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A Review of Secondary Mathematics Textbooks on Families of Functions

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Abstract

In 2008, twelve commonly used textbooks were studied for their use of technology and content across 5 families of functions: polynomial, rational, exponential, logarithmic, and trigonometric. Since 2008, the Common Core State Standards have been released and many textbook companies have used this as an opportunity to change their textbook content. This study reevaluates new high school algebra and precalculus textbooks across many of the same criterion studied in 2008. Some topics such as increasing and decreasing functions significantly increased their presence in the mathematics textbooks while topics such as sketching graphs has decreased. The reason for this shift in the content is unknown, but could be attributed to a change in emphasis of topics in the new Common Core State Standards.

Purpose

In the recent past, mathematics textbooks covered a wide variety of topics mandated by standards from different states and schools, but many schools are currently transitioning to textbooks that concentrate on the Common Core State Standards. In 2008, twelve commonly used high school and college textbooks published between 2000 and 2008 were evaluated for their use of technology in combination with content and were separately evaluated for each family of functions (Quesada & Renker, 2008). They operated under the hypothesis that when the same concepts are studied across multiple families of functions, the readers are able to piece together their knowledge on the subject and create an overall cohesive picture of the topics. Now that many schools have transitioned to the Common Core State Standards and have had the opportunity to adopt new textbooks, the study has been repeated with a new set of algebra and pre-calculus textbooks, focusing on concepts that are universal to different families of functions to see if the textbooks have shifted their content over the past 10 years.

Background

In a 1994 issue of Education Digest, Jan Mokros comments on the quality of mathematics textbooks in her article, “Math Textbooks: Where’s the Math?” She states, “Textbook publishers have persuaded us that their writers have an almost mystical knowledge about what should be covered” (Mokros, 1994, p. 63). Mokros continues on to comment that the information that is covered in mathematics textbooks is covered poorly. The mathematics that appears in textbooks is too clean, and rarely a reflection of real mathematics. She concludes her article by questioning if newer textbooks will focus more on real mathematics and less on figurative, formulaic mathematics. Robert E. Reys (2001) calls for a reform in these same textbooks in his article "The Math Wars." Many companies have come up with excuses regarding the poor quality of their books with the leading excuse being there is no national curriculum on which the books would be based. He comments that in the cycle of publishing, textbooks rarely have the time to be carefully researched and field-tested before they are implemented leading to poor textbooks overall. Despite the difficulties in the process of publishing a new textbook, "the need for significant improvement in student learning requires us to overcome these difficulties" (Reys, 2001, p. 258).

This call for mathematics reform coincided with the creation of the National Common Core State Standards. With this change, there was a shift in emphasis in topics. Some topics that were heavily covered in the 2004 edition of Ohio's Academic Content Standards, like domain and range and local and global behavior, can be found scattered throughout the new Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2008). Conversely topics such as sketching and solving problems algebraically are more explicitly and thoroughly laid out in the Common Core State Standards released in 2008 (2008). With the release of the Common Core State Standards, a publishers' criteria was created by a collection of educational groups; The National Governors Association, The Council of Chief State School Officers, Achieve, The Council of Great City Schools, and The National Association of State Boards of Education. This guide that was created to influence the high school curriculum materials was commented on by Erik W. Robelen in his article, "Common-Core Writers Offer 'Publishers' Criteria' for Math" published in 2013. It encourages textbook companies to focus on the three dimensions of the mathematics standards: focus, coherence, and rigor. On the topic of focus, it urges textbook companies to address fewer topics in greater depth with a "clear eye" on preparing students for postsecondary education (Robelen, 2013, p. 9). This article encourages textbooks to spend more time on the overarching material that is relevant to numerous concepts and families of functions (i.e. domain, range, etc.) and less time on highly specific material. He comments that textbooks that adopt this model will push to the forefront of the mathematics textbook market.

Textbook Structure

It is widely accepted that in order for a mathematics textbook to be considered good in terms of a specific topic, it must include practice regarding the topic. In their article "Practice Makes (Nearly) Perfect," Christianson, Mestre, and Luke (2012) conclude the strongest predictor of students producing correct equations is practice. In the experiment students' scores improved in accuracy as they progressed through practice even though they received no feedback on their work (Christianson et al., 2012). The idea of repetition is present in the average mathematics textbook where the authors present a topic and proceed to have numerous examples on the subject. For example, a precalculus textbook authored by Larson, Hostler and Edwards begins with an introduction to polynomials and polynomial functions by giving a definition followed by two examples and their solutions (Larson et al., 2001). Just beyond this section is the typical homework section full of 65+ problems related to the section. Homework has been one of the most prominent features and an integral part of mathematics textbooks in many educational systems (Cheema & Sheridan, 2015). Homework allows for students to get the practice needed on a specific subject outside of the classroom. It can also facilitate the connections to different families of functions through the review of previously mentioned topics.

Previous Study

The need for changes in mathematics textbooks continues to be discussed in Quesada and Renker's (2008) paper, "The Impact of Technology in Topic Foundational to Calculus at the Precalulus Level." They discuss a series of concepts that should be covered when teaching each of the following family of functions; polynomial functions, rational functions, exponential functions, logarithmic functions, and trigonometric functions. These topics include, but are not limited to, range, domain, concavity, increasing/decreasing, parity, and graph sketching. They found these topics have become increasingly relevant with the introduction and implementation of technology in the classroom. When all of these concepts are studied regarding each family of functions, it allows students to piece together their knowledge of each family of functions and shows how the families of functions are intertwined with each other. Students are then able to take what they have learned and think about the topics at a more abstract level, applying their knowledge to other topics. After reviewing textbooks being used at the time, they concluded that there is a need to reevaluate the essential components of precalculus and make sure textbooks move to reflect this change (Quesada & Renker, 2008).

Methodology

The data was collected by reviewing five textbooks newly adopted in the greater Akron, Ohio area and rating each topic on a scale of 0-3. This was chosen based on the three major parts of the every day mathematics textbook discussed in the original study; discussing a topic, showing examples, and homework problems for practice. If a topic was not discussed at any point in the textbook, it scored a zero. A score of one in a topic area meant only one of the following things occurred: the topic was discussed, there was an example connected to this topic, or there was a homework problem on this topic. For example, one book fails to discuss how to find rational zeros but assigns 8 problems similar to the one below:

> Find the zeros (if any) of the rational function and verify your answer using a graphing utility.

$$g(x) = \frac{x^2 - 9}{x^2 + 5x + 6}$$

If a topic earned a two, the topic was discussed and an example was included, but no homework problems followed up on the topic. This set of circumstances was not common in the analysis of the textbooks but did arise in the graphing of logarithmic functions. One book briefly talks about end behavior by stating, the value of *x* approaches zero as the value of *y* gets smaller, on a diagram of a graph showing that logarithmic and exponential functions are inverses of each other. This is complemented with a small side note about end behavior in a following example problem similar to the one below:

> Graph the function:

$$g(x) = 4\log_{10} x + 7$$

Off to the side of the problem, there is a study tip in a small bubble labeled as end behavior. It tells the students, in this example as *x* approaches infinity, *g(x)* also approaches infinity.

Because of it's lack of homework problems on the subject, Book 2 received a 2 on the topic of end behavior in regards to logarithmic functions. A topic that earned a three was discussed with an example and included homework problems to follow up the concept. A strong example of a topic that earned a three would be Book 3's lesson on polynomial regression. It begin the lesson with an introduction of the subject through the use of calculators, similar to the following:

> Polynomial functions can be used to model many real-world situations. The behavior of different degrees of polynomial functions can suggest which polynomial would be the best fit for a particular data set.
>
> You can use a graphing utility to help you find functions that fit the data set.
>
> Graph the scatter plot of the data and the regression function in the same window.

This is followed by a series of example problems, and a collection of homework problems, both of which cover both numerical and real-world problems. Similar to the ones described below:

> Problem 5: Interpolation and Extrapolation
>
> The table shows average annual consumption of soda per person in the U.S for chosen years from 1930 to 2015

Year	Gallons of Soda Consumed
1930	5

1950	6
1975	15
1998	30
2015	52

Homework Problem 35: The table below shows the percentage of stay at home dads in the U.S. for selected years between 1980 and 2015

Year	1980	1985	1990	1995	2000	2005	2010	2015
%	0.5	0.8	1.0	1.1	1.3	2	3.5	5

a. What is the average rate of change between 1980 and 1995? What about 2000 and 2015?
b. Make a scatter plot of the data. What kind of polynomial appears to fit the data the best?
c. Use a graphing calculator to find the polynomial mentioned in part (b).
d. Use the model you found in part (c) to predict what percent of dada will be stay at home dads in the year 2025.
e. How much confidence do you have in your prediction? Why?

Unlike the previously described scale used to evaluate the examples above, the original 2008 study had analyzed their books on a scale from 0-2. This scale was altered to allow for more flexibility in rating the textbooks. Because of the differing scales the data was linearly translated to a 0-2 scale to aid in the comparison of specific content. Overall Scores were then converted into average percentages by dividing the points earned by the total points possible. For example, the overall average for sketching (equations to graphs) is 2.28 out of 3. This score was then multiplied by two-thirds to linearly translate it to a scale of 2, in order to be compared to the original study, with the final score being 1.52. This process was repeated for individual families of functions and for whole books so that they can be compared against each other and against the previously studied books on a level scale. This scale was used to evaluate the following topics from the previous study:

- Attributes: Range, domain, intercepts/zeros, asymptotes, local behavior, end behavior, concavity, min/max (increasing/decreasing)
- Solving equations: intercepts, difference function, solving algebraically, solving using the intermediate value theorem with data
- Regression
- Connections between representations: Equations to graphs, graphs to equations

These topics were evaluated for each family of functions and across each textbook. The examples of each conceptual category that follow are similar to the ones found in homework section of the textbooks analyzed.

<u>Example 1:</u> Finding a Function's Domain and Asymptotes

a) Find the domain of the function,

b) Decide whether the function is continuous,

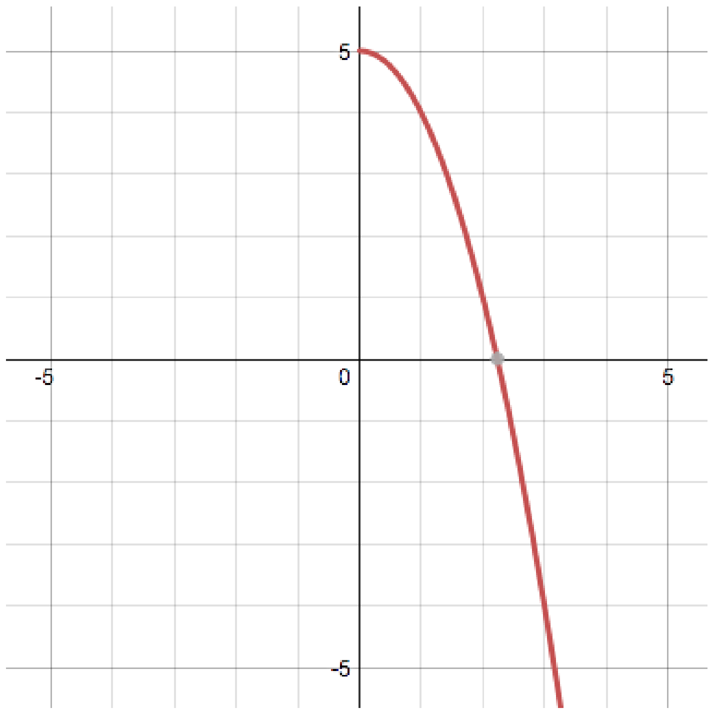
c) Identify any horizontal and vertical asymptotes,

d) Verify your answer to part a both graphically and numerically

$$f(x) = \frac{2x^2 + x - 4}{x^2 + 1}$$

This example covers the topics of domain and asymptotes. Additionally, it has the student check their work by making connections between an equation and a graph, bringing to their attention the potential intercepts/zeros of the problem. Hence, a question such as this would be counted as addressing the following categories: Rational Functions, Attributes (domain, intercepts/zeros, asymptotes) and Connections between Representations (Equations to graphs, graphs to equations).

Example 2: Half of the graph of an even function is shown.



a) Sketch a complete graph of the function

b) Find the domain and the range of the function

c) Identify the open intervals on which the function is increasing, decreasing, or constant

d) Find any relative minimum and relative maximum values of the function.

This example tests the students' knowledge of local and end behavior, domain and range, and increasing and decreasing functions. The reader is also asked to make connections between graphs and equations from the initial description of the problem as an even function. Hence, a question such as this would be counted as addressing the following categories: Polynomial Functions, Attributes (range, domain, intercepts/zeros, asymptotes, local behavior, end behavior, concavity, min/max [increasing/decreasing]).

Example 3: Approximating the zeros of a function.

a) Use the Intermediate Value Theorem and a graphing calculator to graphically find any intervals of length 1 in which the polynomial function is guaranteed to have a zero.

b) Use the zero feature to find the real zeros of the function.

c) Verify your answers by using the table function of your graphing calculator.

$$f(x) = x^3(x-4)^2$$

Example 3 asks the reader to implement the concepts of Intermediate Value Theorem and zeros help to solve the equation given. Hence, a question such as this would be counted as addressing the following categories: Polynomial Functions, Attributes (zeros) and solving equations (Intermediate Value Theorem).

Example 4: Solve the logarithmic equation algebraically. Round your result to three decimal places.

$$\log_5(3x+2) = \log_5(6-x)$$

This example focuses on solving a logarithmic problem algebraically. Hence, a question such as this would be counted as addressing the following categories: Logarithmic functions, solving equations (solving algebraically).

Example 5: The number of subscribers S (in millions) to a cable network from 2008 to 2014 is shown in the table.

Year	Subscribers S (in millions)
2008	139.8
2009	152.7
2010	180.4
2011	209.3
2012	243.6
2013	259.7
2014	268.5

a) Use the graphing calculator to create a scatter plot of the data

b) Use the regression feature of the graphing calculator to find a linear model for the data.

Example 5 concentrates mainly on the concepts of regression and linear functions. Hence, a question such as the one above would be counted as addressing the following categories: Polynomial Functions, Regression.

Results

Focusing on this study alone, the criterion that scored the highest across all textbooks and all families of functions (polynomials, rational functions, exponential functions, logarithmic functions, and trigonometric functions) in the attributes category was domain and intercepts both with a score of 2 out of 3. Books 2, 4 and 5 covered domain over most families of functions while books 1 and 3 barely touched on it outside of general polynomial functions. Intercepts were more common across all families of functions but still only received an average of 2. These were closely followed by range with a score of 1.6.

Domain and range are often times thought of as a pair. This idea is brought down by their differing scores, which stem from the rational function family. When discussing these functions, many textbooks are concerned with the domain of the function and choose not to discuss the range in conjunction. This same type of pairing is found in local and end behavior, both of which received a 1 out of 3. End behavior was discussed more often than local behavior, but was discussed with less caliber, allowing the averages to be the same. Minimums/Maximums and asymptotes both scored below 1 with 0.8 and 0.6 respectively which surpassed concavity's score of 0. Minimums and maximums were only touched on a few times across all families of functions while asymptotes were only talked about in one family, rational functions. Concavity, which plays a major role in understanding the behavior of a function was not discussed by any of the books or mentioned in any homework problems.

Table 1

Scores of textbooks in conceptual categories over five families of functions

	Polynomials					Rational					Exponential					Logarithmic					Trigonometric				
Book Number	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
Attributes																									
Range	3	1	0	0	3	0	3	0	3	0	0	3	1	3	3	0	3	0	3	3	0	3	0	3	3
Domain	3	1	0	0	3	3	3	3	3	3	0	3	1	3	3	0	3	0	3	1	0	3	0	3	3
Intercepts /Zeros	3	3	3	3	3	1	3	3	3	3	0	3	0	3	0	0	3	0	3	1	0	3	0	3	3
Asymptote	0	0	0	0	0	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Local Behavior	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	3	3
End Behavior	3	3	3	3	3	3	0	0	0	0	0	1	0	0	0	0	2	0	0	0	0	0	0	3	3
Concavity	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Max/Min Inc/Dec	3	0	0	3	3	0	0	0	0	0	0	1	3	3	3	0	0	0	0	1	0	0	0	0	0
Solving Equ.																									
Intercepts	3	3	3	3	3	3	3	3	3	3	0	3	0	3	0	0	3	0	3	2	0	3	0	3	3
Diff. Function	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Solving Alg.	3	3	3	3	3	0	0	3	3	3	0	3	3	0	3	0	3	3	0	3	0	0	0	0	0
IVT	2	0	0	0	3	0	0	0	0	3	0	0	0	0	3	0	0	0	0	3	0	0	0	0	0
Regress.																									
Regress.	3	0	3	3	3	0	0	0	3	0	0	3	3	3	3	0	3	0	3	0	0	3	0	0	0
Comp. between Repres.																									
Equ. to Graphs	3	3	3	3	3	0	3	3	3	0	3	3	3	3	3	0	3	0	3	3	0	3	0	3	3
Graphs to Equ.	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0
Average	2.1	1.3	1.4	1.6	2.2	0.9	1.3	1.2	1.6	1.2	0.2	1.5	0.9	1.4	1.4	0	1.5	0.2	1.2	1.3	0	1.6	0	1.4	1.4
Average Percent	0.7	0.4	0.5	0.5	0.7	0.3	0.4	0.4	0.5	0.4	0.1	0.5	0.3	0.5	0.5	0	0.5	0.1	0.4	0.4	0	0.5	0	0.5	0.5

In terms of solving equations the highest scoring criteria is the concept of making graphs from equations with a score of 2.3 out of 3. This was closely followed by the concept of using intercepts to aid in solving equations with a score of 2.1. The common scores are not surprising because in most families of functions intercepts are used in the graphing process. Within the solving equations category using the Intermediate Value Theorem and a collection of data scored a 0.6 out of 3 and using the difference of two functions to solve equations scored no points. The Intermediate Value Theorem was extensively studied across four families of functions in one textbook but not mentioned in any other textbook explaining its barely above zero score. Solving by the difference of two functions was not brought up in any of the five books studied. Solving functions this way involves subtracting two functions and then solving for the zeros of the new function. This makes solving a system of equations very easy, but is currently not taught in most textbooks. The topic of regression is covered in most books in regards to polynomial functions and exponential functions but rarely covers this topic in any other family of functions.

Comparing the newly found data to the previous study, there has been no improvement in the topics of domain and range, local and global behavior, and increasing and decreasing functions. The corresponding new scores were linearly translated to a range of [0,2] for the comparison. The scores have decreased over the past eight years. Previously, domain and range scored a 1.45 out of 2 and currently, it scores a 1.2. This same trend was noticed in local and global behavior, 1.13 to .67, and increasing and decreasing functions, 1.26 to .53. This reduction of content could stem from shift in focus to other topics such as solving algebraically and sketching. Both of these concepts saw an increase in content in the book with jumps from .25 to 1.12 and 1.16 to 1.52 respectively.

Table 2

Comparison of Mathematics Textbooks in 2008 and 2016

Topic	2008 Score (out of 2)	2016 Score (out of 2)
Domain and Range	1.45	1.2
Local and Global Behavior	1.13	0.67
Increasing and Decreasing Functions	1.26	0.53
Solving Algebraically	0.25	1.12
Sketching Graphs	1.16	1.52

In the Quesada-Renker paper, they used the criteria of 50% to establish success in a particular topic (2008). If a topic scored less than 50% they concluded it was a topic in need of work, while those over 50% were in good standing. In general, they came to the conclusion that weak areas included numerical solutions, using graphical representations, relative growth, inequalities, sketching polynomials (turning points and effects of leading coefficient), and optimization. Conversely, they noticed local and global behavior, local extrema and increasing and decreasing functions were strongly emphasized along with transformations and domain and range.

Applying this criterion to the five textbooks recently studied, there were more topics that needed improvement than those that did not. In general, weak areas included local behavior (32%), end behavior (35%), concavity (0%), maximums and minimums (27%), difference function (0%), solving with the intermediate value theorem (19%), regression (48%), translating graphs to equations (5%), and asymptotes (2%). Conversely, strong topics included domain (55%), range (67%), intercepts (68%), solving algebraically (56%), and graphing from equations (76%). These findings parallel some of the findings of the 2008 study. Domain and range have been, and continue to hold a strong part in the mathematics textbook, but conversely local and global behavior appear to have regressed in their textbook presence.

Conclusion

Over the past 8 years there has been an immense change in the curriculum with the introduction and implementation of the Common Core State Standards. With this change, we have also seen a change in mathematics textbooks published and used in the classroom. Textbooks can be easily evaluated over all families using their common structure: concept discussion, examples, and homework. There are a wide variety of scores, but a few specific areas stood out. Local and global behavior and increasing and decreasing functions lost their place in some of the textbooks, solving algebraically and sketching, found a home over the past 10 years. This change could be a reflection of a shift in emphasis of certain concepts in the new Common Core State Standards. Despite the growth in some areas, the mathematics textbooks still need much improvement to produce students that are ready for the rigor of the postsecondary classroom. To be prepared, students need to study concepts across all families of function, learning to make connections in the process. These definitions, skills and connections will help them in their future math careers.

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