely the addition of time indexing.

Powers as Counterfactual Conditionals

The nature of statements asserting the existence of powers indicates that they are in fact counterfactual conditionals. Counterfactual conditionals are statements that assert that "if something which is not the case had been the case, then something else would have been true."[107] Since the assertion of the existence of a power seems to include that the act effecting the change not yet have occurred, to state that a power

[106] *See generally* J. CALAMARI & J. PERILLO, THE LAW OF CONTRACTS 475-515 (2d ed. 1977).

[107] J. POLLOCK, SUBJUNCTIVE REASONING 1 (1976). Counterfactual conditionals need not be quite so limited. It might not be required that the antecedent not have occurred. The counterfactual might be asserted in the situation where the speaker is simply unaware that or whether the antecedent has occurred.

exists is to state that "if something which is not the case had been the case," that is, the act were done, "then something else would have been true," that is, Y's legal relations would have changed.

The major problem in dealing with counterfactual conditionals is the possibility of differing circumstances. One may assert that if he had struck a particular match, it would have lighted, and assuming that the match seems to be properly manufactured, dry, etc., others may agree with the assertion. Yet, one might object to the assertion by questioning what would have been the result if, just at the instance of striking, a large raindrop had hit the match.

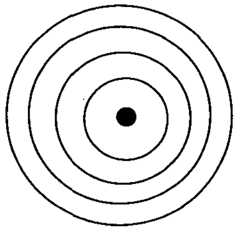
The natural response would seem to be that the speaker meant that, if he had struck the match and the sunny day on which the statement was made did not suddenly turn stormy, the match would have lighted. The particular change in circumstances bringing about the wet condition of the match had to be ruled out. The world in which the match had been struck has to be sufficiently similar to the actual world as to disallow changes in relevant circumstances. At the same time it must be recognized that some circumstances will change. For one thing, under the supposition, the match has been struck. That change must be accompanied by a host of changes regarding the surface against which the match is struck, wind currents stirred up by the act of striking, etc.

Clearly, one asserting the counterfactual conditional is not making the claim that, under any and all circumstances, if the match had been struck it would have lighted. The assertion is more in the nature of a claim that, if all the relevant circumstances remained the same, recognizing that some circumstance must change, and the match had been struck, it would have lighted.

The falsity of such a claim is shown not by postulating changes of circumstances under which the match would have failed to light. Rather, one demonstrates the falsity of this sort of statement by showing under the existing circumstances or circumstances as they would change with the act of striking, even had the match been struck, it would not have lighted. Falsity is demonstrated by showing that the match was improperly manufactured or the truth of the counterfactual is questioned by suggesting that possibility not by hypothesizing what would happen if circumstances were different than in fact they are.

It should be clear that counterfactuals cannot be treated as simple necessary propositions. If the statement regarding the match were interpreted as a claim that, while the match has not been struck in this world, in any world in which the match is struck, it lights, the statement is clearly false. In some of those other possible worlds it is raining or too cold to sustain fire. In some such worlds the match was improperly manufactured, and if logical necessity rather than natural necessity is employed, in some worlds there is no friction and in others friction is unaccompanied by heat.

While a less formal linguistic approach to the analysis of counterfactuals was attempted earlier,[108] the logical analysis of counterfactuals is due in most part to the work of Professors Robert Stalnaker[109] and David Lewis.[110] Lewis' development, as does Stalnaker's, proceeds through a consideration of the possible world semantical analysis of modal logic.[111] Lewis suggests that the set of all worlds accessible to a particular world be arranged in a set of nested spheres. The world in question is placed at the center of the nesting,[112] and the worlds most similar to that world are placed in the smallest sphere[113] drawn around the given world.

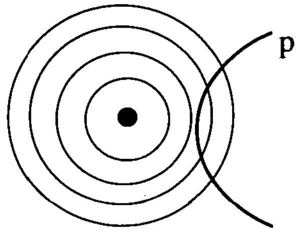


As one proceeds out through larger and larger spheres, the worlds become less similar to the world at the center. That is, more and more propositions that were true in the world in question become false, and more and more propositions that were false become true.

There may be propositions that are true throughout the system of spheres. Those propositions are the necessary propositions. A proposition that is true in some world somewhere within the system of spheres is a possible proposition at the world on which the spheres are centered. The contingent propositions that will represent possible changes in circumstance from those present in the center world will be true at some worlds in the system and false at others. The worlds at which such a proposition is true may be indicated pictorially by cutting through the spheres so that the worlds in which the proposition is true are on one side of the cut and those in which it is false lie on the other.

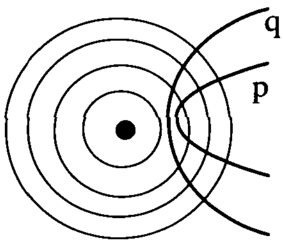
In the diagram below, writing *p* to the right of the cut indicates that those worlds in which *p* is true lie to the right, while those worlds in which *p* is false lie to the left. In this case *p* is false in the world on which the spheres are centered. From the point of view of the center world, *p* is false but possible. A diagram in which the

[108] *See, e.g.*, N. GOODMAN, FACT, FICTION AND FORECAST (1955); Chisholm, *Law Statements and Counterfactual Inference*, 15 ANALYSIS 97 (1955).
[109] *See, e.g.*, Stalnaker, *A Theory of Conditionals*, STUDIES IN LOGICAL THEORY: ESSAYS 98-112 (2 AM. Philosophical Q. Monograph Series 1968).
[110] *See, e.g.*, D. LEWIS, COUNTERFACTUALS (1973).
[111] *See supra* notes 67-68 and accompanying text. *See also, e.g.*, Kripke, *supra* note 67.
[112] There are modal logic systems in which the actual world does not have access to itself. *See, e.g.*, D. SNYDER, *supra* note 68, at 66-77 (presenting his system CMn). In such a logic the center point would not be a part of the set of spheres.
[113] *But see infra* notes 119-20 and accompanying text.



center were to the right of the cut would represent the situation where a proposition was not only possible but also true.

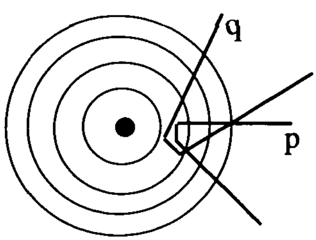
A strict implication, that is, p necessarily implies q, is depicted in the following diagram. The set of accessible worlds in which p is true is a subset of the set of accessible worlds in which q is true. Hence, in every accessible world in which p



is true, q is also true. The material implication is thus never false so there is strict implication.

Once again, counterfactual conditionals can not be represented as strict implications, since there are normally circumstances which, if sufficiently changed, would lead to a true antecedent and false consequent. It is then not true that in every accessible world in which p is true q is true.

What is required to fit counterfactual conditionals into this semantic model is to limit the examination to the most central set of spheres. In analyzing a counterfactual, one is not interested in what happens under every conceivable change of circumstances. Rather, one is interested in whether the consequent holds when the circumstances are changed just enough to make the antecedent true. All other circumstances that can remain the same, that is those circumstances that are not affected by the change in the truth value of the antecedent, should be kept constant. Pictorially, a true conterfactual conditional is represented as follows.



Attention is limited to the smallest sphere containing worlds in which *p* is true. Those are the worlds in which the circumstances are changed only enough so that *p* can be true. While the diagram shows that there are also other worlds in which *p* is true, it is the set of worlds most similar to the actual world but in which the antecedent has occurred that are of interest. The worlds in that smallest sphere in which *p* can be true are examined, and if in that sphere every world in which *p* is true is also a world in which *q* is true, the counterfactual is true at the world on which the system of spheres is centered.

The difficulties with this semantic model are in vagueness in the standard for deciding which worlds are contained in that smallest sphere and indeed whether there is in fact a smallest sphere. With regard to the first problem, Lewis' system is an improvement over Stalnaker's. Stalnaker had assumed that there would be a unique world most similar to the actual world but in which the antecedent is true. However, Lewis shows that two worlds may be tied in terms of which is the most similar to the actual world. He uses as an example the propositions *If Bizet and Verdi were compatriots, Bizet would be Italian* and *If Bizet and Verdi were compatriots, Bizet would not be Italian.*[114] It is not at all clear that a world in which the two were compatriots by being French is more similar to the actual world than a world in which the two are compatriots by being Italian or vice versa. It is, thus, uncertain which world to include within the smallest sphere. Yet, that decision would be determinative with regard to the truth of the two counterfactuals. Lewis would place both within the smallest sphere, and since neither consequent is true in all worlds in the smallest sphere in which the antecedent is true, neither counterfactual is true.

Despite Lewis' improvement, there is still a problem in not having a standard against which to judge similarity. Professor Nute presents an example that demonstrates the problem. He analyzes the counterfactual *If Carter had never served as Governor of Georgia, he would never have been President of the United States.* He then considers four different worlds. In the first, wl, Carter loses two elections for Governor and never goes on to be President. In w2 Carter does not run for Governor but instead runs for and is elected United States Senator and then goes on to be elected President. In w3 Carter runs for the Senate instead of Governor, loses and never runs for President. And, in w4 Carter runs for the Senate instead of Governor, is elected but performs poorly and is never elected President.[115]

In all four worlds the antecedent is true, yet in one of them the consequent is false. Only if we are willing to conclude that w2 is less similar to the actual world than wl, w3 and w4, and hence that w2 is not within the smallest sphere, may we still claim that the counterfactual is true. But, there is no standard against which to judge that degree of similarity.

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[114] *See* D. LEWIS, *supra* note 110, at 80.

[115] D. NUTE, TOPICS IN CONDITIONAL LOGIC 71 (1980).

Of course, in ordinary discourse, one would raise the situation presented in w2 and the person asserting the counterfactual would either agree that the counterfactual expressed is false or would assert that he really intended the antecedent to include not only not being Governor but never having been elected to statewide office. Such an approach may suffice for ordinary discourse, but it does point to a problem in the semantics. Taking the antecedent as stated, one will usually, if not always, be able to show a difficulty similar to that shown here in determining which of two alternative worlds, one with a true consequent and one with a false consequent, is more similar to the actual world.

The lack of an acceptable selection function for most similar worlds will lead to a weaker logic than might be possible, if such a selection function existed. Professor Pollock[116] weakened the logic he presents to avoid theses that are objectionable because they rest on what he sees as untenable assumptions regarding selection of similar worlds. While he still believes the logic to be strong enough to prove all true principles regarding these counterfactual conditionals, he admits that the inadequacy of the semantics makes it impossible to prove that all such principles can be proved.[117]

In the belief that it is better to work with a system that may prove to be too weak and only decide how to strengthen the system if the need arises than to work in a system that is too strong and end up with unacceptable theses creeping into the results, the development here will follow the approach of Professor Pollock. It should, however, be noted that one selection criteria could be added to the semantics when considering powers. For a world to be among those most similar to the actual state of affairs, the law must not change.[118]

Professor Lewis also offers a example to show that there may be cases in which there is no smallest sphere around the center world. There is still a nesting, but there is no minimal change that makes the antecedent true. That is, for any change that makes the antecedent true, there could have been a lesser change that would also have made the antecedent true. Lewis' example[119] involves drawing a line segment slightly less than one inch long. He then considers alternative worlds in which the line is longer than one inch. Among such worlds, there is no most similar world. For any such world, there is a difference between the length of the line in the actual world and the length of the line in that alternative. But, there must be another alternative

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[116] *See* J. POLLOCK, *supra* note 107, at 42-43. *See also infra* notes 118-20 and accompanying text.

[117] *See* J. POLLOCK, *supra* note 107, at 43-44. Such a proof is known as a completeness proof.

[118] Note that "world" is being used in a technical way here. Clearly, the law is not the same everywhere in the physical world. What is meant in text is that in considering the alternative states of affairs represented by the accessible worlds, the alternatives considered should not include alternatives in which the laws, or at least the relevant laws, change. France, in that alternative world, may continue to have laws that differ from those in the state of Missouri, but if a power is asserted as the result of Missouri law, the law of Missouri must be the same in all the worlds most similar to the actual.

[119] *See* D. LEWIS, *supra* note 110, at 20-21.

world in which the difference is less. Since there is no minimal difference, there is no most similar world.

Professor Pollock's approach to this problem is to note that the magnitude of a change does not necessarily determine comparative similarity.[120] What is required is not that the alternative world be that which is most similar to the actual in the sense employed above but rather that it be the world where change is minimal. All worlds in which the length of the line segment is changed and any other changes necessitated by that change have occurred have the same magnitude of change.

The variety of questions at issue in legal analysis might indicate that Lewis' example, resting as it does on mathematical limits and the extreme density of worlds required to handle statements regarding such limits, would not arise. Nonetheless, the conservative approach would be to avoid the chance of unacceptable theses and reject the assumption that there will be a smallest sphere that can handle Lewis' example. Taking instead the similarity relation as based on minimal change, as Pollock does, leads again to a slightly weaker logic but one which should prove sufficiently strong for the purposes here employed.

A FORMAL SYSTEM OF THE ANALYSIS OF POWERS

The logic developed thus far must be extended in several directions, before powers, disabilities, liabilities and immunities may be expressed and analyzed. First, a new connective representing the counterfactual conditional must be introduced and rules for the manipulation of that connective must be added. Second, rules for quantification must be added so that the existence of acts and legal relations can be asserted and analysis involving all acts or legal relations can be undertaken.

The logic for counterfactuals presented is based on that developed by Professor Pollock.[121] Professor Pollock's logic is weaker than systems developed by Professors Stalnaker[122] and Lewis;[123] that is, any theorem in Pollock's logic is also a theorem in Stalnaker's and Lewis', but each of the latter contain theorems not included in Pollock's logic.[124] Pollock finds some of the additional theorems present in the Stalnaker's and Lewis' logics to be objectional.[125] While those theorems might not raise difficulties in Hohfeldian analysis,[126] the safest approach is to use the weaker system, so long as it proves to be adequate.[127]

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[120] *See* J. POLLOCK, *supra* note 107, at 21-23.
[121] *See id.* at 42, 43.
[122] *See* Stalnaker, *supra* note 109.
[123] *See* Lewis, *Completeness and Decidability of Three Logics of Counterfactual Conditionals*, 37 THEORIA 74 (1972).
[124] J. POLLOCK, *supra* note 107, at 43.
[125] *Id.*
[126] *See supra* notes 119-20 and accompanying text.
[127] The weaker the system, the fewer theses it contains. With fewer theses there is less chance of an objectional thesis. The safest approach would then be to start with the weakest system that appears capable of providing

A symbol for the counterfactual conditional must be added to the vocabulary, and the grammar (the formation rules) must be extended to include the new symbol. The vocabulary is expanded by adding the symbol =>. The additional formation rule required is:

FORMATION RULE

5. If u and v are wffs, then u=>v is a wff.[128]

The formula *u=>v* is read as "If *u* were the case, then *v* would be the case."

Rules for the manipulation of => in conjunction with the other connectives must then be added. Rather than proceeding through the use of transformation rules, Professor Pollock stated axioms for his system. Since that part of the logic already presented employs transformation rules, that approach will continue. A set of such rules slightly weaker than Pollock's axioms[129] will be included in the logic.

TRANSFORMATION RULES

34. From (p=>q)&(p=>r) derive p=>(q&r).
35. From (p=>r)&(q=>r) derive (p⋁q)=>r.
36. From (p=>q)&(p=>r) derive (p&q)=>r.
37. From p&q derive p=>q.
38. From p=>q derive p->q.
39. From L(p->q) or from M'p&(p->>q) derive p=>q.[130]

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a framework for analysis. If that system proves to be inadequate, that is, if there are statements that ought to be theses but are unproveable, the system can be extended. Even then the conservative approach is to extend the system only to the point needed to resolve the difficulty.

[128] Formation rule (4) must be amended to state that, if a string of symbols is not a wff by a combination of rules (1), (2), (3) and (5), it is not a wff.

[129] Professor Pollock also includes all tautologies as axioms and Modus Ponens as a rule. The effect of that inclusion is to cause the logic to contain propositional logic. Since a slightly weaker form of propositional logic is already contained in Professor Allen's system, this development will continue along those weaker lines and build on the logic as developed rather than the stronger base Pollock uses.

[130] Pollock's rule states that if *p->q* is a thesis, then so is *p=>q*. That rule expresses a relationship between strict implication and counterfactual conditionals. If *p->q* is a thesis, then *(p->q)* is necessary; that is, *p* strictly implies *q*. Since *p->q* is then true in every possible world, then in all those closest worlds in which *p* is true the material implication holds and *q* is true. Hence, if *p* were true, *q* would be true or if *p* were the case *q* would be the case.

The logic developed here employs two levels of necessity, logical necessity and natural necessity. Even if we are willing to postulate that the laws of nature could be different, a world not adhering to the laws of nature would not be among those worlds closest in similarity to the actual world. So, if *p->>q* holds in the actual world, then *p->q* is true in all worlds with the same laws of nature and hence also true in all those the most similar world in which *p* is true. *g* is then also true in all those worlds, so *p=>q* is true in the actual world. The only exception is the case in which *p* represents a naturally impossible proposition. In such a case, the closest worlds in which *p* is true do not adhere to the laws of nature. *p->q* may not hold in some of those worlds, and hence *g* may be false in one of the most similar worlds in which *p* is true. *p=>q* would then not hold in the actual world. By requiring *M'p*, that possibility is eliminated.

40. From L(q->r) or from M'p&(q->>r) derive (p=>q) ->(p=>r).[131]
41. From L(p<->q) or from M'p&(p<<->>q)[132] derive (p=>r)->(q=>r).[133]

Since the acceptability of the transformation rules may not be obvious, an explanation of the rules within the context of the semantics seems warranted. Rule (34) considers the case in which *if it were the case that p then it would be the case that q and if it were the case that p then it would be the case that r* are both true. In all the worlds most similar to the actual world in which *p* is true *q* is true, and in all those worlds *r* is true. Hence, in all the worlds most similar to the actual world in which *p* is true *q&r* is true. Thus, if it were the case that p then it would be the case that (q&r). From *(p=>q)&(p=>r)* we may derive *p=>(q&r).*

To justify rule (35) assume *(p=>r)&(q=>r).* In all the most similar worlds in which *p* is true, *r* is true. Also, in all the most similar worlds in which *q* is true, *r* is true. Consider all those worlds in the innermost sphere in which *pVq* is true. In each such world, at least one of *p* or *q* is true. If *p* is true, then since that world is a most similar world in which *p* is true, *r* must be true. If *q* is true, then similarly, since that world is a most similar world in which *q* is true, *r* must be true. Thus,

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Note that the change in Pollock's logic has strenghtened the system. The counterfactual can be derived from an implication of less universality than the strict implication required by Pollock. Some expansion was necessary in order to explicate the relationship between ->> and =>. The discussion of the change within the context of the semantics indicates that the expansion is reasonable and unlikely to lead to objectional theses.

[131] Pollock's rule states that if *q->r* is a thesis, then so is *(p=>q)->(p=>r).* If *q->r* is a thesis, then *q->r* is necessary. Since *q->r* is true in every possible world, then in every world in which *q* is true *r* is also true. If *p=>q* is taken as true, then in all the most similar worlds to the actual in which *p* is true *q* is also true. But, wherever *q* is true, so is *r*. Hence, in all the most similar worlds in which *p* is true, *r* is true. Thus, *p=>r* is true, as is *(p=>q)->(p=>r).*

As with rule (39), the fact that the logic developed here employs two levels of necessity requires an examination of the relationship between *q->>r* and *(p=>q)->(p=>r).* Once again, even if we are willing to postulate that the laws of nature could be different, a world not adhering to the laws of nature would not be among those worlds closest in similarity to the actual world. If *q->>r* is true in the actual world, then *q->r* is true in all worlds having the same laws of nature. If *p=>q* is true in the actual world, then in all the closest worlds in which *p* is true, *q* is true. If those worlds are worlds in which the laws of nature hold, then *r* is true in them as well. Hence, *p=>r* will be true in the actual world, as will *(p=>q)->(p=>r).* Once again, what is required is that the most similar worlds in which *p* is true be worlds that adhere to the laws of nature, so that the inference from *p* through *q* to *r* may be made. That will be the case so long as *p* is naturally possible.

Again, the change in Pollock's logic has strengthened the system, but the discussion of the change within the context of the semantics indicates that the expansion is reasonable and unlikely to lead to objectional theses.

[132] *p<<->>q* was not included in the vocabulary but was introduced as shorthand for *(p->>q)&(q->>p).*

[133] Pollock's rule states that if *(p<->q)* is a thesis, then so is *(p=>r)->(q=>r).* If *p<->q* is a thesis, then *p<->q* is necessary. Since *p<->q* is true in every possible world, then in every world in which *p* is true *q* is true and vice versa. If *p=>r* is taken as true, then in all the most similar worlds to the actual in which *p* is true *r* is also true. But, since *p* and *q* are true in the same worlds, the set making up all most similar worlds in which *p* is true is also the set of all the most similar worlds in which *q* is true, and *r* is true in those worlds. Thus, *q=>r* is true, as is *(p=>r)->(q=>r).*

As with rules (39) and (40), the fact that the logic used here has two levels of necessity requires that

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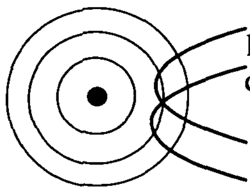
for any of the most similar worlds in which $p\vee q$ is true, r is true.[134] $((p=>r)\&(q=>r))$ $->((p\vee q)->r)$ is then true in the semantics.

Rule (36) considers the case in which $(p=>q)\&(p=>r)$ is true. In all the worlds most similar to the actual world in which p is true, q and r are both true. Consider, then, all the most similar worlds in which $p\&q$ is true. The inference to r in those worlds might seem simple, since p is true whenever $p\&q$ is true and since $(p=>r)$ follows from the assumption in rule (36). The problem with this argument is that the most similar worlds in which $p\&q$ is true may not be as similar to the actual world as the most similar worlds in which p is true, as shown in the following diagram. In order to prevent the situation depicted, the transformation rule requires that $p=>q$ be true. So, in the most similar worlds in which p is true, q is also true. Thus, the most



similar worlds in which $p\&q$ is true are also the most similar worlds in which p is true. Since, r is true in all such worlds, $((p=>q)\&(p=>r))->((p\&q)=>r)$ holds in the semantics.

Rule (37) considers the case in which $p\&q$ is true in the actual world. The actual world is then the world most similar to itself in which p is true. Since q is also true in that world, $p=>q$ is also true in the actual world.

Lastly,[135] in justifying rule (38), $p=>q$ is taken as true in the actual world. Then, in all those worlds most similar to the actual world, but in which p is true, q is true. If p is taken as true in the actual world, then since the actual world is now the most similar world in which p is true, q must be true in the actual world. Hence, $p->q$ is true in the actual world,[136] and $(p=>q)->(p->q)$ holds in the semantics.

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we examine the relationship between $p<<->>q$ and $(p=>r)->(q=>r)$. Once again, as long as p is not naturally impossible, then the set of most similar worlds in which p is true will be worlds in which the laws of nature hold. But, in all worlds adhering to the laws of nature, p is true if and only if q is true. Thus, if we assume that in all the most similar worlds in which p is true r is also true, then since those worlds are the same worlds as the most similar worlds in which q is true, $q=>r$ is true as is $(p=>r)>(q=>r)$.

Again, the change in Pollock's logic has strengthened the system, but the discussion of the change within the context of the semantics indicates that the expansion is reasonable and unlikely to lead to objectional theses.

[134] This conclusion requires the recognition of the fact that the smallest sphere in which $p\vee q$ is true is the smaller of the spheres in which p is true or q is true. The problem presented by $p\&q$ in justifying rule (36) does not arise here.

[135] The justification within the semantics for transformation rules (39) through (41) is presented *supra* notes 130-33.

[136] It must be recognized that the $p->q$ derived in rule (38) is not the material implication used by Pollock,

One last set of additions to the vocabulary, formation rules and transformation rules is required, before powers, disabilities, liabilities and immunities can be expressed within the logic. This last extension incorporates aspects of predicate logic by allowing quantification over certain variables. Symbols for the existential and universal quantifiers must be added to the vocabulary. *S* will represent the phrase *there is* or *there exists* or *for some* or any other formulation of the existential quantifier. *U* will represent the phrase *for each* or *for all* or *every* or any other formulation of the universal quantifier.

In addition, the variables *LR*, *LR'*, *LR''*, etc. must be added to. the vocabulary. Such variables represent any of the eight legal relations defined by Hohfeld: right, duty, privilege, no-right, power, disability, liability or immunity. The formation rules will allow the use of *LR* in a wff only when one party to the legal relation is identified.

The formation rules for incorporating these new symbols into the grammar are as follows.

FORMATION RULES

6. If LR is a legal relation variable and y is a person variable, then LRy is a wff.[137]

7. If a is and act variable and p is a proposition, then (Sa)p and (Ua)p are wffs.

8. If p is a proposition and LRy is a variable over y's legal relations for some person y, then (SLRy)p and (ULRy)p are wffs.[138]

(Sa)p may be read in various ways, including *there exists an a such that p* and *for some a, p*. *(Ua)p* may also be read in various ways, among them *for all a, p* and *for each a,p*. Similar readings are given for existential and universal quantification over legal relations. For example, *(SLPy)p* is read as *for some legal relation to which y is a party, p*.

The increased vocabulary also requires the addition of transformation rules for the manipulation of the symbols.

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but the genuine implication of Allen's logic. The rule is thus somewhat stronger than Pollock's, since an implication with a relevance condition is derived. The derivation is, nonetheless, justified in the semantics. It was not simply the case that when *p* was taken as true, *q* happened to be true. Rather, it was the fact that *p* was true that lead to the truth of *q*. Thus, the dependence requirement for the Deduction Theorem in Allen's logic is satisfied, and *p->q* may be derived.

[137] The wff *LRy* is then a variable over the legal relations of y.

[138] Once again, formation rule (4) must be amended and should now read: "If a string of symbols is not a wff by a combination of (1), (2), (3), (5), (6), (7) and (8) is not a wff."

TRANSFORMATION RULES

42. If p is a proposition containing the act variable a, then from (Ua)p derive p(b), where p(b) is the wff obtained by replacing all occurrences of a in p by b. Similarly, if p is a proposition containing the legal relation variable LRy, then from (ULRy)p derive p(LR'y), where p(LR'y) is the wff obtained by uniformly replacing all occurrences of LRy in p by a particular legal relation to which y is a party.

43. If p is a proposition containing the act variable a, then from (Sa)p derive p(b). The symbol b, chosen to replace a, must be one that does not already occur unbound[139] in the proof. Similarly, if p is a proposition containing the legal relation variable LRy, then from (SLRy)p derive p(LR'y). Again, any prior occurrences of LR'y in the proof may not have been unbound.[140]

44. If p is a proposition containing an unbound act variable, derive (Ua)p. Similarly, if p is a proposition containing an unbound legal relation variable, derive (ULRy)p. The unbound variable must not have entered the proof unbound as part of an assumption or through the operation of rule (43).

45. If p is a proposition containing an unbound act variable, derive (Sa)p. Similarly, if p is a proposition containing an unbound legal relation variable, derive (SLRy)p.

46. From ~(Ua)p or ~(ULRy)p derive (Sa) ~p or (SLRy) ~p, respectively, and vice versa, and from ~(Sa)p or ~(SLRy)p derive (Ua) ~p or (ULRy) ~p, respectively, and vice versa. Furthermore, substitution of ~(Ua) ~ for (Sa) and vice versa, and of ~(Sa) ~ for (Ua) and vice versa is permitted, as are the equivalent substitutions for quantification over LRy.

With regard to rule (45), it should be noted that the derivation of the existential

[139] A variable is unbound when it is outside the scope of a quantifier over that variable. For example, *a* is unbound in *Bax*, while *a* is bound in *(Ua)Fax* and in *(Sa)Da*. Note that *a* is unbound in *(Ub)(Bax & Bbx)*, because *b* is the variable that is quantified.

[140] It should be noted that if a variable occurs unbound in an assumption, rule (43) may not be circumvented by waiting to introduce the assumption into the proof. That is, if an assumption contains the unbound variable *b* or *y* or *LR'y*, *p(b)* may not be derived from *(Sa)p*, nor *p(y)* from *(Sx)p*, nor *p(LR'y)* from *(SLRy)p*, despite the fact that the assumption has not yet been listed as a step in the proof.

The same limitation does not exist with regard to a variable that occurs bound in an assumption or earlier step. For example, if the proof includes *(Ub)p* and *(Sa)q*,*q(b)* might first be derived, so long as *b* does not occur unbound elsewhere. At that point *p(b)* may also be derived, since rule (42) does not require that the replacement variable not have occurred unbound.

may proceed from an identified individual rather than a variable. For example, from *x has accepted the contract* one may derive *there exists an act a such that a has been done by x* or *Bax*. Similarly, from a proposition containing *right-xay* unbound, one may derive *there is some legal relation to which y is a party such that p* or *(SLRy)p*.

The logical context finally exists within which to define power, disability, liability and immunity. To say that x has a power over y is to say that there is some legal relation to which y is a party such that the legal relationship does not pertain, but there is an act that while it is naturally possible to do the act it has not been done by x nor is it obligatory on x,[141] but if x were to do the act then the legal relation involving y would hold. Expressed within the grammar of the logic, *Power-xy* is defined as

(SLRy)[~LRy&(Sa)[~Bax&M'Bax& ~OBax&(Bax=>LRy)]].

The statement of a power has been presented as the power to create rather than terminate a legal relation. However, the termination of any legal relation is identical to the creation of its opposite.[142] Thus, the characterization of the power to create is an adequate characterization of the concept of power.[143]

The definition of liability is, naturally, quite similar to that of power. Hohfeld stated that power and liability are correlatives.[144] As such, they must be logically identical or at least logically equivalent. In fact the only difference is an interchange of symbols. The difference between powers and liabilities, as between rights and duties and between disabilities and immunities, is the point of view of the parties to the relationship.[145] That change in point of view is reflected in the interchange of person variables. *Liability-xy* is defined as

(SLRx)[~LRx&(Sa)[~Bay&M'Bay& ~OBay&(Bay=>LRx)]].

To say that x is under a disability with regard to y is to say that for any of y's legal relations, it can not be true that the legal relation does not pertain but that there is an act that x has not done, can do and is not obligated to do that will create that legal relation. *Disability-xy* is defined as

(ULRy) ~[~LRy&(Sa)[~Bax&M'Bax& ~OBax&(Bax=>LRy)]].

Once again, the definition of immunity will be quite similar to that of disability

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[141] The requirement that the act not be obligatory keeps the act within the volitional control of x. *See supra* notes 26-28 and accompanying text.

[142] *See supra* notes 20-31 and accompanying text.

[143] Powers do not require that the legal relation by between y and a third party. The relation created may be between x and y.

[144] *See supra* note 29 and accompanying text.

[145] *See supra* note 29 and accompanying text.

with the only difference being an interchange of symbols to reflect the change in point of view between the two relations. *Immunity-xy* is defined as

(ULRx) ~[~LRx&(Sa)[~Bay&M'Bay& ~OBay&(Bay=>LRx)]}.

It remains to be shown that the correlation and opposition that Hohfeld posited between the relations can be proved in the logic. To show that *Power-xy* and *Liability-yx* are correlatives *Power-xy<->Liability-yx* must be proved. The key to the proof is the recognition that the definition given for liabilities was that of *Liability-xy*. The symbolization of *Liability-wz* would be obtained by replacing all occurrences of x in the symbolization of *Liability-xy* with *w* and all occurrences of *y* with *z*. Thus, the symbolization of *Liability-yx* requires that the occurrences of *x* and *y* in *Liability-xy* be interchanged. *Liability-yx* is thus

(SLRy)[~LRy&(Sa)[~Bax&M'Bax& ~OBax&(Bax=>LRy)]].

The proof may then proceed.

PROOF:|- Power-xy<->Liability-yx

	STEP	JUSTIFICATION	ASSUMPTIONS
1.	Power-xy	Ass. for DT	1
2.	(SLRy)[~LRy&(Sa) [~Bax&M'Bax& ~OBax&(Bax=>LRy)]]	Def. Power, 1	1
3.	Liability-yx	Def. Liability, 2	1
4.	Power-xy->Liability-yx	DT, 1 -3	
5.	Liability -yx	Ass. for DT	2
6.	(SLRy)[~LRy&(Sa) [~Bax&M'Bax& ~OBax&(Bax=>LRy)]]	Def. Liability, 5	2
7.	Power-xy	Def. Power, 6	2
8.	Liability-yx->Power-xy	DT, 5-7	
9.	(Power-xy->Liability-yx)& (Liability-yx->Power-xy)	TR 1, 4, 8	

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10.	Power-xy<->Liability-yx	Def. < ->, 9	

Similarly, to show that *Disability-xy* and *Immunity-yx* are correlatives, *Disability-xy<->Immunity-yx* must be proved. Once again the *x* and *y* interchange in going from *Immunity-xy* to *Immunity-yx* is the key.

PROOF:|- Disability-xy<->Immunity-yx

	STEP	JUSTIFICATION	ASSUMPTIONS
1.	Disability-xy	Ass. for DT	1
2.	(ULRy) ~[~LRy&(Sa) [~Bax&M'Bax& ~OBax&(Bax=>LRy)]]	Def. Disability	1
3.	Immunity-yx	Def. Immunity	1
4.	Disability-xy->Immunity-yx	DT, 1-3	
5.	Immunity-yx	Ass. for DT	2
6.	(ULRy) ~[~LRy&(Sa) [~Bax&M'Bax& ~OBax&(Bax=>LRy)]]	Def. Immunity	2
7.	Disability-xy	Def. Disability	2
8.	Immunity-yx->Disability-xy	DT, 5-7	
9.	(Disability-xy-> Immunity-yx)&(Immunity-yx ->Disability-xy)	TR 1, 4, 8	
10.	Disability-xy<->Immunity-yx	Def. <->, 9	

The proofs of the oppositions expected to hold among the concepts is made more difficult by the fact that Professor Allen's system does not allow the inference from *p* to *~(~p)*. Without the availablity of that inference, proof of the opposition of *Power-xy* and *Disability-xy* requires proof of all of *Power-xy-> ~Disability-xy*, *Disability-xy-> ~Power-xy*, *~Power-xy->Disability-xy* and *~Disability-xy->Power-xy*.

PROOF:|- Power-xy -> ~Disability-xy

	STEP	JUSTIFICATION	ASSUMPTIONS
1.	Power-xy	Ass. for DT	1
2.	(SLRy)[~LRy&(Sa) [~Bax&M'Bax& ~OBax&(Bax=>LRy)]]	Def. Power, 1	1
3.	~(ULRy) ~[~LRy&(Sa)[~Ba M'Bax& ~OBax&(Bax=>LRy)]]	TR 46, 2	1
4.	~Disability-xy	Def. Disability, 3[146]	1
5.	Power-xy-> ~Disability-xy	DT, 1-4	

PROOF:|- Disability-xy -> ~power-xy

	STEP	JUSTIFICATION	ASSUMPTIONS
1.	Disability-xy	Ass. for DT	1
2.	(ULRy) ~[~LRy&(Sa) [~Bax&M'Bax& ~OBax&(Bax=>LRy)]]	Def. Disability, 1	1
3.	~(SLRy)[~LRy&(Sa) [~Bax&M'Bax& ~OBax&(Bax=>LRy)]]	TR 46, 2	1
4.	~Power-xy	Def. Power, 3	1
5.	Disability-xy-> ~Power-xy	DT, 1-4	

[146] This step actually requires that the defined term imply the definition. That is easily shown by taking the defined term as an assumption, replacing it with its definition, and applying the Deduction Theorem. An application of Modus Tollens then allows the inference of the negative of the defined term from the negative of the defining formula.

PROOF:|- ~Power-xy->Disability-xy

	STEP	JUSTIFICATION	ASSUMPTIONS
1.	~Power-xy	Ass. for DT	1
2.	~(SLRy)[~LRy&(Sa) [~Bax&M'Bax& ~OBax&(Bax=>LRy)]]	Def. Power, 1[147]	1
3.	(ULRy) ~[~LRy&(Sa) [~Bax&M'Bax& ~OBax&(Bax=>LRy)]]	TR 46, 2	1
4.	Disability-xy	Def. Disability, 3	1
8.	~Power-xy->Disability-xy	DT, 1-4	

PROOF:|- ~Disability->Power-xy

	STEP	JUSTIFICATION	ASSUMPTIONS
1.	~Disability-xy	Ass. for DT	1
2.	~(ULRy) ~[~LRy&(Sa) [~Bax&M'Bax& ~OBax&(Bax=>LRy)]]	Def. Disability, 1	1
3.	(SLRy)[~LRy&(Sa) [~Bax&M'Bax& ~OBax&(Bax=>LRy)]]	TR 46, 2	1
4.	Power-xy	Def. Power, 3	1
6.	~Disability-xy->Power-xy	DT, 1-4	

The proofs of the remaining required oppositions proceed rather easily from those correlative relations already proved and from the oppositions also already proved. The following remain to be proved: *Immunity-xy-> ~Liability-xy*, *Liability-xy-> ~Immunity-xy*, *~Immunity-xy->Liability-xy* and *~Liability-xy-> Immu-*

[147] Added justification is needed here, as it was *supra* note 146. First, the implication from the defining formula to the defined term is required, but that is a simple application of the Deduction Theorem. Modus Tollens is then applied.

nity-xy. Those proofs follow. They could actually be accomplished with fewer steps by using the methods of the preceeding four proofs. However, the somewhat longer proofs presented provide examples of the manipulation of the terms of the power set.

PROOF:|- Immunity-xy-> ~Liability-xy

	STEP	JUSTIFICATION	ASSUMPTIONS
1.	Immunity-xy	Ass. for DT	1
2.	Immunity-xy<->Disability-yx	Proved above	
3.	Immunity-xy->Disability-yx	Def <->, TRl, 2	
4.	Disability-yx	MP, 3, 1	1
5.	Disability-yx-> ~Power-yx	Proved above	
6.	~Power-yx	MP, 5, 4	1
7.	Power-yx<->Liability-xy	Proved above	
8.	Liability-xy->Power-yx	Def <->, TRl, 7	
9.	~Liability-xy	MT, 8, 6	1
10.	Immunity-xy-> ~Liability-xy	DT, 1-9	

PROOF:|- Liability-xy-> ~Immunity-xy

	STEP	JUSTIFICATION	ASSUMPTIONS
1.	Liability-xy	Ass. for DT	1
2.	Liability-xy<->Power-yx	Proved above	
3.	Liability-xy->Power-yx	Def <->, TR 1, 2	
4.	Power-yx	MP, 3, 1	1
5.	Power-yx-> ~Disability-yx	Proved above	
6.	~Disability-yx	MP, 5, 4	1

7.	Disability-yx<->Immunity-xy	Proved above	
8.	Immunity-xy->Disability-yx	Def <->, TR 1, 7	
9.	~Immunity-xy	MT, 8, 6	1
10.	Liability-xy-> ~Immunity-xy	DT, 1-9	

PROOF:|- ~Immunity-xy->Liability-xy

	STEP	JUSTIFICATION	ASSUMPTIONS
1.	~Immunity-xy	Ass. for DT	1
2.	Immunity-xy<->Disability-yx	Proved above	
3.	Disability-yx->Immunity-xy	Def <->, TR 1, 2	
4.	~Disability-yx	MT, 3, 1	1
5.	~Disability-yx->Power-yx	Proved above	
6.	Power-yx	MP, 5, 4	1
7.	Power-yx<->Liability-xy	Proved above	
8.	Power-yx->Liability-xy	Def <->, TR 1, 7	
9.	Liability-xy	MP, 8, 6	1
10.	~Immunity-xy->Liability-xy	DT, 1 ~9	

PROOF:|- ~Liability-xy->Immunity-xy

	STEP	JUSTIFICATION	ASSUMPTIONS
1.	~Liability-xy	Ass. for DT	1
2.	Liability-xy<->Power-yx	Proved above	
3.	Power-yx->Liability-xy	Def <->, TR 1, 2	

4.	~Power-yx	MT, 3, 1	1
5.	~Power-yx->Disability-yx	Proved above	
6.	Disability-yx	MP, 5, 4	1
7.	Disability-yx<->Immunity-xy	Proved above	
8.	Disability-yx->Immunity-xy	Def <->, TR 1, 7	
9.	Immunity-xy	Mp, 8, 6	1
10.	~Liability-xy->Immunity-xy	DT, 1-9	

Thus, *,Power-xy<->~Disability-xy, Disability-xy<->~Power-xy, Immunity-xy<-> ~Liability-xy* and *Liability-xy<-> ~Immunity-xy* all hold within the system. The relationships Hohfeld posited for his legal relations are proveable in the logic.

Before concluding, the direction for one further expansion should be indicated. Hohfeld's relations all exist, or fail to exist, between exactly two persons. For that reason, the analysis presented has not included quantification over persons. However, multital rights between an individual and a group have been recognized.[148] If multital rights are to be considered, quantification over person variables within particular legal relations must be allowed. That is, the formation rules must allow expressions such as *(Uy)Right-xy* or, if the right holds only against individuals belonging to a particular set Y, *(Uy)((y belongs to Y)->Right-xv).*[149]

CONCLUSION

This article has expanded on the logic developed by Professor Allen so as to

[148] *See supra* notes 32-34 and accompanying text.

[149] The formation rules would have to be amended to include: If x is a person variable, and p is a proposition, then (Sx)p and (Ux)p are wffs.

Transformation rule (42) would have to be amended to include: If p is a proposition containing the person variable x, then from (Ux)p derive p(y). Transformation rule (43) would include: If p is a proposition containing the person variable x, then from (Sx)p drive p(y). And again, y must not have yet occurred unbound in the proof.

Transformation rule (44) would include: If p is a proposition containing an unbound person variable, derive (Ux)p. Again, the unbound variable must not have entered the proof as part of an assumpion of through the operation of rule (43). Transformation rule (45) would be amended to include: If p is a proposition containing an unbound person variable, derive (Sx)p.

Lastly, the manipulations of rule (46) could also be performed on quantifications over person variables. From ~(Ux)p derive (Sx)~p, and vice versa, and from ~(Sx)p derive (Ux)~p, and vice versa. Substitution of ~(Ua)~ for (Sa) and vice versa, and of ~(Sa)~ for (Ua) and vice versa would also be permitted for quantification over person variables.

provide a formal system in which all the Hohfeldian terms are formalized and in which all the relations Hohfeld requires among his concepts may be proved to hold. The value behind this effort may be both theoretical and practical. On the theoretical side, the effort to present a formalization required an analysis of the concepts involved. It is hoped that that examination has added to the informal understanding of the terms. On the practical side, if Hohfeldian relations are ever to be included in computer analyses and are ever to be a part of artificial intelligence programs, the symbolization of the concepts and the development of a logic for the manipulation of those symbolizations are required.