Spring 2016

Wavelet Transform of Optical Conductivity of Bi2Sr2CaCu2O8+x in the Normal and Superconducting State

Patrick Janovick
pjj7@zips.uakron.edu

Wavelet Transform of Optical Conductivity of $Bi_2Sr_2CaCu_2O_{8+x}$ in the Normal and Superconducting State

Patrick Janovick

Department of Physics

Honors Research Project

Submitted to

The Honors College

Approved: Date 05/04/16
Honors Project Sponsor (signed)

SASA DORDEVIC
Honors Project Sponsor (printed)

Date 5/4/16
Reader (signed)

Jutta Luettmer-Strathmann
Reader (printed)

Date 5/5/16
Reader (signed)

ROBERT MALLIK
Reader (printed)

Accepted: Date 5/4/16
Department Head (signed)

David Steer
Department Head (printed)

Date 5/5/16
Honors Faculty Advisor (signed)

ROBERT MALLIK
Honors Faculty Advisor (printed)

Date
Dean, Honors College

Abstract

This paper compares the wavelet transform of the optical conductivity of $Bi_2Sr_2CaCu_2O_{8+x}$ in both the normal and the superconducting state. The wavelet transforms of the normal and superconducting state are also compared with wavelet transforms of the Drude model as well as the Lorentz model. The wavelet transform of the collected data for $Bi_2Sr_2CaCu_2O_{8+x}$ is similar in shape to the Drude model. The wavelet transform of the collected data in the normal state corresponds to the Drude model with a high scattering rate and the collected data in the superconducting state corresponds to the Drude model with a low scattering rate.

Introduction

Wavelet transforms use small waves (wavelets) to process signals and compress images [1]. The use of small waves is a big difference between wavelet transforms and Fourier transforms. This allows the wavelet transform to be done more locally than the Fourier transform. A wavelet transform allows for the unfolding of a signal into space (or in this case frequency) and scale. The scale is obtained from the dilation (or constriction) of the selected wavelet [1]. This paper uses a wavelet transform on the optical conductivity of the superconductor $Bi_2Sr_2CaCu_2O_{8+x}$ in both the normal state at a temperature of 300K and in the superconducting state at 10K. The critical temperature for $Bi_2Sr_2CaCu_2O_{8+x}$ is T_c = 92 K so the experimental data in the normal state was collected significantly above the critical temperature and the data in the superconducting state was collected significantly below the critical temperature. The wavelet transforms of the experimental data in the normal state and in the superconducting state are compared to the wavelet transforms of model data.

Transform

The optical conductivity is written as $\sigma(\omega) = \sigma_1(\omega) + i\sigma_2(\omega)$, where $\sigma_1(\omega)$ is the absorptive part and $\sigma_2(\omega)$ is the dispersive part of the complex optical conductivity. The continuous wavelet transform is done using MATLAB. In MATLAB the function *cwtext (signal, scales, wname, 'extmode','sp0', 'extLen', 2000, 'plotMode', 'absglb')* is used to complete the wavelet transform. The *signal* is the input signal, the *scales* are the scales used for the wavelet transform (y axis), *wname* is the name of the wavelet transform, *extmode* is the extension parameter, *sp0* is a smooth extension of order 0, *extLen* is the length extension for the x-axis which is set to 2000, *plotMode* is what sets the plot mode to *absglb* which plots the absolute values of the coefficients of the continuous wavelet transform for all the scales [2].

The general form of a continuous wavelet transform is given by the formula

$$[W_\psi f](a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} \psi\left(\frac{x-b}{a}\right) f(x)dx\ ,$$

Where $\psi\left(\frac{x-b}{a}\right)$ is the wavelet used for the transform and f(x) is the function that is being transformed. In a Fourier transform the wavelet has the form $e^{-2\pi i x\xi}$. So a Fourier transform has the form

$$f(\xi) = \int_{-\infty}^{\infty} e^{-2\pi i x\xi}\ f(x)dx\ .$$

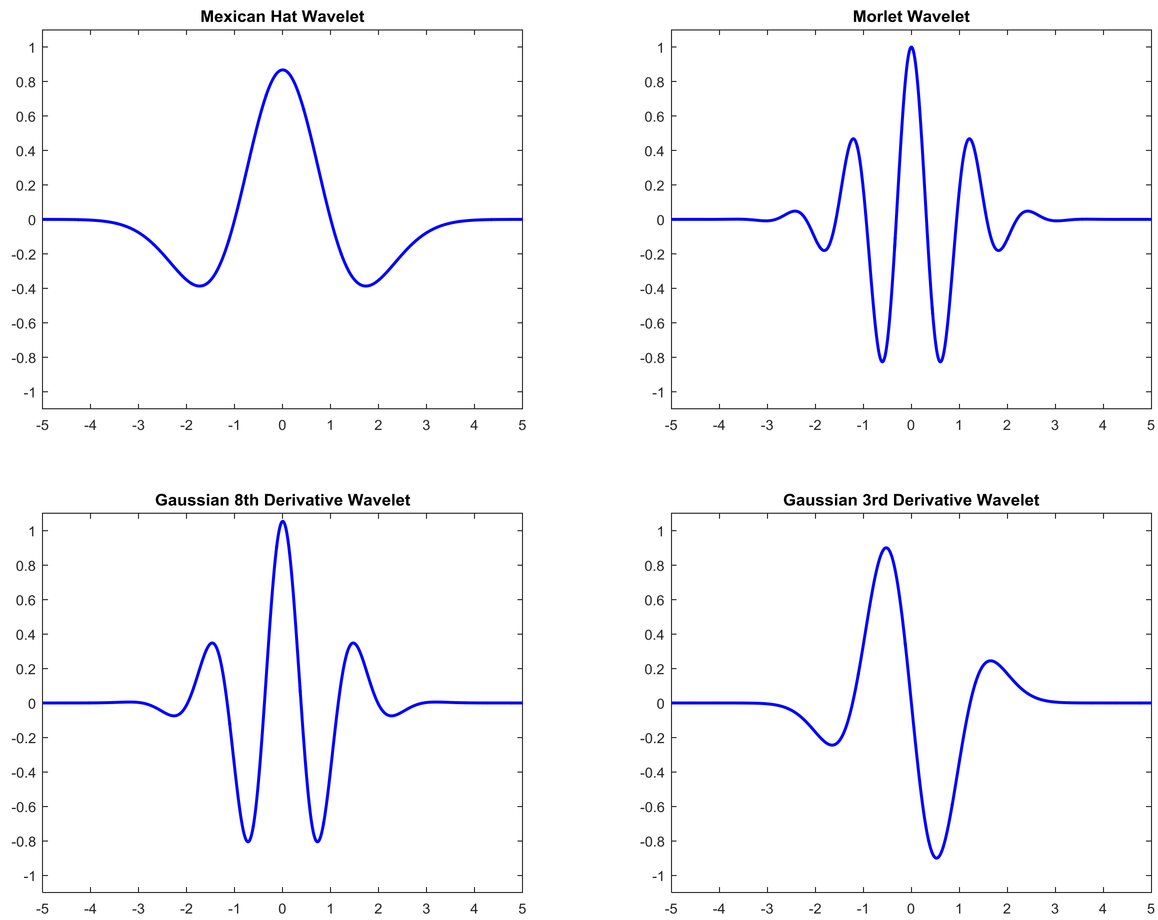

Figure 1 shows a series of wavelets that can be used to complete a continuous wavelet transform.

The continuous wavelet transforms used in this paper are done using the "Mexican Hat" wavelet which is proportional to the second derivative of the Gaussian probability [3]. The Mexican Hat wavelet is represented by the following equation

$$\psi = \frac{2}{\sqrt{3}\pi^{\frac{1}{4}}} e^{\frac{-x^2}{2}}(1 - x^2) .$$

The "Mexican Hat" wavelet is shown if Figure 1 along with several other wavelets. All the wavelets in Figure 1 have a limit of zero as x goes to infinity in both the positive and negative direction. The equation for the Mexican Hat wavelet can be obtained by typing "waveinfo('mexh')" into the command window in MATLAB.

The Drude Model

The first set of data that is analyzed using the continuous wavelet transform is a set of model data constructed using the Drude Model. The Drude Model is given by the following equation [4-6].

$$\sigma(\omega) = \sigma_1(\omega) + i\sigma_2(\omega) = \frac{\sigma_0}{1-i\omega\tau} = \left(\frac{1}{4\pi}\right)\frac{\omega_p^2\tau}{1-i\omega\tau}$$

In the Drude Model the only parameter that is adjusted to construct the model data is twice the mean free time between collisions, τ which is also called the relaxation time and $1 / \tau$ is the scattering rate. The DC conductivity, σ_0, and the plasma frequency, ω_p, are both kept constant. The separation between the optical conductivity and the y axis is related to the scattering rate, therefore the larger the scattering rate, the greater the separation between the y axis and the curve as shown in the left column of Figure 2.

The first graph in the left column of Figure 2 shows the real and imaginary parts of the optical conductivity as a function of frequency for a scattering rate of $1 / \tau = 100$. The second and third graphs in the left column in Figure 2 show the real and imaginary parts of the optical conductivity for scattering rates of $1 / \tau = 200$ and $1 / \tau = 500$ respectively.

The first graph in the right column of Figure 2 shows the continuous wavelet transform of the optical conductivity for the corresponding graph in the left column ($1 / \tau = 100$). The second and third graphs in the right column are the continuous wavelet transforms of the optical conductivity with scattering rates of $1 / \tau = 200$ and $1 / \tau = 500$. The continuous wavelet transform of the optical conductivity is done with respect to the frequency and the scales. The scales of a wavelet transform are obtained from the dilatation (or constriction) of the wavelet [1]. The color bar on the right of each of the continuous wavelet transforms is used to denote the absolute value of the coefficient of the wavelet transform, $C_{a,b}$ at a given scale and frequency (red denotes a large absolute value and blue denotes a small absolute value).

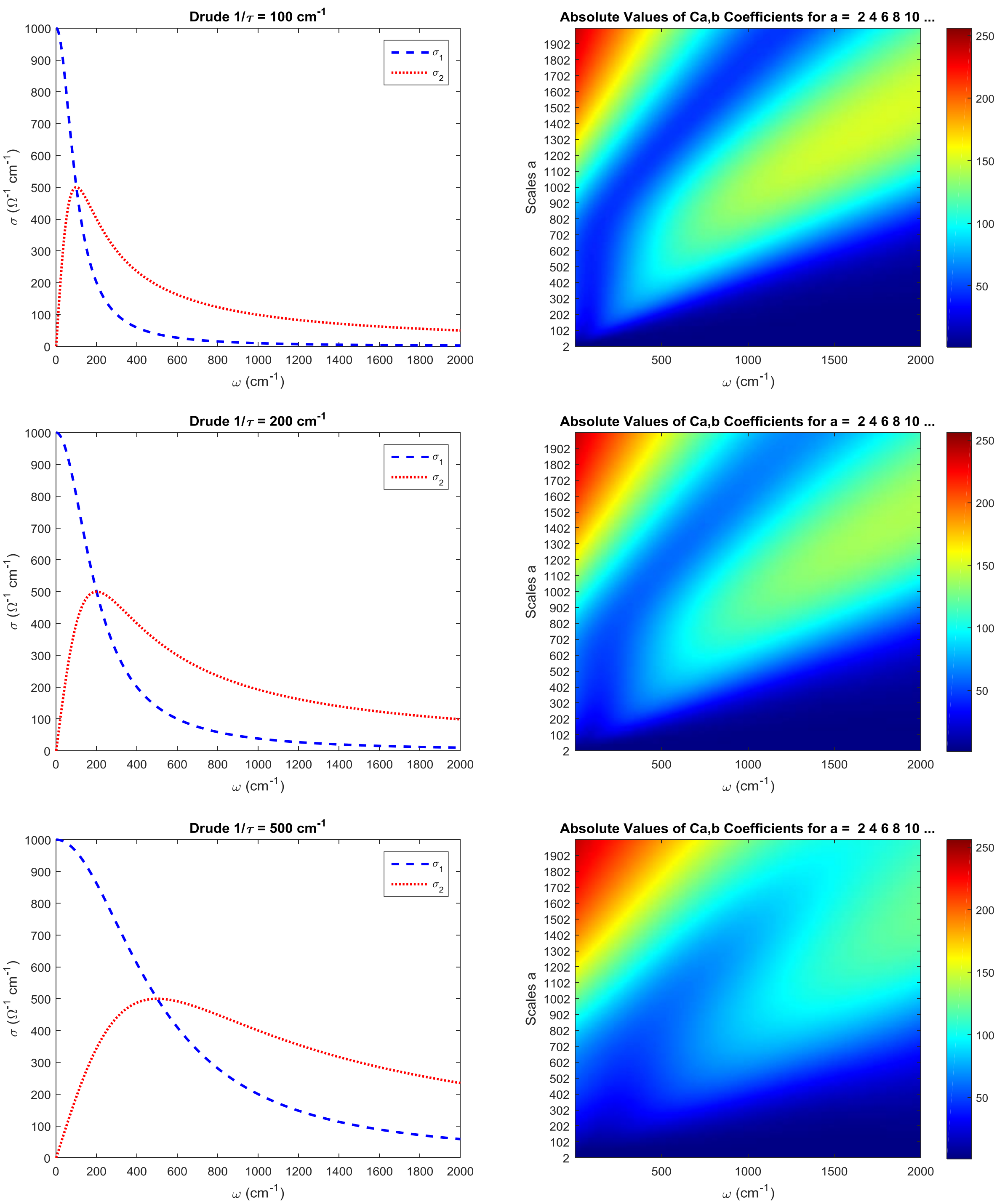

Figure 2: The left column shows σ_1 (blue lines) and σ_2 (red lines) created using the Drude Model for $1 / \tau = 100$, $1 / \tau = 200$, and $1 / \tau = 500$ from top to bottom. The right column shows the corresponding wavelet transform for each value of $1/\tau$. For the wavelet transform red indicates higher absolute values for the wavelet transform coefficients and blue indicates lower values. For all three cases shown $\sigma_0 = 1000$.

As the value of $1 / \tau$ increases the separation of σ_1 and σ_2 from the x-axis increases and the wavelet transform begins to show less variance (a larger part of the wavelet transform is blue so the range of values of the coefficients is smaller) . The yellow area in the wavelet transform for $1 / \tau = 100$ grows smaller as $1 / \tau$ increases, therefore as the scattering rate increases the values of the wavelet transform coefficients decrease.

The Lorentz Model

The Lorentz Model can also be used to create a model of optical conductivity. For the Lorentz Model the optical conductivity is given by [4][5]

$$\sigma(\omega) = \left(\frac{1}{4\pi}\right)\frac{i\omega\omega_p^2}{\omega^2-\omega_0^2+i\gamma\omega}\ .$$

In the Lorentz Model, ω_0 is the position, ω_p is the strength of the Lorentz oscillator and γ is the width of the Lorentz oscillator. For the two sets of model data created for the Lorentz Model, two of the parameters are kept constant while one is changed. In the first set of model data created using the Lorentz Model ω_0 is changed while ω_p is kept constant and γ is kept constant.

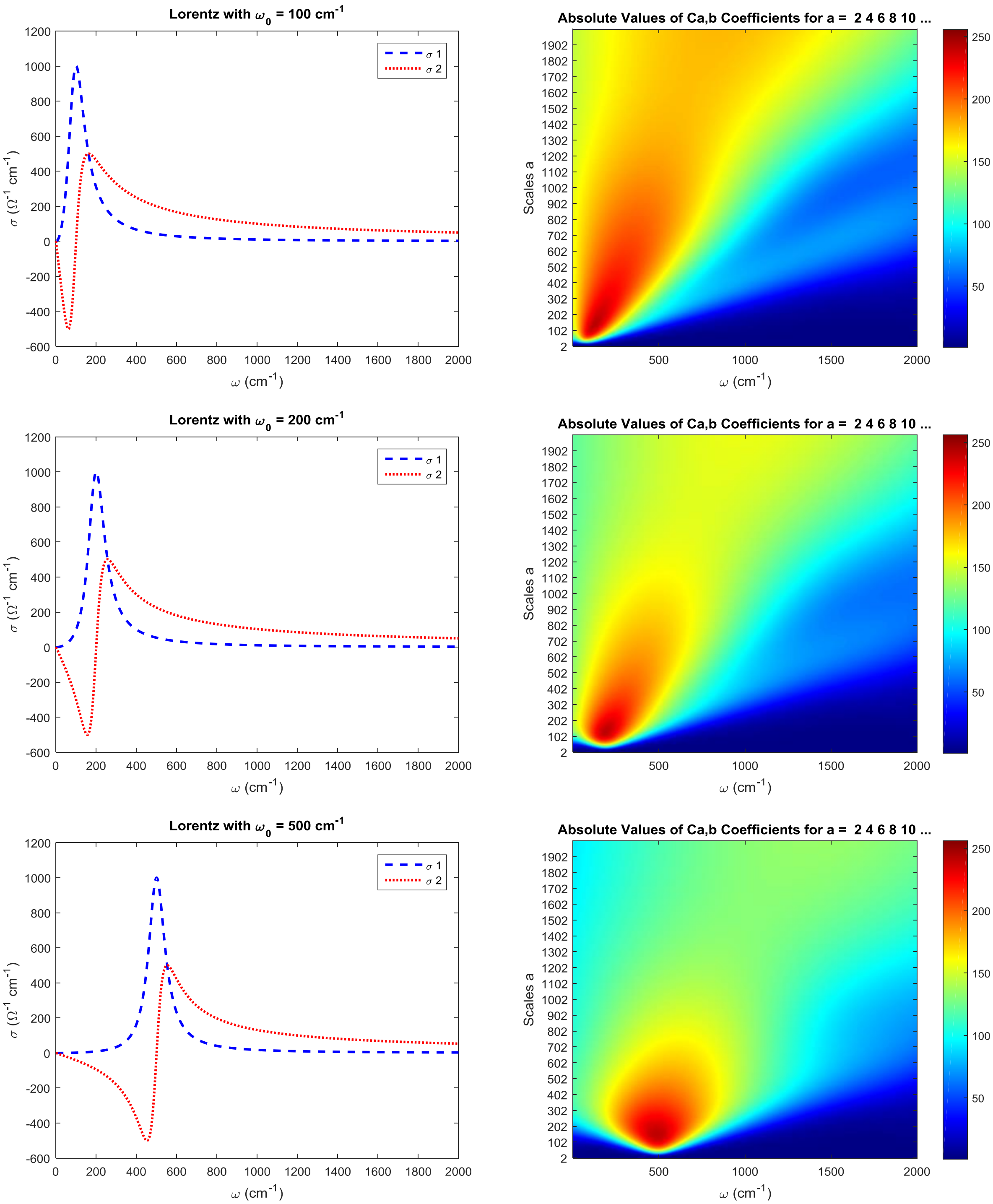

Figure 3: The left column shows σ_1 (blue lines) and σ_2 (red lines) created using the Lorentz Model for ω_0 = 100, ω_0 = 200 and ω_0 = 500 and the corresponding continuous wavelet transforms in the right column. For the wavelet transform red indicates higher absolute values for the wavelet transform coefficients and blue indicates lower values.

As ω_0 increases in Figure 3 the red area in the continuous wavelet transform shifts along the x-axis to be centered at ω_0. As the value of ω_0 increases the height of the red spot in the wavelet transform decreases in height. As the value of ω_0 increases the variation in the wavelet transform decreases.

In the second set of data constructed using the Lorentz Model the width, γ, is changed while both ω_0, and ω_p are kept constant.

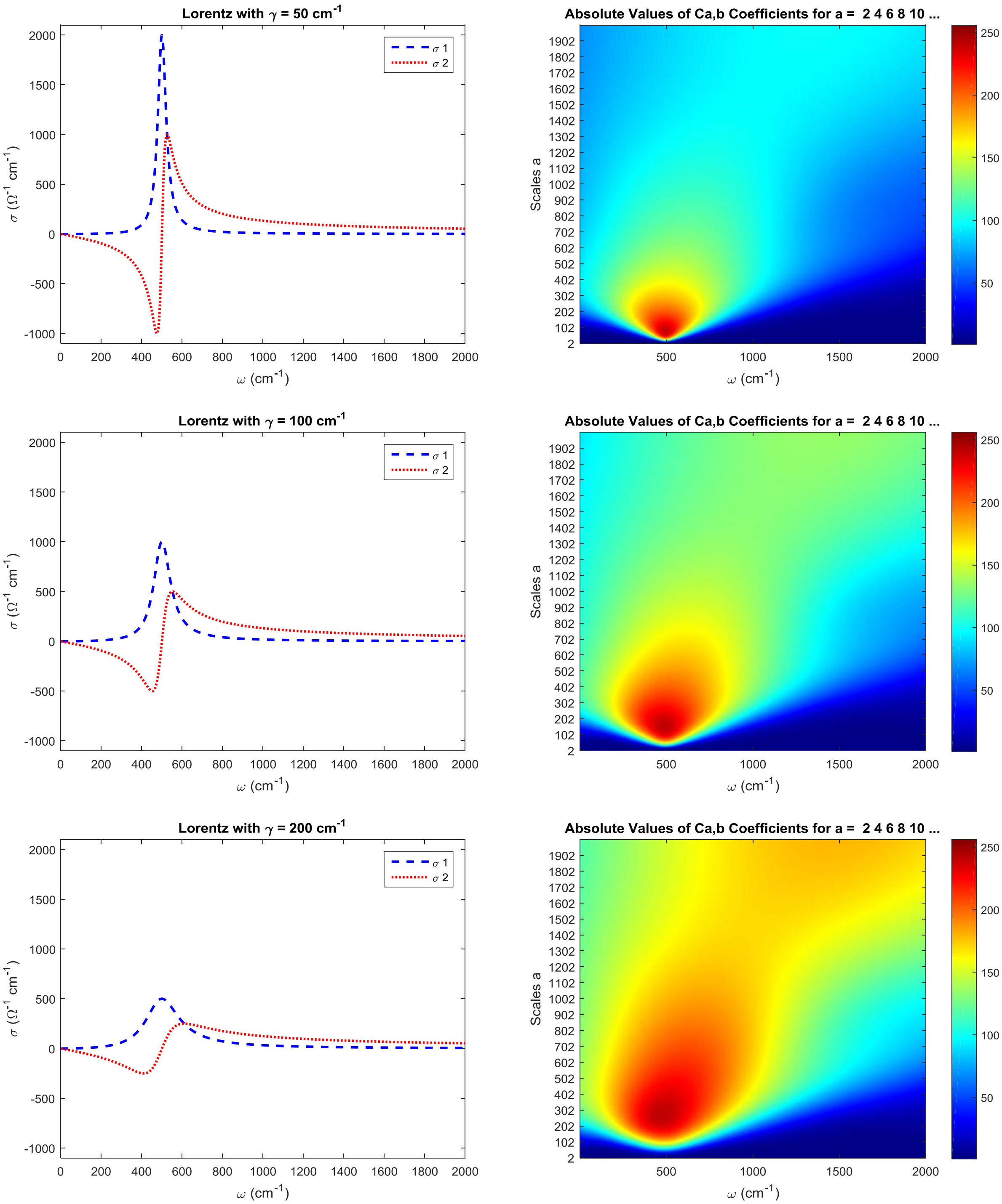

Figure 4: The left column shows the optical conductivity created using the Lorentz model where only γ is changed. The values are γ = 50, γ = 100, and γ = 200. The right column shows the corresponding continuous wavelet transform for each value of γ. For the wavelet transform red indicates higher absolute values for the wavelet transform coefficients and blue indicates lower values.

As the values for γ increase the width of the optical conductivity increases as well as the width of the wavelet transform. The increase in γ also causes the red area at a frequency of 500 to increase in height as well as width.

Experimental Data

The experimental data in Figure 5 is for the optical conductivity of the superconductor $Bi_2Sr_2CaCu_2O_{8+x}$ in the normal state (300 K) and the superconducting state (10 K). The critical temperature is T_c = 92 K so the data for the normal state was collected well above the critical temperature and the data for the superconducting state was collected well below the critical temperature. In order to complete the wavelet transform the data need to be evenly spaced, however the experimental data are not evenly spaced. To fix this issue the data need to be interpolated. This fix is implemented using MATLAB which has several ways to interpolate data. In this case the *spline(x ,y, xx)* function was used to interpolate the data. The input values for the function are the x and y coordinates of the data points and the xx vector which is the vector of evenly spaced x coordinates. The *spline* function fits a different cubic function between the points in the data set [7]. Once the data are interpolated the wavelet transform is completed.

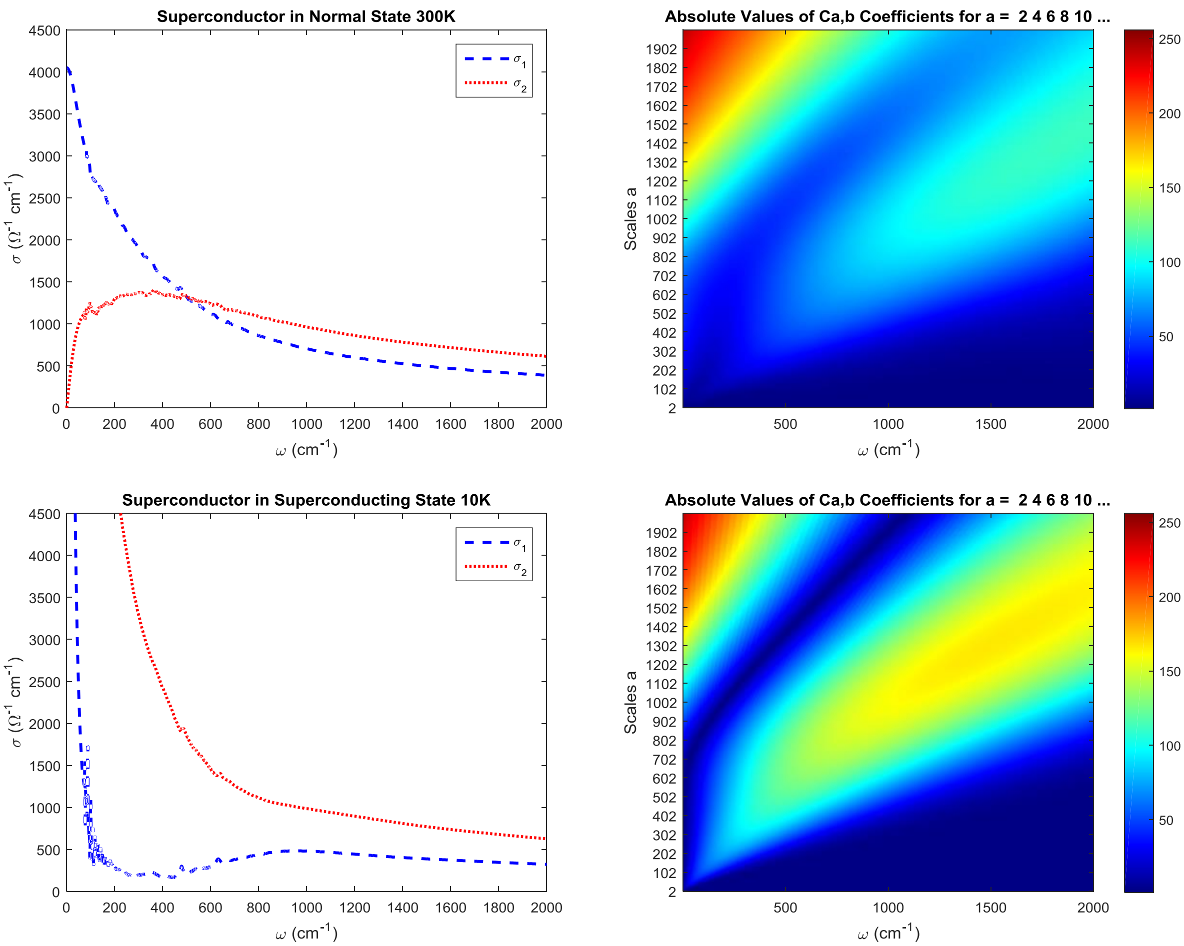

Figure 5: The left column shows experimental data collected in the normal state (300 K) and in the superconducting state (10K) on the left and the corresponding wavelet transform on the right. For the wavelet transform red indicates higher absolute values for the wavelet transform coefficients and blue indicates lower values.

In both the normal and the superconducting state, in the graphs of the optical conductivity as a function of the frequency, the optical conductivity decreases as the frequency increases in a manner more like the Drude Model than the Lorentz Model. This gives the shape of the wavelet transform a similar shape to the wavelet transform of data created using the Drude model. The position (the optical conductivity is not shifted along the x axis) of the optical conductivity in both states is also closer to the Drude model than the Lorentz model. The scattering rate decreases in the superconducting state which leads to a high variance in the wavelet transform that creates a distinct yellow section on the graph. The wavelet transform of the normal state has less variance as a result of a higher scattering rate. The higher scattering rate in the normal state results in the pale blue section which is smaller and less defined than the yellow section of the wavelet transform of the superconducting state. The dark blue section at high frequencies and low values on the scale is smaller in the normal state than the superconducting state.

Conclusion

The wavelet transforms of the experimental data match the wavelet transforms of the data created using the Drude Model better than they match the wavelet transforms of the data created using the Lorentz Model. The wavelet transform of the experimental data in the normal state is best represented by the wavelet transform of data created using the Drude Model with a high scattering rate. The wavelet transform of the experimental data in the superconducting state is best represented by the wavelet transform of data created using the Drude Model with a low scattering rate.

References

[1] Marie Farge, Annu. Rev. Fluid Mech. **24**, 395 (1992).

[2] MathWorks Documentation *cwtext*, accessed March 2016.

[3] Math Works Documentation *Wavelet Families*, accessed March 2016.

[4] S.V. Dordevic and D.N.Basov, Annalen der Physik **15**, 545 (2006).

[5] S.V. Dordevic, Physica B **407**, 4354 (2012).

[6] D.N.Basov and T.Timusk, Rev.Mod.Phys.**77**, 721 (2005).

[7] MathWorks Documentation *Interpolants*, accessed March 2016.

Appendix

Below is the program for the wavelet transform of the data from the Drude Model. The rest of the wavelet transforms are completed using a version of this program.

```
% PJJ 12/17/15
% CWText_Drude.m
% This Program plots the continuous wavelet transforms with extensions
% parameters of model data created using the Drude model

clear all;

% This first section of the program is to expand the color bar for the
% wavelet transform. To do this a color map is created but the wavelet
% transform is done using the jet option in MATLAB, but the extended
% colorbar is kept. The default color bar does not go high enough for the
% wavelet transform.

m = 240; % number of colors
map = zeros(m,3);
R = linspace(0,1,m)';
map(1:m, 1) = R;

% Set a matrix of colors each set of three numbers is a color
T = [0,0,143
    0,0,255
    0,255,255
    255,255,0
    255,0,0
    128,0,0
    0,0,0]./255;

% Set the starting values for each color in T
x = [0 40 80 120 160 200 240];

% Interpolate the areas between the points in x
map = interp1(x/255,T,linspace(0,1,255));

% Plot the colormap
figure(1);clf
I = linspace(0,1,255);
imagesc(I(ones(1,10),:)')
colormap(map)


% Wavelet transform
% Load the data form the file
myfid100 = fopen('Drude100.dat','r');
[A,countD100] = fscanf(myfid100,'%f %f %f  \n',[3,inf]);
fclose(myfid100);
countD100;
```

```
omegaD100 = A(1,:);
sigma1D100 = A(2,:);
sigma2D100 = A(3,:);

% Calculate the complex optical conductivity
sigmaD100 = sigma1D100 + 1i*sigma2D100;

myfid200 = fopen('Drude200.dat','r');
[B,countD200] = fscanf(myfid200,'%f %f %f  \n',[3,inf]);
fclose(myfid200);
countD200;

omegaD200 = B(1,:);
sigma1D200 = B(2,:);
sigma2D200 = B(3,:);

% Calculate the complex optical conductivity
sigmaD200 = sigma1D200 + 1i*sigma2D200;

myfid500 = fopen('Drude500.dat','r');
[C,countD500] = fscanf(myfid500,'%f %f %f  \n',[3,inf]);
fclose(myfid500);
countD500;

omegaD500 = C(1,:);
sigma1D500 = C(2,:);
sigma2D500 = C(3,:);

% Calculate the complex optical conductivity
sigmaD500 = sigma1D500 + 1i*sigma2D500;

% Evenly space the data for Drude100
t = linspace(0,2000,2000);
splD100 = spline(omegaD100,sigmaD100,t); % Optical conductivity
spl1D100=spline(omegaD100,sigma1D100,t); % Sigma 1
spl2D100=spline(omegaD100,sigma2D100,t); % Sigma 2

% Evenly space the data for Drude200
splD200 = spline(omegaD200,sigmaD200,t); % Optical conductivity
spl1D200=spline(omegaD200,sigma1D200,t); % Sigma 1
spl2D200=spline(omegaD200,sigma2D200,t); % Sigma 2

% Evenly space the data for Drude500
splD500 = spline(omegaD500,sigmaD500,t); % Optical conductivity
spl1D500=spline(omegaD500,sigma1D500,t); % Sigma 1
spl2D500=spline(omegaD500,sigma2D500,t); % Sigma 2

% Set scales for cwtext
scales = 2:2:2000;

% Type of CWT
wname = 'mexh';

% Type of CWT at 300K
```

```
wnameD100 = wname;
% Set scale for CWT at 300K
scalesD100 = scales;

% Type of CWT at 10K
wnameD200 = wname;
% Set scale for CWT at 10K
scalesD200 = scales;

% Type of CWT at 10K
wnameD500 = wname;
% Set scale for CWT at 10K
scalesD500 = scales;

% Set color scale
color = jet;

% Plot the graphs
% Drude 100
figure(2);clf
plot(t,spl1D100,'b--',t,spl2D100,'r:','LineWidth',2);
xlabel('\omega');
ylabel('\sigma');
title('Drude 100')
legend('\sigma_1','\sigma_2')
ylim([0 1000])

% Wavelet Transform for Drude model 100
figure(3);clf
cwtext(splD100,scalesD100,wnameD100,'extmode','sp0','extLen',2000,'plotMode',
'absglb');
colormap(color);
colorbar;
xlabel('Frequency');

% Wavelet Transform for sigma 1 in Drude model 100
figure(4);clf
cwtext(spl1D100,scalesD100,wnameD100,'extmode','sp0','extLen',2000,'plotMode'
,'absglb');
colormap(color);
colorbar;

% Wavelet Transform for sigma 2 in Drude model 100
figure(5);clf
cwtext(spl2D100,scalesD100,wnameD100,'extmode','sp0','extLen',2000,'plotMode'
,'absglb');
colormap(color);
colorbar;

% Drude 200
figure(6);clf
plot(t,spl1D200,'b--',t,spl2D200,'r:','LineWidth',2)
xlabel('\omega');
ylabel('\sigma');
title('Drude 200')
```

```
legend('\sigma_1','\sigma_2')

% Wavelet Transform for Drude 200
figure(7);clf
cwtext(splD200,scalesD200,wnameD200,'extmode','sp0','extLen',2000,'plotMode',
'absglb');
colormap(color);
colorbar;
xlabel('Frequency');

% Drude 500
figure(8);clf
plot(t,spl1D500,'b--',t,spl2D500,'r:','LineWidth',2)
xlabel('\omega');
ylabel('\sigma')
title('Drude 500')
legend('\sigma_1','\sigma_2')

% Wavelet Transform Drude 500
figure(9);clf
cwtext(splD500,scalesD500,wnameD500,'extmode','sp0','extLen',2000,'plotMode',
'absglb');
% xlim([0 200]);
colormap(color);
colorbar;
xlabel('Frequency');

% Drude 100, Drude 200, Drude 500
figure(10);clf
plot(omegaD100,sigmaD100,'b-',omegaD200, sigmaD200,'r--',...
    omegaD500, sigmaD500, 'g:', 'LineWidth',2)
xlabel('omega')
ylabel('sigma')
legend('Drude100','Drude200','Drude500')
title('Drude Model')
```